LECTURE NOTES ON

MECHANICS OF MATERIAL (Th 3)

Diploma 3rd Semester

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MECHANICS OF MATERIAL

Analysis of Stress

Sign Convention Tension +ve, Compression –ve, θ anticlockwise +ve, $\sigma_x > \sigma_y$, shear clockwise positive.

If a element is applied by stresses as shown in Fig.1 and we need to find the normal stress and tangential stress on a plane, making angle θ with the plane on which σ_x is applied. θ is also the angle between the axis on which σ_x

- $P_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta$
- $P_t = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - q\cos 2\theta$

$$P_r = \sqrt{P_n^2 + P_t^2}$$

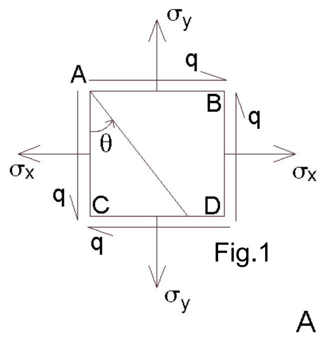

Fig.1

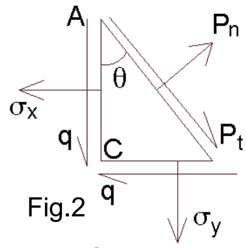

Fig.2

The sum of normal stress acting on perpendicular faces of a plane stresses element is always constant and independent of θ.

As shown in Fig.3 $\sigma_x + \sigma_y = P_{n1} + P_{n2}$

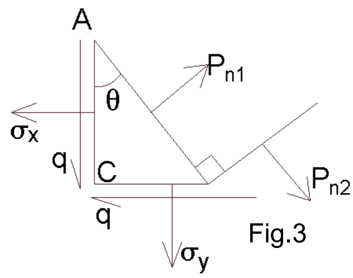

Fig.3

Transformation equation for plane stress

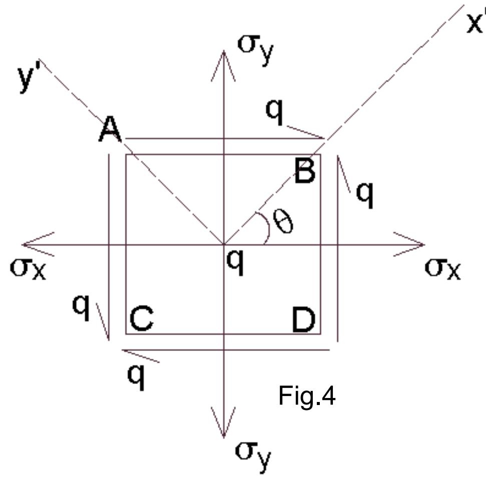

Fig.4

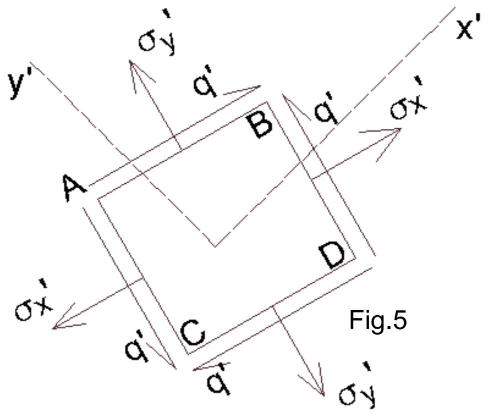

Fig.5

- Fig.4 shows the normal stress condition but if the stress block is rotated by an angle θ, than the value of the stresses will change and the new values will be σ_x', σ_y' & q' as shown in Fig.5 the values will be as follows

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta \qquad q' = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - q\cos 2\theta$$

- σ_y' will be calculated by the same principal of as discussed above that the sum of the normal stress is constant.

  $\sigma_x + \sigma_y = \sigma_x' + \sigma_y'$ or by putting $\theta = \theta + 90$ in above equation

Principal Stresses and Principal Planes

- The value of normal stress changes with change in angle θ, at any certain angle the value of P_n will be either maximum or minimum. The plane on which the value of P_n is either maximum or minimum are known as Principal planes and the stress are known as Principal stresses.

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- Principal Stress $P_n = P_n, \text{Max (Major Principal stresses) or } P_n = P_n, \text{Min (Minor Principal stresses)}$ than these stresses are known as Principal stresses and the planes on which they acted are known as Principal planes.

$$P_n, \text{Max} = \frac{\sigma_x + \sigma_y}{2} + R \qquad P_n, \text{Min} = \frac{\sigma_x + \sigma_y}{2} - R \qquad \text{where } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2}$$

The angel $'\theta'$ on which major principal plane exist is given by $\tan 2\theta = \frac{2q}{\sigma_x - \sigma_y}$ and the minor plane angel is $\theta + 90^0$

Principal Planes are those planes where normal stresses are either maximum or minimum and shear stress is zero.

The tangential stress on Principal plane is 0.

Maximum Shear Stress

$$(P_t), \text{Max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2} = R = \frac{P_n, \text{Max} - P_n, \text{Min}}{2}$$

Value of normal stress on the plane having maximum shear $= (P_n)_{Pt,Max} = \frac{P_n,\text{Max} + P_n,\text{Min}}{2}$

Maximum shear stress will occur at a plane making an angle of 45^0 or 135^0 with the principal planes. Hence, first find the principal plane and then plane having maximum shear.

Mohr Circle

- For mohr circle, axial stress on x-axis and shear stresses on y-axis.
- The radius of mohr circle is $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2} = R$ The coordinate of center of mohr circle is $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$
- Each line joining the center of mohr circle and point on mohr circle represents a plane

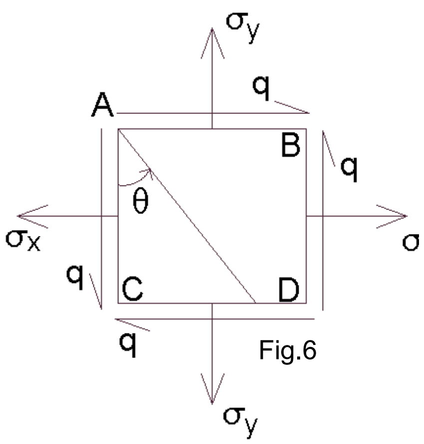

Fig.6

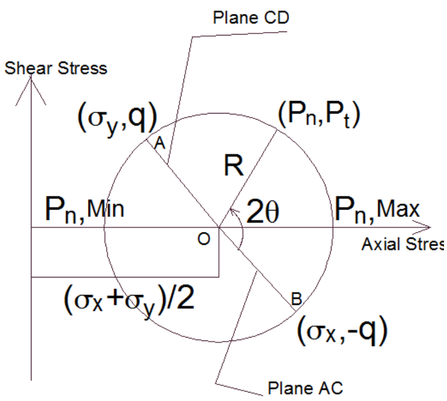

Fig.7

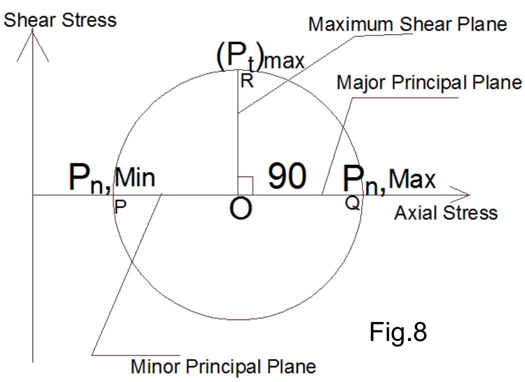

Fig.8

- The line joining center and point A $(\sigma_y, +q)$ and point B $(\sigma_x, -q)$ are planes represented by OA and OB respectively. Join the line AB.
- The point O is the bisector of line join A and B. Now make a angle of 2θ, with the line OB to find σ_n & σ_t
- If we take angle from the center of the mohr circle than angle will be always twice the actual angle.
- As shown in Fig.8 the angle between planes having $P_n, \text{Max (OQ) and } P_n, \text{Min(OP)}$ is 180 on mohr circle hence the original angle between the planes having $P_n, \text{Max (OP) and } P_n, \text{Min(OQ)}$ is 90^0

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- As shown in Fig.8 the angle between P_n, Max (OQ)and P_t, Max(OR) $= 90^0$ on mohr circle. So actual angle is 45^0 the angle between P_n, Min (OP)and P_t, Max(OR) $= 90^0$ on mohr circle. So actual angle is 45^0.
- As shown in Fig.8 the shear stress on OQ planes (Major principal Plane) and shear stress on OP plane (Minor Principal Plane) is zero.
- From above discussion it is clear that the center of the circle will always lie on Axial stress axis, Hence to draw the mohr circle we require two point on circle (either diametrically opposite or not diametrically opposite).

Example1. Find the intensity of normal stress, shear stress, and resultant stress on a plane, the normal of which is inclined 30^0, to the axis of the bar. Find maximum shear stress and draw mohr circle.

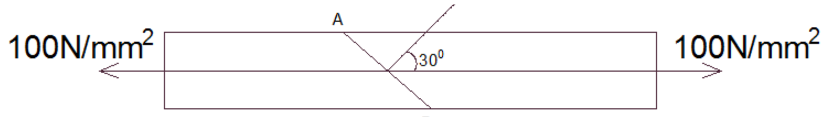
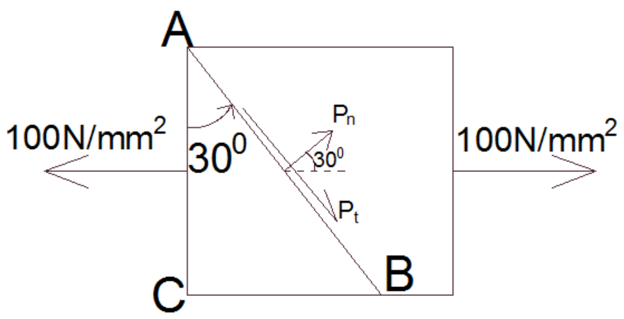
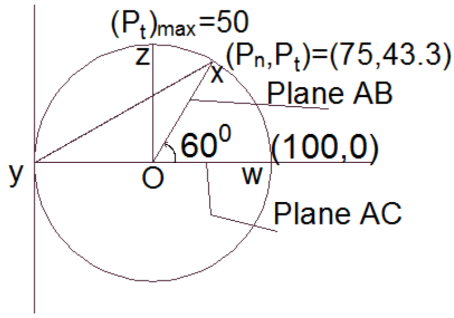


Fig.9





Solution As shown in Fig.10 AB is the plane on which we need to find normal and tangenial stresses.

$$P_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta \quad P_t = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - q\cos 2\theta \quad P_r = \sqrt{P_n^2 + P_t^2}$$

$$(P_t), \text{Max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + q^2}$$

Substitute $\sigma_x = 100, \sigma_y = 0, q = 0, \theta = 30^0$ we get $P_n = \frac{75N}{mm^2}$, $P_t = \frac{43.3N}{mm^2}$, & $P_r = \frac{86.6N}{mm^2}$

$$(P_t), \text{Max} = 50N/mm^2$$

Mohr Circle⟹ As shown in Fig.11 the stress coordinate is w= (100,0) and other stress co-ordinate is y=(0,0), join wy the mid point of wy will be center of the mohr circle, O=(50,0).

The plane on which tangential and normal stress is required is AB (as shown in Fig.11) the angle between plane AB and vertical plane (AC) is 30^0. Hence the angle between OW and OX is 60^0. (the AC plane on stress element is OW plane on mohr circle and AB plane on stress element is OX plane on mohr circle.

The coordinate of 'X' (75,43.3) represent the P_n & P_t on plane AB.

Plane OZ on mohr circle represent the plane of maximum shear stress and the value of max shear stress is $50N/mm^2$

Example2 Find the stresses on a plane for the plane stresses as shown , the normal of which is inclined 45^0 clockwise. (all units in N/mm²)

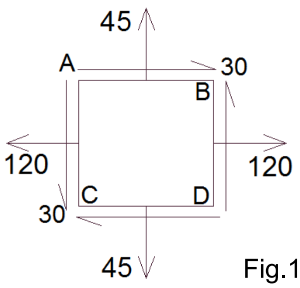


Fig.12

Solution

$$P_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta \quad P_t = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - q\cos 2\theta$$

Substitute $\sigma_x = 120, \sigma_y = 45, q = 30, \theta = -45^0 \;\&\; -16^0$ we get

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For $\theta = -45^0, P_n = 52.5\ P_t = -37.5$ For $\theta = -16\ P_n = 98.41\ P_t = -45.31$

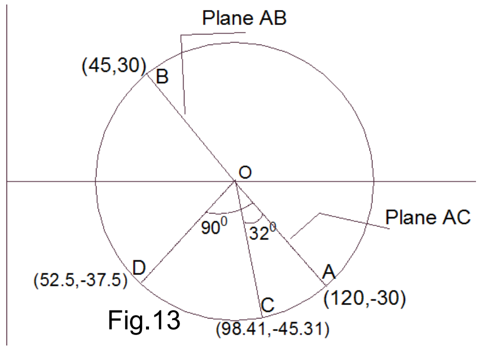


Fig.13

For Mohr circle

Plane AC & BD on stress element is plane OA &OB on mohr circle. Point 'O' is the mid point of line AB.

From OA draw the line OC at a angle of 32^0 clockwise (actual angle is 16^0)

The ordinate of point C will give $P_n, P_t = (98.42, -45.31)$

From OA draw the line OD at a angle of 90^0 clockwise (actual angle is 45^0)

The ordinate of point D will give $P_n, P_t = (52.5, -37.5)$

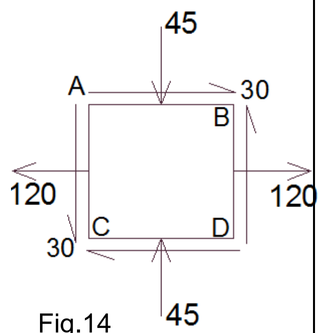


Fig.14

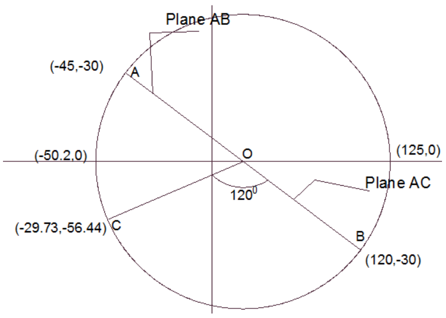
Example3. Find the stress component on a plane 60^0 to that of the tensile stress clockwise

Solution $P_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta \quad P_t = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - q\cos 2\theta$

Substitute $\sigma_x = 120, \sigma_y = -45, q = 30, \theta = -60^0$

we get $P_n = -29.73\ P_t = -56.44$

First put the coordinate A (-45,-30) & B (120,-30). Join AB. The point where line cut the x-axis is center of the circle. With center as O and OA or OB as radius draw the circle. As shown in Fig. 15



OA is plane CD in stress element and OB is plane AC in stress plane. Draw a line at an angle of 120^0 from OB, which gives a clockwise plane OC where we need to find the stress, the coordinate of C $P_n, P_t = -29.73, 56.44$ are the stress on the plane.

Fig.15

Example4 For the given stress element find the principal stresses and find the linear strain in diagonal.

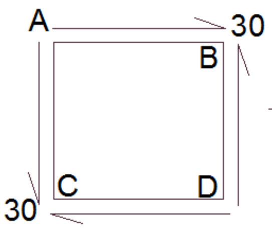


Fig.16

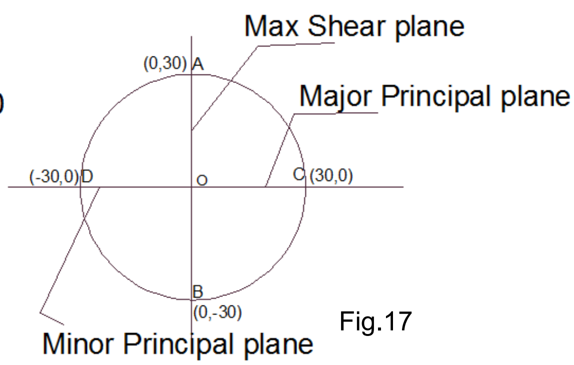


Fig.17

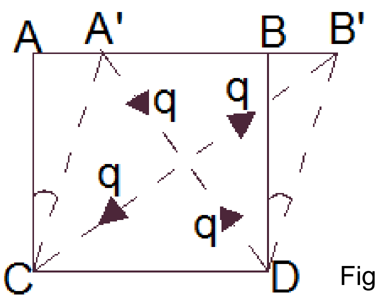


Fig.18

Solution the given stress element is a case of pure shear. the plane AB on mohr circle is OA and the plane BD on mohr circle is OB. The principal stress plane is OC and OD. the principal stress is 30 and at an angle of 45^0 clockwise from AC which is the diagonal joining AD. Similarly the minor principal stress is also 30 but compressive and at an angle of 45^0 from BD anticlockwise.

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The linear strain in diagonal is $e = \frac{q}{2N}$

Example5 At a point in a material under stress, the intensity of resultant stress on a certain plane is 60N/mm^2 (tensile) inclined at 30 to the normal of that plane. The stresses on a plane at right angles to this has a normal tensile component of intensity of 40N/mm^2 as shown in Fig.19. Find (1) the resultant stress on the second plane (2) the principal planes and stresses and (3) plane of maximum shear stress.

.Solution On the plane BD the tangential stress is 60sin30^0=30N/mm^2. Hence on the plane AB, the complimentary shear stress of magnitude 30N/mm^2 will develop. The normal stress on plane BD is 60cos30^0=51.2N/mm^2. The final stresses element is shown in Fig.20 with stress element as shown in Fig.20 draw the mohr circle and find the stresses. Mohr circle is shown in Fig.21

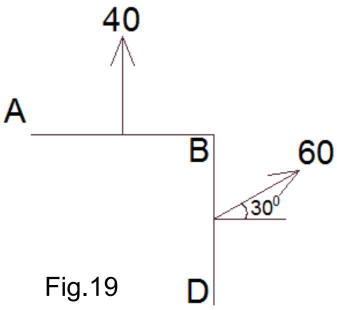


Fig.19

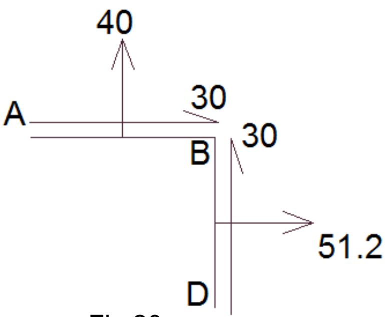


Fig.20

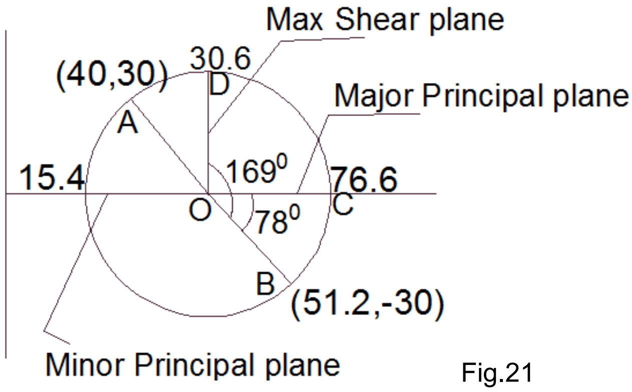


Fig.21

Example6 Find the principal stresses and the angle between the plane PQ & PR as shown in Fig.22

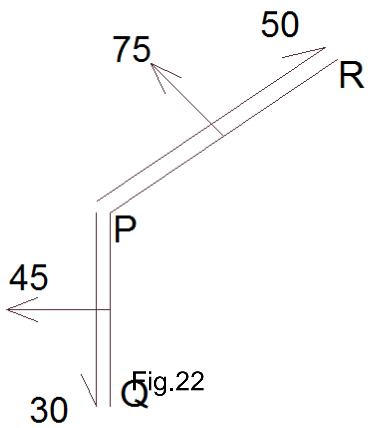


Fig.22

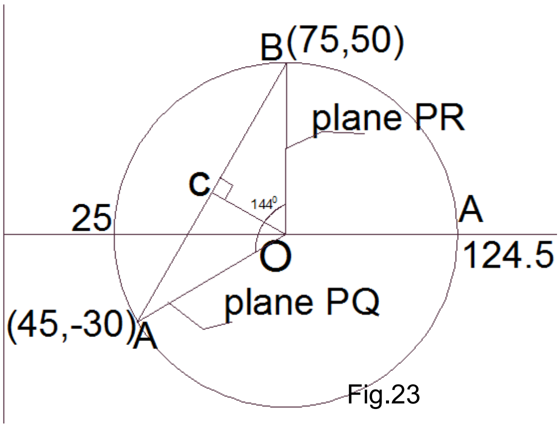


Fig.23

Solution these type of questions are can best be solved by Mohr circle.

Plot the coordinate A(45,-30) and B(75,50) both the points lie on the circle but they are not diametrically opposite (the chord joining AB is not the diameter because the y cordiante is not same). But we know that the line joining the mid point of chord and center of the circle is perpendicular to the chord. We will apply this property to find the center of the mohr circle.

Find the mid point of chord AB which is ‘C’ from ‘C’ draw the perpendicular on x-axis. The point where perpendicular cut the x-axis is the center of mohr circe. As shown in Fig.23 the ‘O’ is the center of the circle. Taking OB as radius draw the circle as shown in Fig.23. Plane OA on mohr circle is plane PQ, plane OB on mohr circle is plane PR. The angle between the plane on mohr circle is 144^0, so actual angle is 72^0. The principal stress are 124.5 and 25.

ANALAYSIS OF STRAIN

- Analysis of strain is same as analysis of stress, just replace the axial stress by axial strain and shear stress by shear strain/2 and θ is the angle measured by x-axis instead of y-axis.
- Sign convention will be same, Tensile strain +ve, compressive strain –ve, θ clockwise +ve, shear strain clockwise +ve
- All the formula and mohr circle will be same just replace $\sigma_x = e_x, \sigma_y = e_y \; q = \emptyset \frac{1}{2}$
- Please note that shear stress , is replace by shear strain /2 (ø /2)

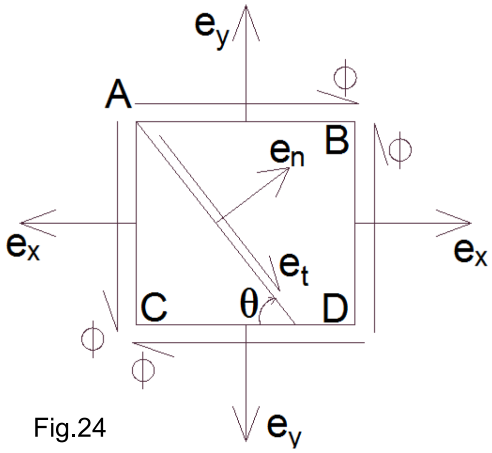


Fig.24

$$e_n = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 2\theta + \emptyset \frac{1}{2}\sin 2\theta$$

$$\frac{e_p}{2} = \frac{e_x - e_y}{2}\sin 2\theta - \emptyset \frac{1}{2}\cos 2\theta$$

$$(e_n, Max) = \frac{e_x + e_y}{2} + R \qquad e_n, Min = \frac{e_x + e_y}{2} - R$$

$$\text{where } R = \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + (\emptyset/2)^2}$$

The angel $'\theta'$ on which major principal plane exist is given by $\tan 2\theta = \frac{\emptyset}{\sigma_x - \sigma_y}$ and the minor plane angel is $\theta + 90^0$

$$(e_t, Max)\frac{1}{2} = \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + \frac{\emptyset^2}{4}} = R = \frac{e_n, Max - e_n, Min}{2}$$

Value of normal stress on the plane having maximum shear $= (e_n)_{@e_t, Max} = \frac{e_n, Max + e_n, Min}{2}$ 6

- In experimental works, strains are measured with the help of strain gauges or strain rosettes. Hence, stresses are calculated by the strain obtained from strain rosettes.
- If e_x & e_y are perpendicular strain obtained by strain rosettes than respective stresses will be given by

$$P_x = \frac{E(e_x + \mu e_2)}{1 - \mu^2} \quad \& \; P_y = \frac{E(e_y + \mu e_x)}{1 - \mu^2}$$

Example1 A flat brass plate was stretched by tensile force acing in direction x and y at right angle. Strain gauges show that strains in x direction was .00108 and in the y-direction 0.00024. Find (1) stresses acing in x-direction and y direction (2) direct and shearing stains on a plane inclined at 40^0 to the x-direction (3) normal and shearing stresses on that plane. Take E=80KN/mm^2 and μ=0.3.

Solution Given $e_x = .00108 \;\&\; e_y = 0.00024$ ø=0 θ=40^0

- $e_n = \frac{e_x+e_y}{2} + \frac{e_x-e_y}{2}\cos 2\theta + \emptyset\frac{1}{2}\sin 2\theta \Longrightarrow e_n = 7.33 * 10^{-4}$

$\frac{e_p}{2} = \frac{e_x-e_y}{2}\sin 2\theta - \emptyset\frac{1}{2}\cos 2\theta \Longrightarrow e_p = 8.28 * 10^{-4}$ radian

- $P_x = \frac{E(e_x+\mu e_2)}{1-\mu^2} \Longrightarrow P_x = 80 * \frac{10^3(0.00108+.3*.00024)}{1-0.3^2} = \frac{101.27N}{mm^2}$
- $P_y = \frac{E(e_y+\mu e_x)}{1-\mu^2} = 80 * \frac{10^3(0.00024+.3*.00108)}{1-0.3^2} = \frac{49.58N}{mm^2}$

The find normal stress and shear stress we will use analysis of stress but the angle will be 90^0-40^0=50^0 (in analysis of stress the angle is with the y-axis) $P_x = 101.27 \;\&\; P_y = 49.58 \;\&\; q = 0 \; P_n = \frac{70.94N}{mm^2} \;\&\; P_t = \frac{25.45N}{mm^2}$

Example2 A brass plate 3mm thick is stretched in the plane of the plate in two direction at right angles. An extensometer, arranged in X-direction gave an extension of 0.0413mm on a gauge length of 50mm, while another arranged in Y direction gave an extension of 0.0187mm on a gauge length of 100mm.if the normal stress on a plane making angle θ with Y-axis is 66N/mm^2, find angle θ. Find also the decrease in the thickness of the plate. E of the brass=80N/mm^2 and μ=0.3.

Solution Strain in X & Y direction $e_x = \frac{0.0413}{50} = 8.25 * 10^{-4}$ & $e_y = \frac{0.0187}{100} = 1.87 * 10^{-4}$

$\sigma_x = \frac{E(e_x+\mu e_2)}{1-\mu^2} \Longrightarrow \sigma_x = 75.66\text{N/mm}^2$ $\sigma_y = \frac{E(e_y+\mu e_x)}{1-\mu^2} \Longrightarrow \sigma_y = \frac{31.91\text{N}}{\text{mm}^2}$

θ is the angle between normal stress and Y-axis hence angle between Y-axis and plane is 90-θ.

Substituting σ_x=75.66 σ_y=31.91, q=0 and θ =90-θ P_n=66

$P_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + q\sin 2\theta \Longrightarrow \theta = 620$

Strain in the direction of thickness

$e_z = -\frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} = -\frac{0.3}{80000} * (75.65 - 31.91) = -1.64 * 10^{-4} \Longrightarrow \frac{\Delta t}{t} = \frac{\Delta t}{3} = 1.64 * 10^{-4}$

Hence change in thickness is -4.92x10^{-4}mm (decrease in thickness)

Example3 A piece of material is subjected to three perpendicular tensile stresses and the strains in the three mutual direction are in ratio of 2:3:4. If μ=0.286. Find the ratio of the stresses and their value if the greatest is 100N/mm^2.

Solution $e_1 = \sigma_1 - \mu(\sigma_2 - \sigma_3)$ $e_2 = \sigma_2 - \mu(\sigma_1 - \sigma_3)$ $e_3 = \sigma_3 - \mu(\sigma_1 - \sigma_2)$

Given $e_1 : e_2 : e_3 = 2:3:4$ or $e_1 = 2X$ $e_2 = 3X$ $e_3 = 4X$ (X = constant) 7

- $2X = \sigma_1 - \mu(\sigma_2 - \sigma_3) \ldots\ldots(1)$ $3X = \sigma_2 - \mu(\sigma_1 - \sigma_3) \ldots\ldots\ldots(2)$ $4X = \sigma_3 - \mu(\sigma_1 - \sigma_2) \ldots\ldots(3)$

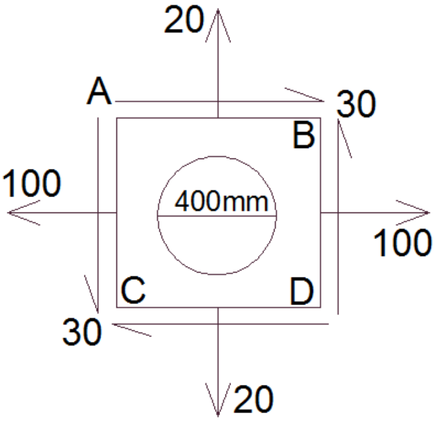
Solving above equations we get $\sigma_1 = 5.97XE$ $\sigma_2 = 6.86XE$ $\sigma_3 = 7.52XE$

$\sigma_1 : \sigma_2 : \sigma_3 = 5.97 : 6.86 : 7.52 = 0.79 : 0.911 : 1$

Hence σ_3 is the greatest =100N/mm^2, $\sigma_2 = \frac{91.1\text{N}}{\text{mm}^2}$ $\sigma_1 = \frac{79.34\text{N}}{\text{mm}^2}$

Example4 A plate as shown in Fig.24 has a circle of 400mm diameter. Find the length of the major and minor axis if μ =0.286 and E=205KN/mm^2.

Solution



$$P_n, \text{Max} = \frac{\sigma_x + \sigma_y}{2} + R \qquad P_n, \text{Min} = \frac{\sigma_x + \sigma_y}{2} - R$$

$$\text{where } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + q^2}$$

$\sigma_x = 100$ $\sigma_y = 20$ $q = 30$ Substituing in the above equation

$P_n, \text{Max} = \frac{110\text{N}}{\text{mm}^2}$ $P_n, \text{Min} = 10\text{N/mm}^2$

Major Principal Strain $e_{max} = \frac{110-0.286*10}{205000} = 5.22 * 10^{-4}$ (increases)

Minor Princiapal Strain $e_{min} = \frac{10-0.286*110}{205000} = -1.047$ (minus shows decrease)

Major Axis length= $(400 + 400 * e_{max}) = 400(1 + 5.226 * 10^{-4}) = 400.209\text{mm}$

Minor Axis length $= (400 - 400 * e_{min}) = 400(1 - 1.047 * 10^{-4}) = 399.958\text{mm}$

MECHANICS OF MATERIAL

STRESS-STRAIN

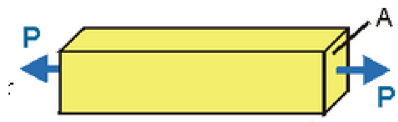


Fig.1

- Stress is defined as force per unit area $\sigma = \frac{\text{Force}}{\text{Average Area}}$.
- The unit of stress in N/m^2 or Pascal (1 N/m^2 =1 Pascal)
- as shown in Fig.1 if a force ‘P’ is applied at a cross section ‘A’ than

Longitudinal Stress $\sigma = P/A$

- To define stress we need magnitude of force, direction of force and area on which this force is applied (plane on which the stress is acted).
- Stress is a two rank tensor. Vector is a one rank tensor. Scalar quantities is a zero rank tensor.

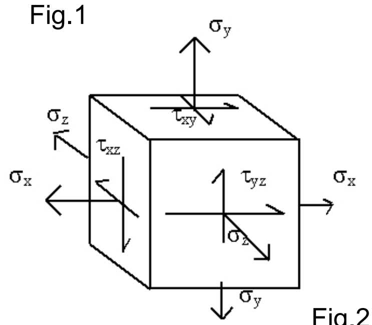


Fig.2

- Stress in 3 dimension Stress = $\begin{Bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{Bmatrix}$

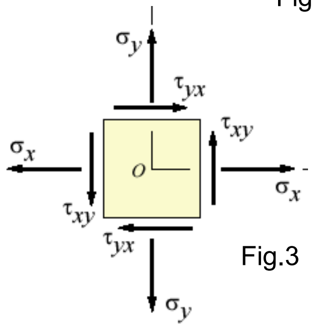


Fig.3

where $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ are axial stresses and $\tau_{xy}, \tau_{xz}, \tau_{yx}, \tau_{yz}, \tau_{zx}, \tau_{zx}$ are shear stresses.

- Plain stress ⇒Stress in 2 dimension is known as Plain stress $\begin{Bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{Bmatrix}$
- For equilibrium $\tau_{yx} = \tau_{xy}$ (obtained by moment equilibrium equation) and this is known as complimentary shear stress.
- For shear stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angle to it.

Strain

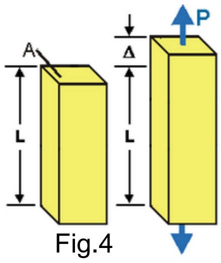


Fig.4

- When a material is loaded with a force, it produces a stress, which then causes a material to deform.
- Longitudinal Strain or Strain is defined as the amount of deformation in the direction of the applied force divided by the initial length of the material. The deformation can be elongation or contraction.
- Strain has no unit. It is denoted by change in length per unit length.
- As shown in Fig.4 a force ‘P’ is applied over area ‘A’ thus longitudinal stress $\sigma = \frac{P}{A}$ which causes elongation of the bar by Δ,. The initial length of the bar is “L’ than

Longitudinal strain $\varepsilon = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta}{L}$

If deformation is elongation than force applied is tensile or vice-versa

Lateral Strain ⟹ When a sample of material is stretched in one direction, it tends to get thinner in the other two directions. Similarly, When a sample of material is compressed in one direction, it tends to get thicker in the other two directions. Lateral strain of a deformed body is defined as the ratio of the change in length (breadth of a rectangular bar or diameter of a circular bar) of the body due to the deformation to its original length (breadth of a rectangular bar or diameter of a circular bar) in a direction perpendicular to the force

Lateral Strain ε_{yy} or $\varepsilon_{zz} = \frac{\Delta D}{D}$

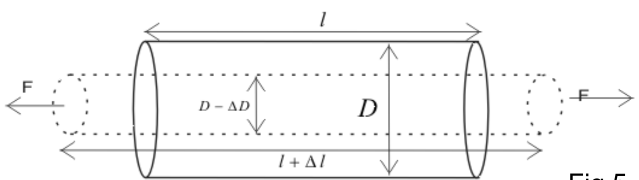


Fig.5

MECHANICS OF MATERIAL

Longitudinal strain $\varepsilon_{xx} = \frac{\Delta L}{L}$

Shear Strain ⇒this is measure of change in angle when shear force/shear stress is applied. When shear stress is applied than the shape of the body changes.

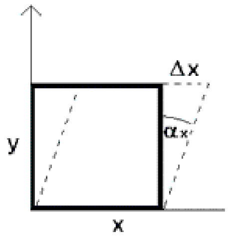


Fig.6

$\alpha_x = \frac{\Delta x}{y}$ the shear strain measures the distortion of the angle.

Volumetric Strain⇒ It is the ratio of change in volume to the original volume.$= \Delta V/V$

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

A cube merged at the center of the sea is the example of three mutually perpendicular like and equal stresses(as per pascal law). When these stresses are applied there will be no change in shape of the body (cube will remain cube) only its dimension will change i.e no effect of shear.

Stress-Strain Curve

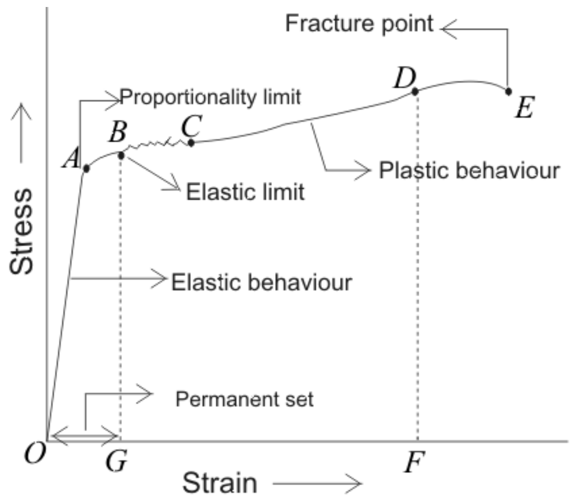


Fig.7

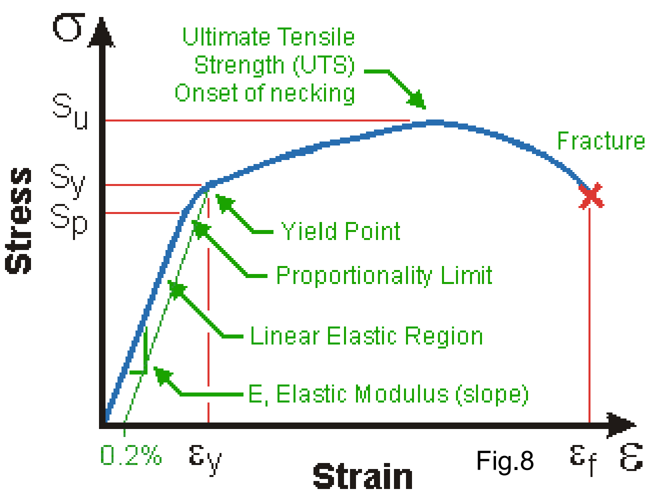


Fig.8

Portion OA is the straight line which clearly shows that stress produced is directly proportional to strain i.e., Hook's law is perfectly obeyed upto A and on removal of stress wire or bar will recover its original condition. Point A is called Proportionality limit

(ii). As soon as proportionality limit is crossed beyond point A, the strain increases more rapidly than stress and curve AB in graph shows that extension of wire in this limit is partly elastic and partly plastic and point B is the elastic limit of the material. Thus if we start decreasing load from point B the graph does not come to O via path BAO instead it traces straight line BG. So that there remains a residual strain. This is called permanent set.

(iii). If we continue to increases the stress beyond point B then for little or no increase in stress the strain increases rapidly upto point C.

(iv). Further increase of stress beyond point C produces a large increase in strain untill a point E is reached at which fracture takes place and from B to D material is said to undergo plastic flow which is irreversible.

- The wire exhibits elasticity from O to b and plasticity from b to d. If the distance between b and d is more, then the metal is ductile. If the distance between b and d is small, then metal is brittle
- The substances which break as soon as the stress is increased beyond elastic limit are called brittle substances eg: glass, cast iron, high carbon steel.
- The substances which have a large plastic range are called ductile substances. Eg: copper, lead, gold, silver, iron, aluminum. Ductile materials can be drawn into wires.
- **Malleable materials** can be hammered into thin sheets. Eg: gold, silver, lead.

MECHANICS OF MATERIAL

Homogenous Material⟹ In engineering a material is said to be homogenous, if the material's elastic property are same, at all the points .

Isotropic Material⟹ An isotropic material is a material having the same elastic properties in all direction of the body.

Anisotropic Material⟹ An anisotropic material is a material having no elastic property same in any direction of the material.

Orthotropic Material ⟹ An orthotropic material is a material having different elastic properties in three mutually perpendicular direction at any one point of the body.

- Total independent elastic constant for isotropic and homogenous material is 2.
- Total independent elastic constant for an orthotropic material is 9.

Elastic constant

- Elastic constants give the relationship between stress and strain. Elastic constant value is constant within the limit of Hook's law and their value only depends on material.

1) **Modulus of Elasticity** E_s ⇒ it is a ratio of axial stress and longitudinal stress $E_s = \frac{\sigma}{\varepsilon} = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} = \frac{PL}{A\Delta L}$

Or change in length along the applied force is $\Delta L = \frac{PL}{AE}$.

If force is tensile than final length=$L + \frac{PL}{AE}$ if force is compressive than final length $L - \frac{PL}{AE}$

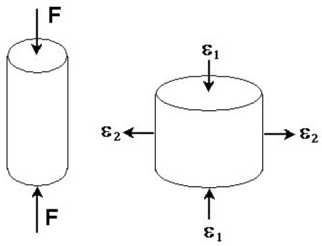


Fig.9

2) **Poisson Ratio** $\mu = \frac{\text{Laterial Strain}}{\text{Longidinal Strain}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$

- As shown in Fig.9 if longitudinal strain is compressive than lateral strain is tensile and vice-versa.
- Theoretically the range of Poisson ratio is -1 to 0.5 but practically its range is 0 to 0.5.

3) **Bulk Modulus** (K) = $\frac{\text{Stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\frac{\Delta V}{V}}$

- When a body is subjected to three mutually perpendicular like stress σ, than the ratio of stress to the volumetric strain is known as Bulk Modulus.

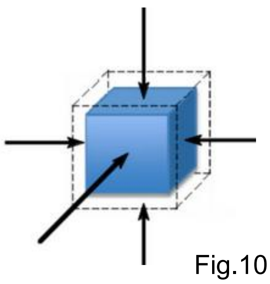
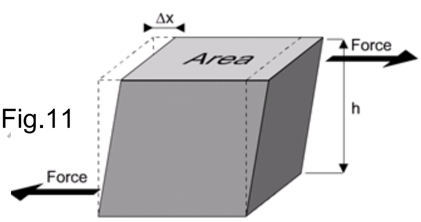


Fig.10

4) **Modulus of Rigidity or Shear Modulus** (G or N)

Fig.11


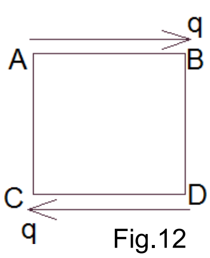


Fig.12

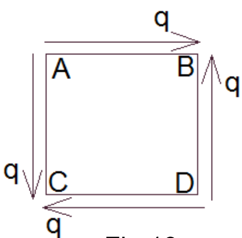


Fig.13

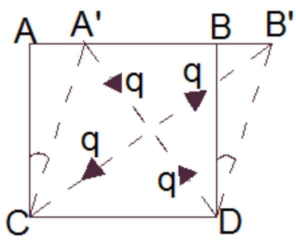


Fig.14

$G \text{ or } N = \frac{\frac{\text{Shear force}}{\text{Area}}}{\frac{\Delta x}{h}} = \frac{q}{\emptyset}$ where $q = \frac{F}{A}$ & $\emptyset = \frac{\Delta x}{h}$

- By applying shear, as shown in Fig.12 the complimentary shear stress are produced at faces AC and BD as shown in Fig.13

- This is a case of pure shear, across the diagonal 'BC' will be in tension and diagonal AD will be compression as shown in Fig.14 the stress generated in both the diagonal will be q as shown in Fig.14 (this will be more covered in mohr circle chapter)
- The longitudinal strain because of q stress in both the diagonal will be $\frac{q}{2N}$.
- The longitudinal strain in both the diagonal is half the shear stain.

Relationship between E, K, G, μ

1) $E = 2N(1+\mu)$ 2) $E = 3K(1-2\mu)$ 3) $E = \frac{9KG}{(3K+G)}$ 4) $\mu = \frac{3K-2G}{6K+2G}$

- **E>K>G**

Example1. Find the change in volume per unit volume, if a axial force is applied as shown in Fig.15 the cross section area is A.

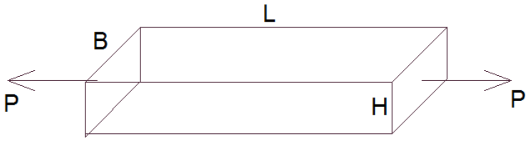


Fig.15

Solution $V = LBH$ & $\delta V = \delta L * BH + \delta B * LH + \delta H * LB$

$$\frac{\delta V}{V} = \frac{\delta L}{L} + \frac{\delta B}{B} + \frac{\delta H}{H} = e_x + e_y + e_z$$

Increase in length $\frac{\delta L}{L} = \frac{P}{A} * \frac{1}{E}$ Decrease in width $\frac{\delta B}{B} = \mu\left(\frac{P}{A} * \frac{1}{E}\right)$ Decrease in height $\frac{\delta H}{H} = \mu\left(\frac{P}{A} * \frac{1}{E}\right)$

$$\frac{\delta V}{V} = \frac{P}{AE} - \mu\frac{P}{AE} - \mu\frac{P}{AE} = \frac{P}{AE}(1-2\mu)$$

Example2 Find the change in volume per unit volume or dilation as shown in Fig.16

Solution As discussed in example1,

$\frac{\delta V}{V} = \frac{\delta L}{L} + \frac{\delta B}{B} + \frac{\delta H}{H} = e_x + e_y + e_z$ ($e_x + e_y + e_z$ is known as dilation)

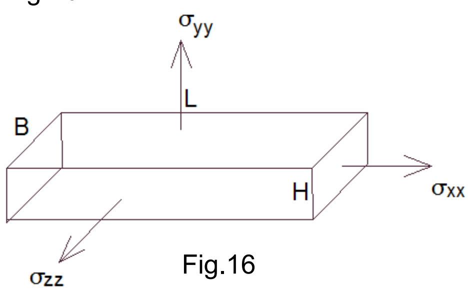


Fig.16

For the given fig.16

$$e_x = \frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{zz}}{E}$$

$$e_y = \frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{xx}}{E} - \mu\frac{\sigma_{zz}}{E} \quad \& \quad e_z = \frac{\sigma_{zz}}{E} - \mu\frac{\sigma_{yy}}{E} - \mu\frac{\sigma_{xx}}{E}$$

$$\frac{\delta V}{V} = e_x + e_y + e_z = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{E}(1-2\mu)$$

Example3. A bar of 25mm diameter is subjected to a pull of 40KN. The length of the bar is 200mm and extension is 0.085 and change in Ø is 0.003mm. Calculate all the constant.

Solution Given L=200mm ΔL=0.085 Ø=25mm ΔØ =0.003mm A=0.7854*25^2=490.87mm^2 P=40KN

Linear Strain= $\frac{\Delta L}{L} = \frac{P}{AE} \Longrightarrow E = \frac{PL}{A\Delta L} \Longrightarrow E = 1.917 * 10^5 N/mm^2$

$$\mu = \frac{\frac{\Delta L}{L}}{\Delta\emptyset \backslash \emptyset} = \frac{4.25*10^{-4}}{1.2*10^{-4}} = 0.28 \qquad E = 3K(1-2\mu) \Longrightarrow K = 1.468 * 10^5 \frac{N}{mm^2}$$

$$E = 2N(1+\mu) \Longrightarrow N = 0.748 * 10^5 N/mm^2$$

Shear Force

Shear force is the algebraic sum of all the transverse forces acting on either side of the section. (Remember, after finding the reaction, treat reaction as a externally applied load).

Sign convention :-

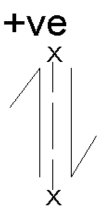


Fig 1

x-x is a section

The upward force on left hand side of section is +ve

The downward force on Right hand side of section is +ve

Example :-

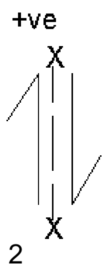


Fig 2

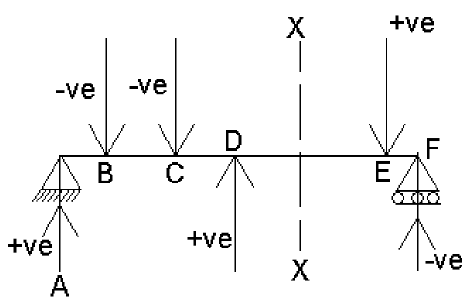


Fig 3

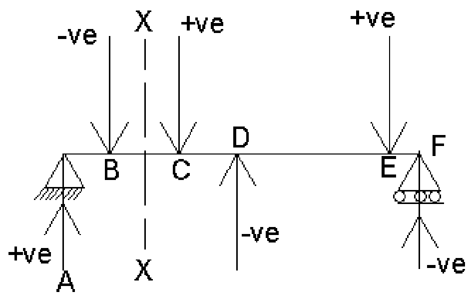


Fig 4

From Fig3 , if we take a section X-X and consider left hand side than force at C is –ve (because as per sign convention all the vertical downward force on left hand side is –ve) but for the same beam and same loading in Fig 4, if the position of the section X-X changes than the same force at C is +ve (because as per our sign convention all the vertical downward force on right hand side is +ve) which mean that selection of section decides the sign of shear force. Similarly check can be done for force at D.

- understanding of sign is very important.

Bending moment

Bending moment is the algebraic sum of all the moments on either side of the section.

Sign convention:-

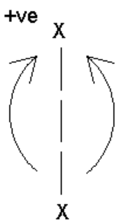


Fig 5

x-x is a section

All the clockwise moment on left side of the section is +ve

All the anticlockwise moment on right side of the section is -ve

Example

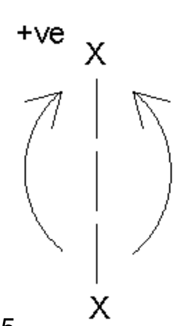


Fig 5

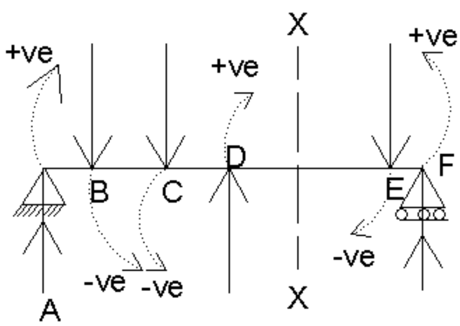


Fig 6

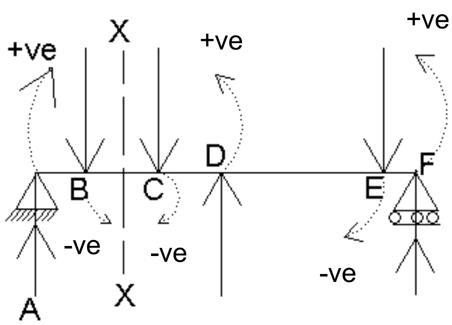


Fig 7

From Fig6 & Fig7 it is clear that the bending moment induced by force at C and force at D does not change with change in section. Hence the bending moment induced by vertical forces does not change with change in section.

Example

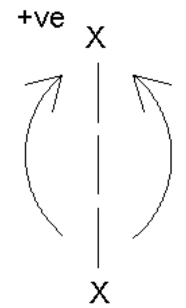


Fig 8

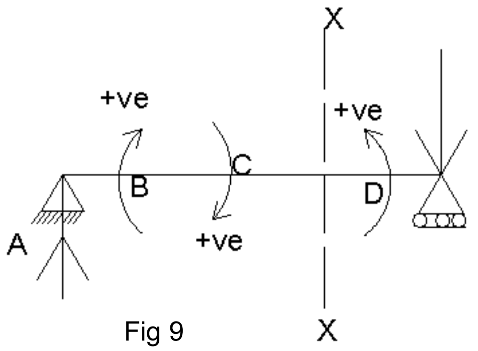


Fig 9

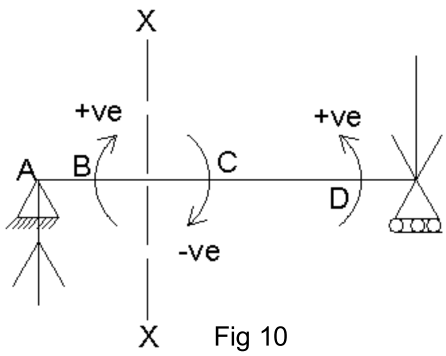


Fig 10

From Fig 9, moment at B is +ve, moment at C is +ve, moment at D is +ve, from Fig10, moment at B is +ve , moment at C is –ve, moment at D is +ve. Hence, the sign of applied moment changes with change in section.

How to write Shear Force and Bending Moment Equation.

Step1, Find all the reactions and treat all the reactions as externally applied loads.

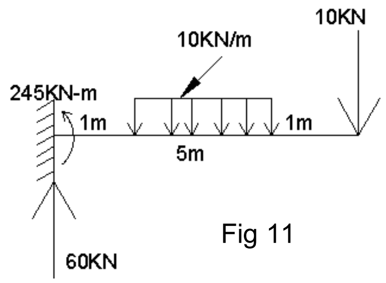


Fig 11

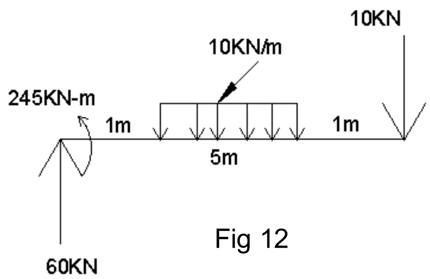


Fig 12

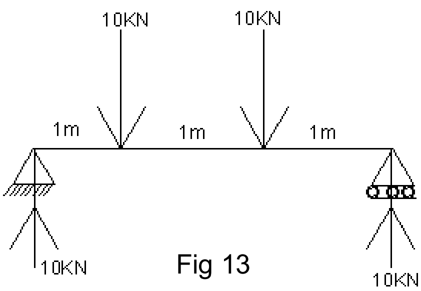


Fig 13

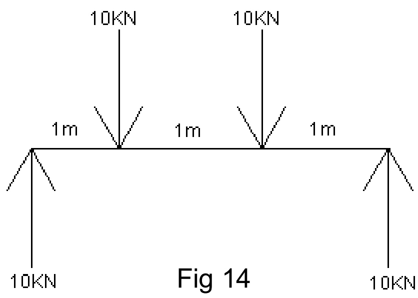


Fig 14

Step2,.. Label all the important points on the beam.,

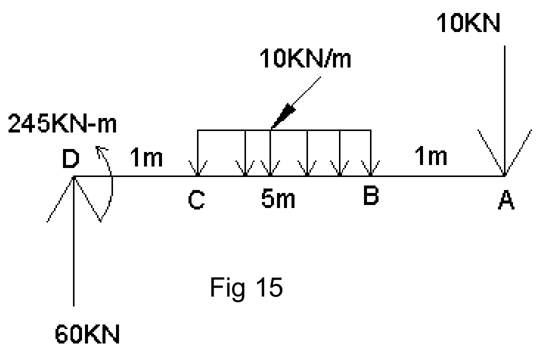


Fig 15

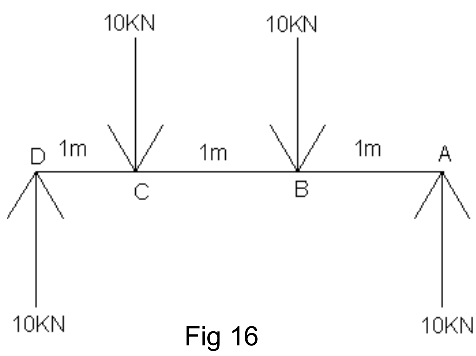


Fig 16

Step 3,. Choose the Sign Convention as discussed

Fig 17

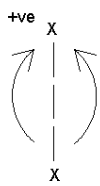


Fig 18

Step 4, Now in between the two labeled points draw a section, take either left or right side of the section (Tip:- try to take a side having less loads than the other side) For Shear Force, Sum algebraically all the vertical force on chosen side (i.e one by one take each load and write sign and value of load Ex +15KN, or $-10x^2$ etc) and for Bending Moment, sum algebraically all the moment on chosen side.(i.e one by one take each load and write sign, value of load and lever arm if any Ex +15*4 or - (15*x)*x/2) (for UDL and VDL convert them into point loads).

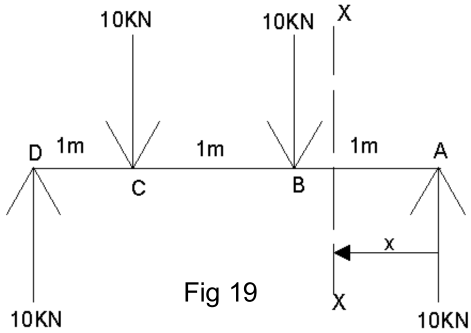
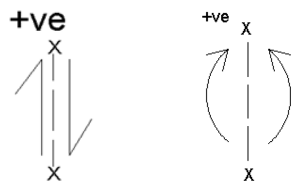


Fig 19

For region AB, section x-x taking Right hand side (x varies from 0 to 1)

(S.F) = -10 S.F is constant in region AB

(B.M)= 10(x) B.M is varying linear in region AB.



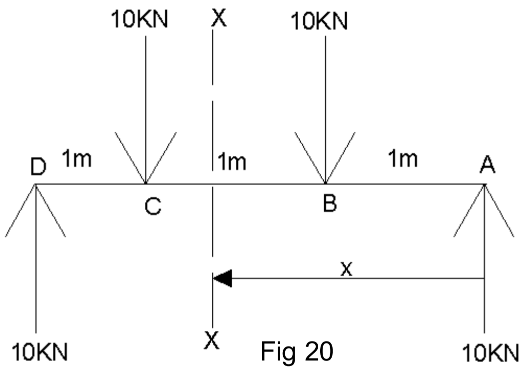


Fig 20

For region BC, section x-x taking Right hand side(x varies from 1 to 2)

(S.F) = -10+10=0 S.F is constant in region CB

(B.M)= 10(x) -10(x-1) =10 B.M is constant in region CB

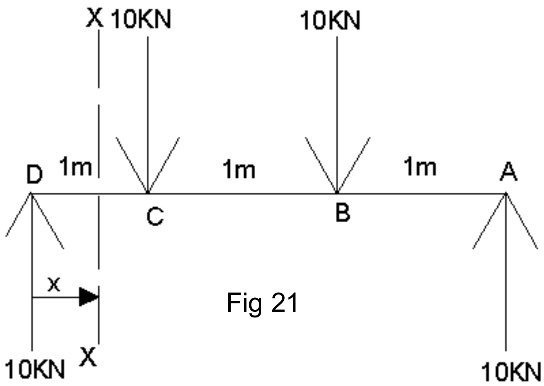


Fig 21

For region DC, section x-x taking Left hand side (less force on left side) (x varies from 0 to 1)

(S.F) = +10 S.F is constant in region DC

(B.M)= 10(x) B.M is varying linear in region DC

Table 1

REGION	LIMIT OF X	SHEAR FORCE	BENDING MOMENT	Variation of S.F & B.M	Boundary Value of S.F	Boundary Value of B.M
A-B	0 to 1	-10	10(x)	Constant & linear	-10, -10	0 ,-10
B-C	1 to 2	0	10	Constant & Constant	0,0	10,10
C-D	0 to 1	10	10(x)	Constant & linear	10,10	0,10

For drawing the Shear force diagram & Bending moment diagram use table above, i.e for each region find the boundary value and the variation of shear force and bending moment along the length in that particular region.

Important:- outside the beam both shear force and bending moment is 0. Hence at the ends of beam we can have two values of Shear force and bending moment, one value is always zero and the other is the boundary values obtain from table.

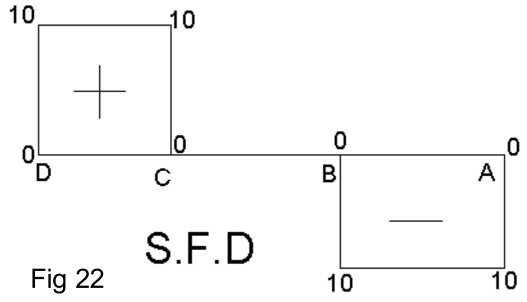


Fig 22

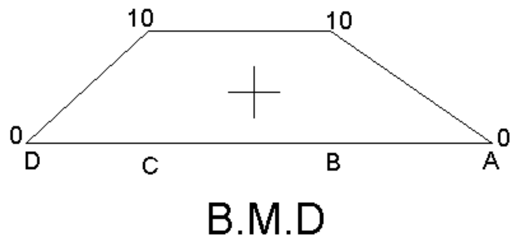


Fig 23

At point A,B,C,D we have two value of shear force. The explanation is as follows:-

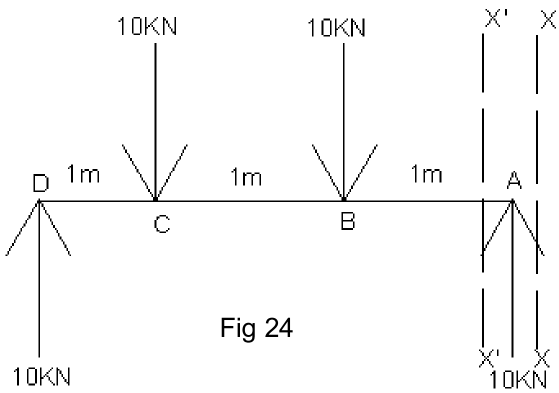
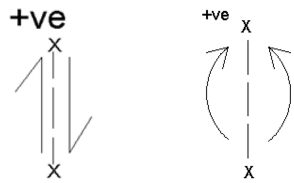


Fig 24

If we consider section X-X which is just right of point A than the value of S.F is 0 but if we consider the section X'-X' which is just left of point A, than the value of S.F is -10. Hence, suddenly the value of Shear Force changes by 10 due to point load.



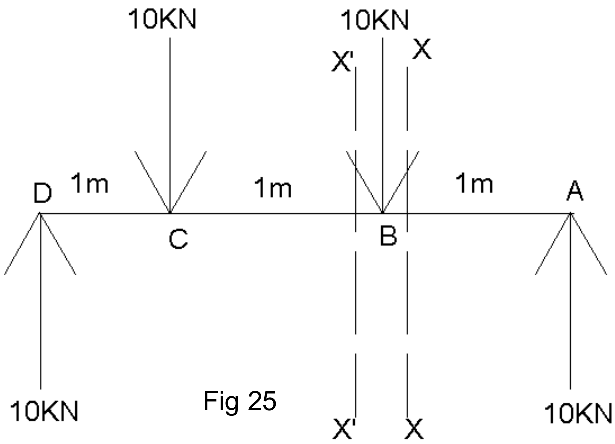


Fig 25

If we consider section X-X which is just right of point B than the value of S.F is -10 but if we consider the section X'-X' which is just left of point B, than the value of S.F is 0. Hence, suddenly the value of Shear Force changes by 10 due to point load

Similarly of point C, D.

Important:- Hence, at every point load the value of Shear Force changes suddenly or even we can say that at every point load we have two values of shear force this means that in Shear force diagram there will be a vertical ordinate corresponding to point load/ reaction in loading diagram. It can be seen in the fig.21 there are 4 point load (2 loading and 2 reaction) hence there will be 4 vertical ordinate in the shear force diagram as seem in Fig 22.

Example 1

Write the shear force and bending moment and draw SFD and BMD.

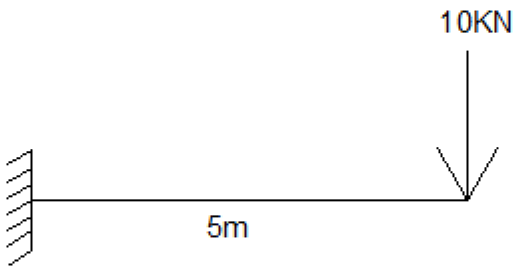
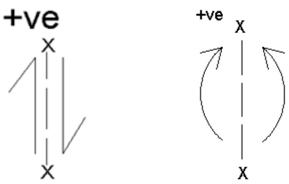


Fig 26



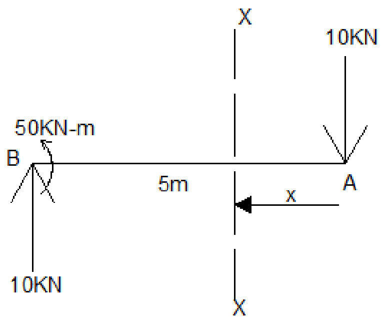


Fig 27

Table 2

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variartion of B.M
A-B	0-5	+10	(0,10),(10,0)	constant	-10x	0,(10,0)	linear

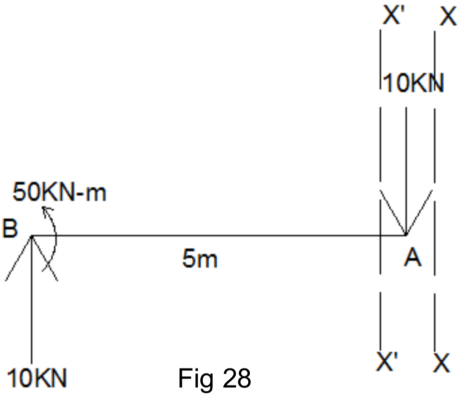


Fig 28

At point A, we have a point load of 10KN hence, the shear force at point A will have two values one 10KN at section x'-x '(just left of A) and other 0 at section x-x (just right of A) as shown in Fig 28 Similarly for B we have two values of shear force.

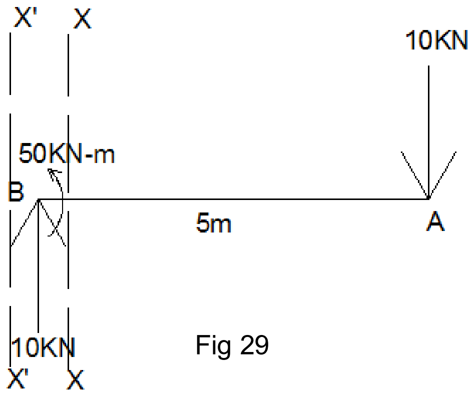


Fig 29

At point B , we have a moment of 50KN-m hence, the bending moment at point B will have two values one 0 at section x'-x' (just left of A) and other 50KN-m at section x-x (just right of A) as shown in Fig 29

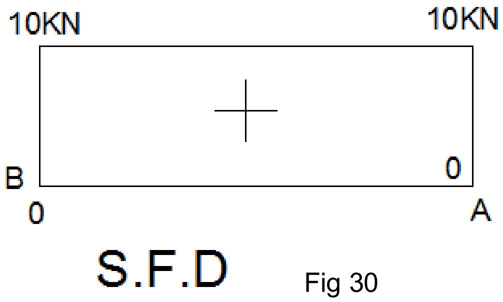


Fig 30

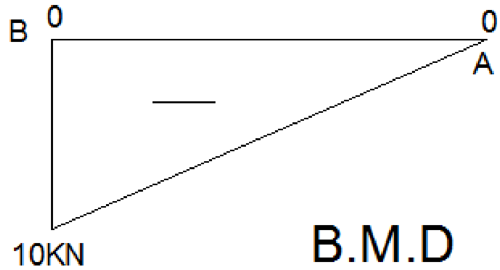
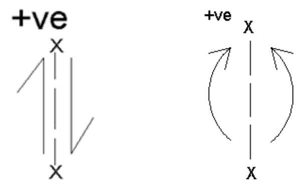


Fig 31



- **If the loading is having a point load than the shear force diagram will have a vertical ordinate (shear force will change suddenly)**
- **As per our sign convention if you move from left to right and a point load is downward than the value of shear force will decrease and if you move from right to left and a point load is downward than shear force will increase.**

Similarly,

- **if the loading is having a concentrated moment than the bending moment diagram will have a vertical ordinate. (bending moment will change suddenly)**
- **As per our sign convention if you move from left to right and a concentrated moment is clockwise than the value of bending movement will suddenly increase and if you are moving from right to left and a concentrated movement is clockwise than the value of bending moment will decrease suddenly.**

Example 2

Write the shear force and bending moment equations and draw SFD and BMD

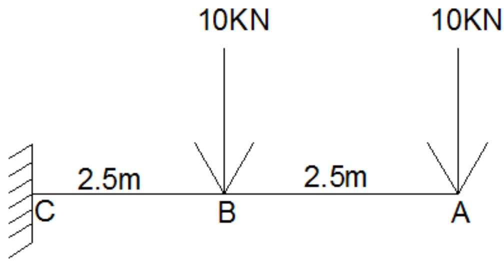


Fig 32

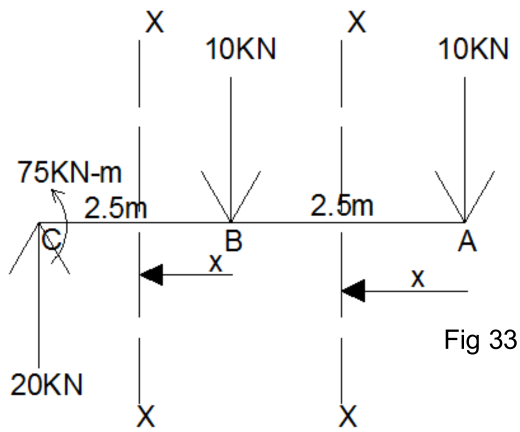


Fig 33

Table 3

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-2.5	+10	(0,10),(10,20)	constant	-10x	0,-25	linear
B-C	0-2.5	+10+10	(10,20),(20,0)	constant	-10(2.5+x)-10(x)	-25,(-75,0)	linear

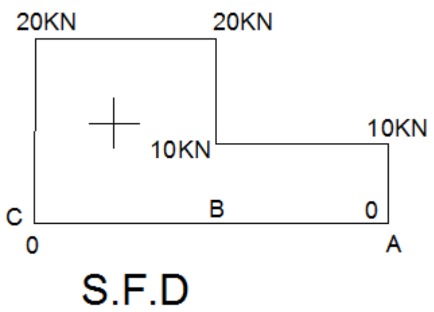


Fig 34

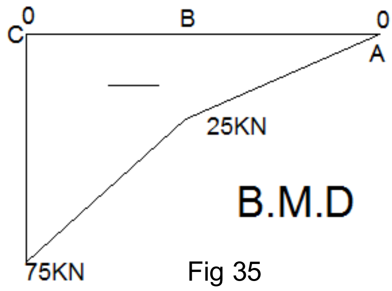
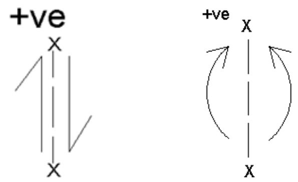


Fig 35



Example 3

and BMD

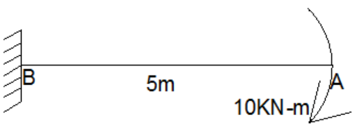


Fig 36

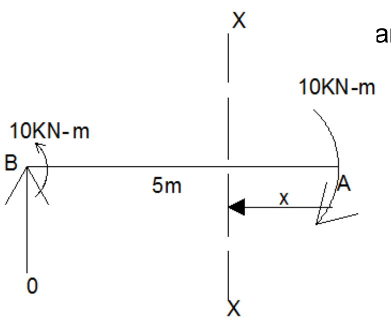


Fig 37

Table 4

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-5	0	0,0	constant	-10	(0,-10),(-10,0)	constant

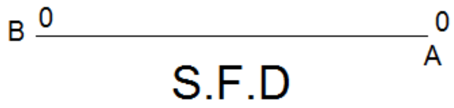


Fig 38

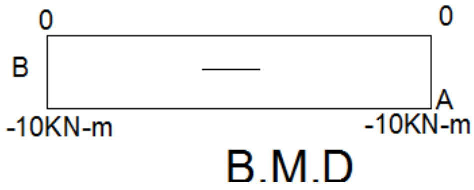
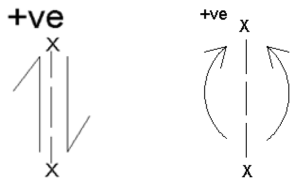


Fig 39



Example 4

Write the shear force and bending moment equations and draw SFD and BMD

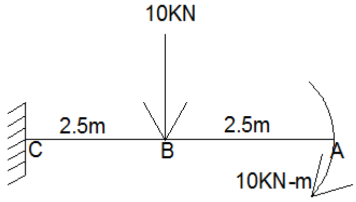


Fig 40

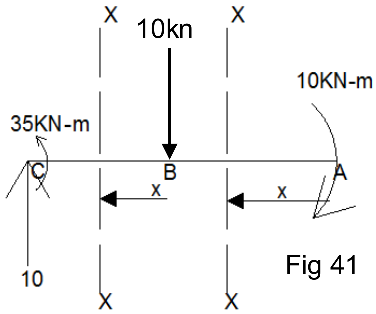


Fig 41

Table 5

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-2.5	0	0	constant	-10	(0,-10),-10	constant
B-C	0-2.5	+10	(0,10),(10,0)	constant	-10-10(x)	-10,(-35,0)	linear

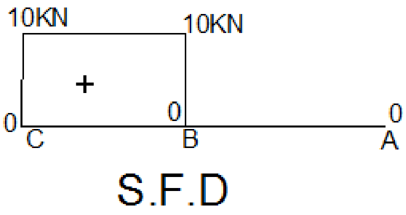


Fig 42

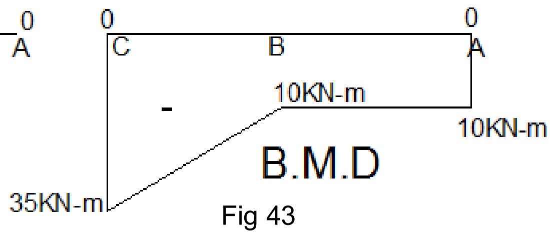
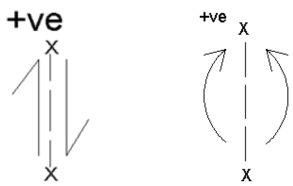


Fig 43



Example 5

Write the shear force and bending moment equations and draw SFD and BMD

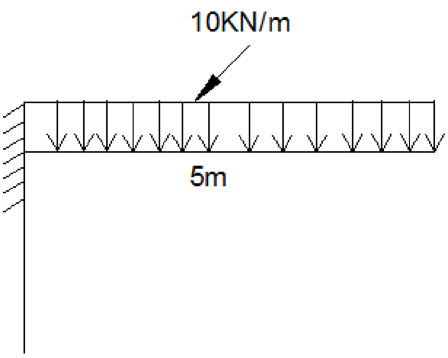


Fig 44

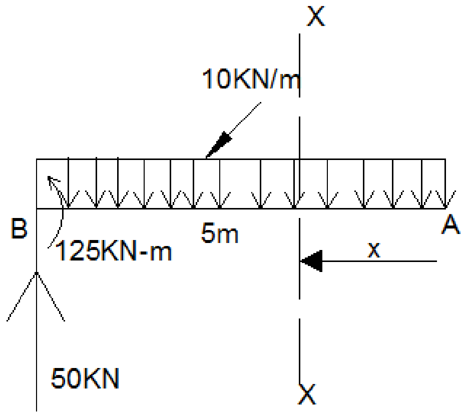


Fig 45

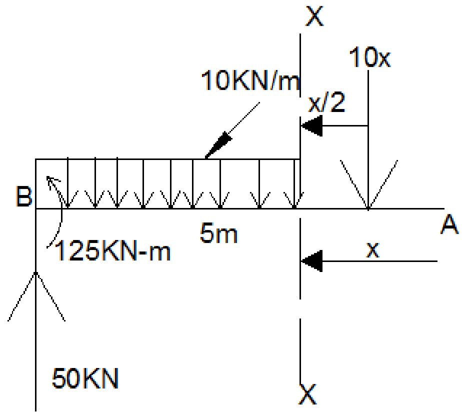


Fig 46

Table 6

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-5	10x	0, (50,0)	linear	-10x(x/2)	0,(-125,0)	parabolic

The variation of shear force is linear but the variation of bending moment is parabolic, we can draw parabola in 2 ways as shown if Fig 48 and Fig 49

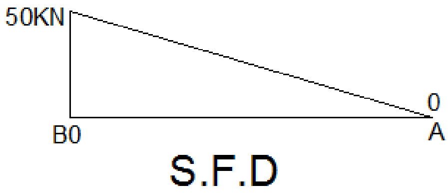


Fig 47

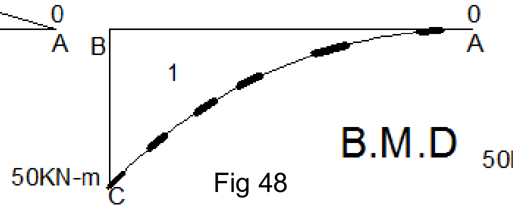


Fig 48

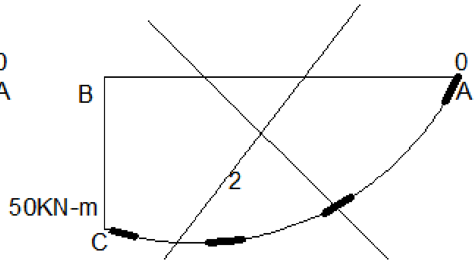


Fig 49

The "2" parabola is not correct, the reason is :

- **Relationship Between Shear force and Bending moment** $\frac{\mathbf{d(B.M)}}{\mathbf{dx}} = \mathbf{S.F},$
- Slope of bending moment diagram at any section = shear force at that section.
- Shear force is positive than slope of bending moment diagram is anticlockwise.
- Value of shear force is increasing between two points than in between those two points the value of slope of bending moment will increase.

The value of shear force from A to B increases which means that the slope of bending moment diagram must increase from A to B. In 1st diagram the slope of bending is increasing but in 2nd diagram the slope of bending moment diagram is decreasing. Similarly, the value of shear force at A is 0 which means the slope of bending moment diagram must be 0 at point B, which is only possible in 1st diagram not is second diagram.

- The shape of bending moment diagram is governed by shear force diagram.
  - $\Delta M=\int V.dx$ change in bending moment diagram between two points=(M_{final}-$M_{initial}$)= Area under the SF diagram between those two points. (not applicable for concentrated point moment)

Example 6

Write the shear force and bending moment equations and draw SFD and BMD

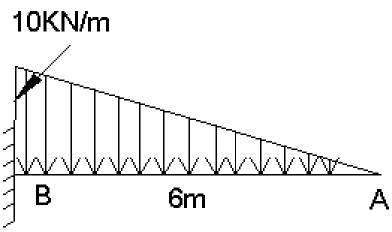


Fig 50

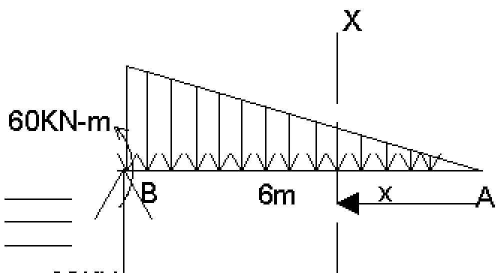


Fig 51

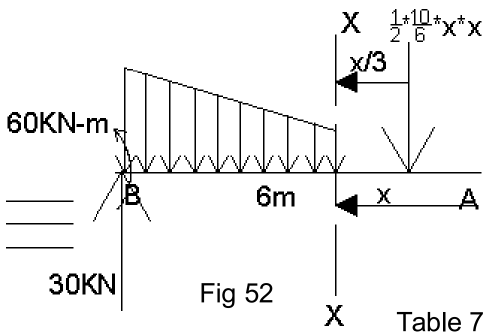


Fig 52

Table 7

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variartion of B.M
A-B	0-6	$(5/6) * x^2$	0, (30,0)	parabolic	$-(5/6) * x^2$*x/3	0,(-60,0)	3 degree parabolic

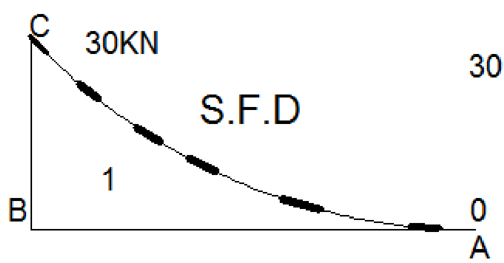


Fig 53

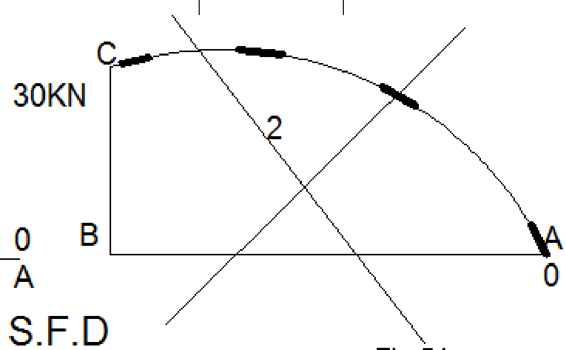


Fig 54

The variation of shear force is parabolic, we can draw parabola in 2 ways as shown if Fig 53 and Fig 54.

The “2” parabola is not correct, the reason is :

- **Relationship Between Shear force and Rate of Loading** $\frac{d(S.F)}{dx} = \textbf{Rate of loading},$
- Slope of shear force diagram at any section = rate of loading at that section.
- If force is downward than slope of shear force diagram is clockwise.
- Value of intensity of loading is increasing between two points than in between those two points the value of slope of shear force will increase.

The value of rate of loading from A to B increases from 0 to 10KN/m, which means that the slope of shear force diagram must increase from A to B. In 1st diagram the slope of shear forces is increasing but in 2nd diagram the slope of shear force diagram is decreasing. Similarly, the value of rate of loading at A is 0 which means the slope of shear force diagram must be 0 at point B, which is only possible in 1st diagram not is second diagram.

- The shape of shear force diagram is governed by rate of loading.
- $\Delta V=\int W_X.dx$ change in shear force diagram between two points=(V_{final}-$V_{initial}$)= Area under the loading diagram between those two points. (not applicable for concentrated point load)

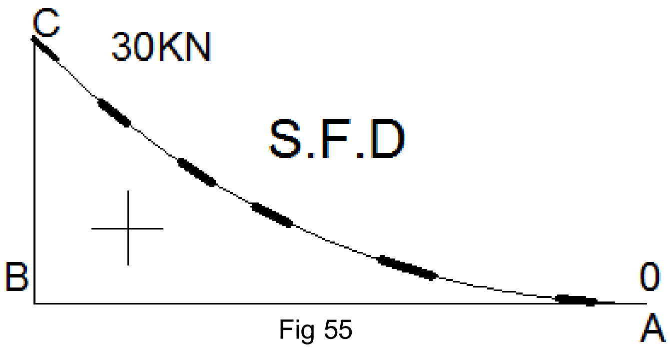


Fig 55

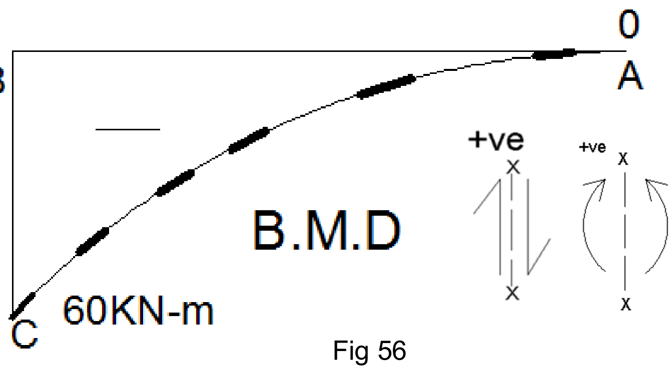


Fig 56

Summary

Table 8

Loading	S.F. diagram	B.M. diagram
Point load (downward)	Vertical ordinate	Change in slope at that point
Rectangular loading (downward)	Incline line, slope clockwise	2nd degree parabola, slope anticlockwise
Triangular loading (downward)	2nd degree parabola, slope clockwise	3rd degree parabola, slope anticlockwise
Concentrate Moment	No effect	Vertical ordinate
No loading	Horizontal line	Incline line
Symmetrical Beam	Skew – symmetrical diagram	Symmetrical diagram

Point of Contraflexure;- Point where bending moment changes its sign.(B.M=0 at the point)

* → It is not the point where B.M is zero but B.M need to change the sign (it’s a important point in RCC design as at this point the direction of reinforcement changes from top to bottom or bottom to top.)

- To find point of contraflexure write the equation of bending moment for the particular region put that to zero.
- **The point where shear force changes its sign (V=0) bending moment will be maximum (+value).**
- **Point of inflection**;- Point where deflected shape changes its curvature (B.M=0 at that point)

Example7 Write the shear force and bending moment equations and draw SFD and BMD.

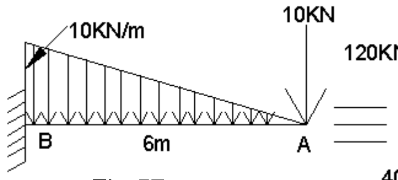


Fig 57

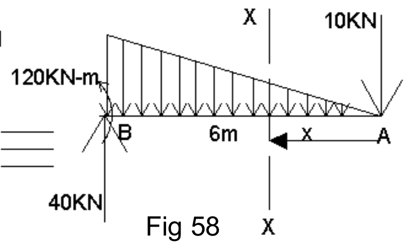


Fig 58

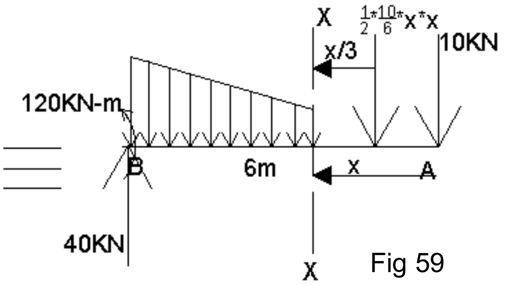


Fig 59

Table 9

Regi on	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-6	$10+(5/6)*x^2$	0, (40,0)	parabolic	$-(5/6)*x^2*x/3-10x$	0,(120,0)	3 degree parabolic

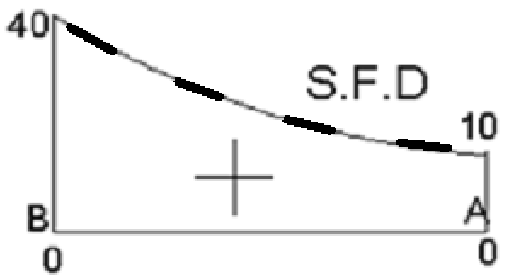


Fig 60

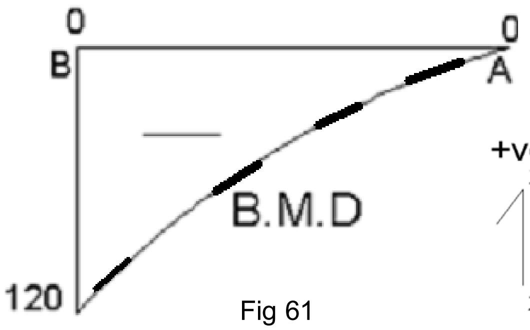
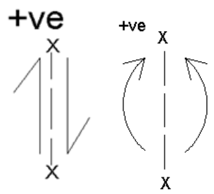


Fig 61



- The loading is downward and increasing from point A to B hence slope of shear force is clockwise and increasing . Shear force is positive and increasing hence slope of bending moment diagram is anticlockwise and increasing.
- At point A rate of loading is 0, hence the slope of SFD is 0 at point A but the slope of bending point will not be 0 at point A because the value of shear force at A is 10.

Example8 Write the shear force and bending moment equations and draw SFD and BMD

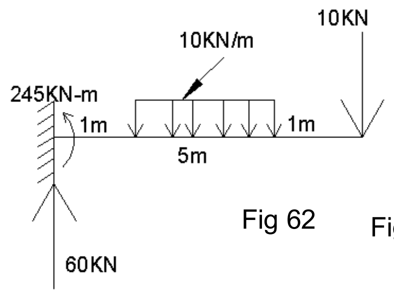


Fig 62

Fig 67

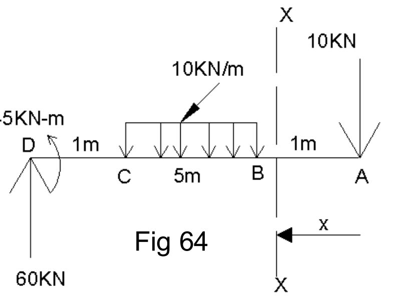


Fig 64

Fig 66

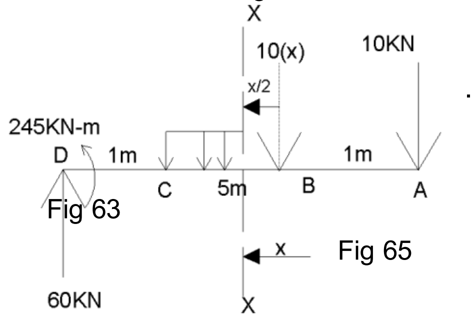
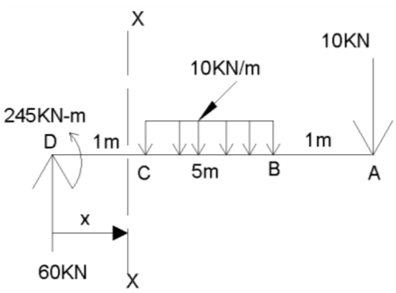


Fig 63

Fig 65



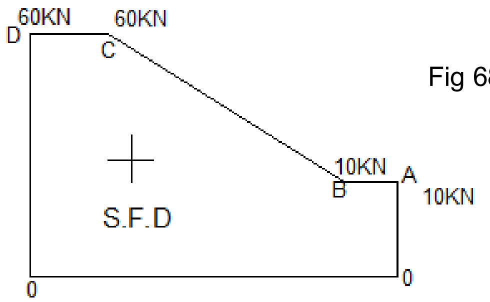
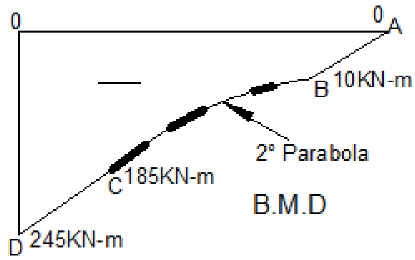


Fig 68



Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-1	+10	(0,10),10	constant	-10x	0,10	linear
B-C	0-5	+10+10(x)	10,60	linear	$-10(x+1)-5x^2$	-10,-185	2^o Parabola
D-C	0-1	+60	(0,60), 60	constant	+60(x)-245	(0,-245),-185	Linear

Example9

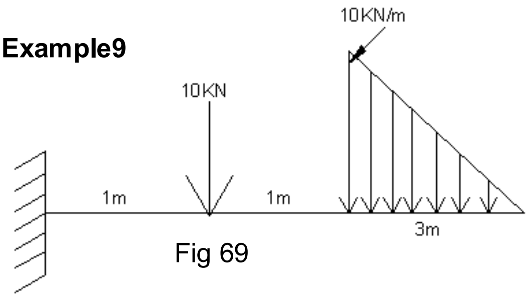


Fig 69

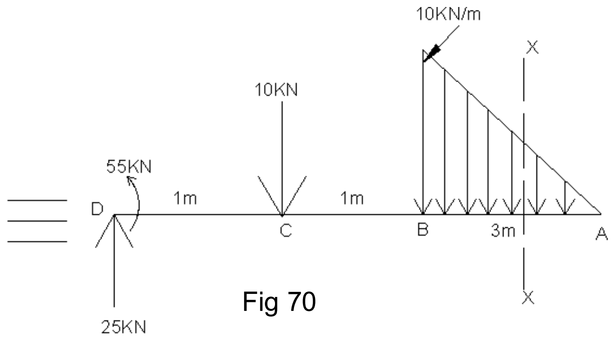


Fig 70

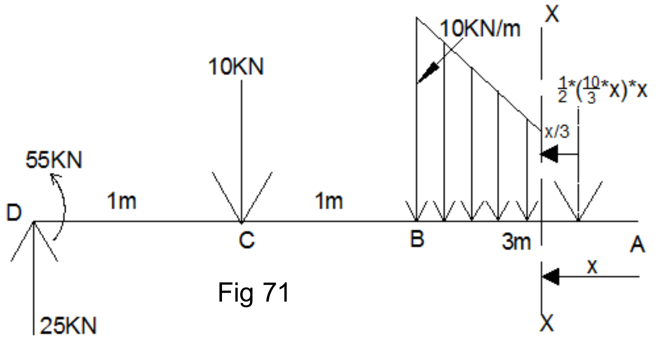


Fig 71

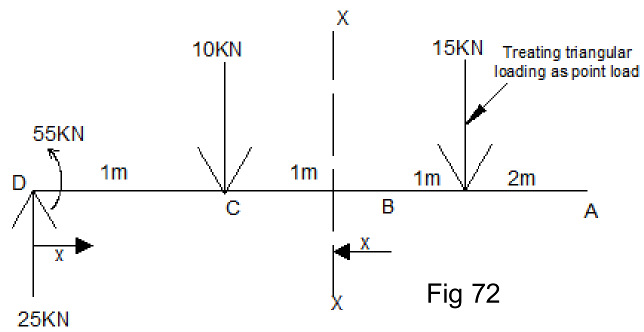


Fig 72

Table 11

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
A-B	0-3	$\left(\frac{1}{2}\right)*\left(\frac{10}{3}\right)*x^2$	0,15	2^o Parabola	$\left(\frac{1}{2}\right)*\left(\frac{10}{3}\right)*x^2*x/3$	0,-15	3^o Parabola
B-C	0-1	15	15, (15,25)	constant	$-15(x+1)$	-15,-30	Linear
D-C	0-1	+25	(0,25),(25)	constant	+25(x)-55	(0,-55),-30	Linear

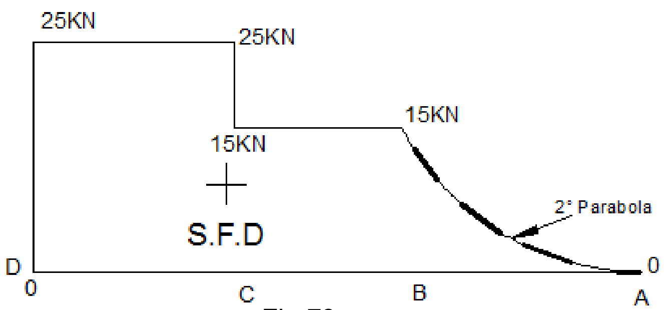


Fig 73

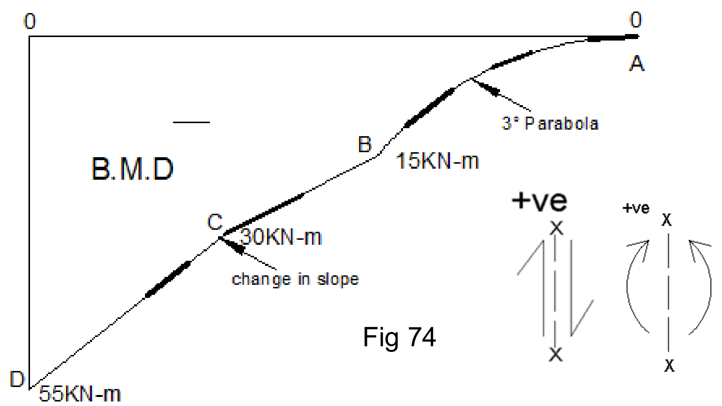


Fig 74

SHEAR FORCE & BENDING MOMENT DIAGRAM

Example10

Draw the shear force and bending moment diagram for Trapezoid loading.

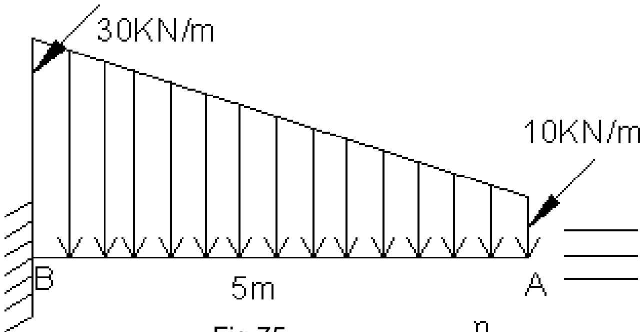


Fig 75

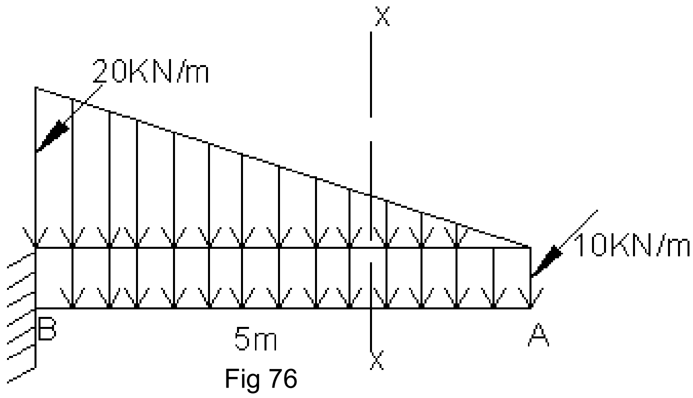


Fig 76

Trapezoid loading can be break into triangular and rectangular loading as shown in Fig 76

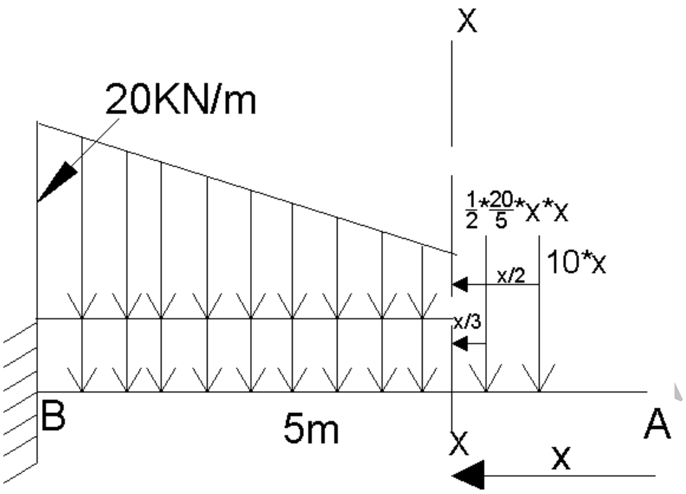


Fig 77

Table 12

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values	Variation of B.M
A-B	0-5	$\left(\frac{1}{2}\right)*\left(\frac{20}{5}\right)*x^2$ +10*x	0,50	2° Parabola	$-\left(\frac{1}{2}\right)*\left(\frac{20}{5}\right)*x^2*x/3$ -10*x*x/2	0,-95	3° Parabola

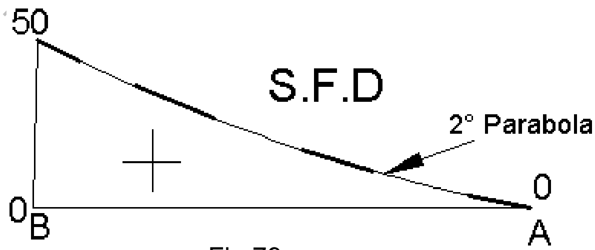


Fig 79

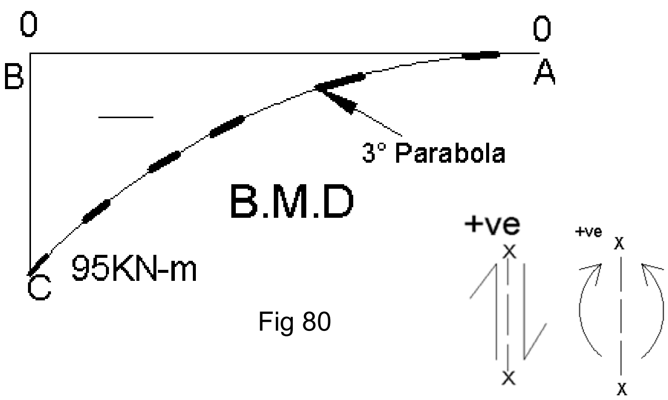


Fig 80

SHEAR FORCE & BENDING MOMENT DIAGRAM

Example11 Draw the shear force and bending moment diagram for Trapezoid loading.

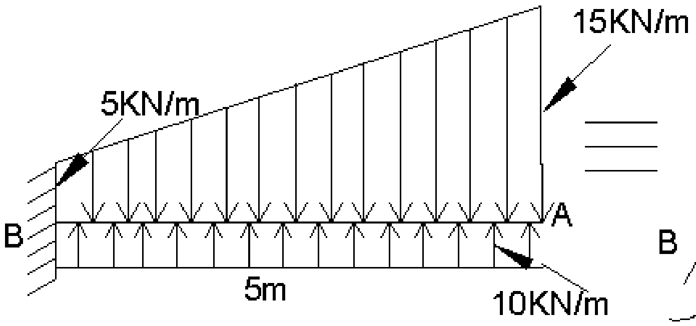


Fig 81

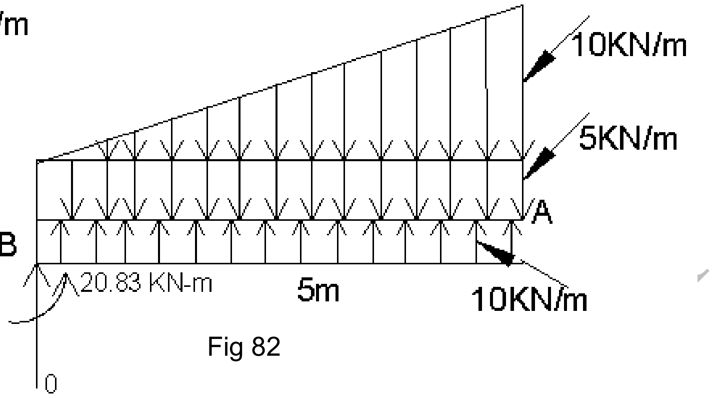


Fig 82

For the given loading, the rate of loading is varying , at point A the rate of loading is 15KN/m-10KN/m=5KN/m downward, but at point B the rate of loading is 5KN/m-10KN/m= -5KN/m. the variation in the rate of loading leads to change is slope of shear force diagram. At 2.5 m from point A, the rate of loading is 0 i.e slope of shear force diagram is 0 at that point.

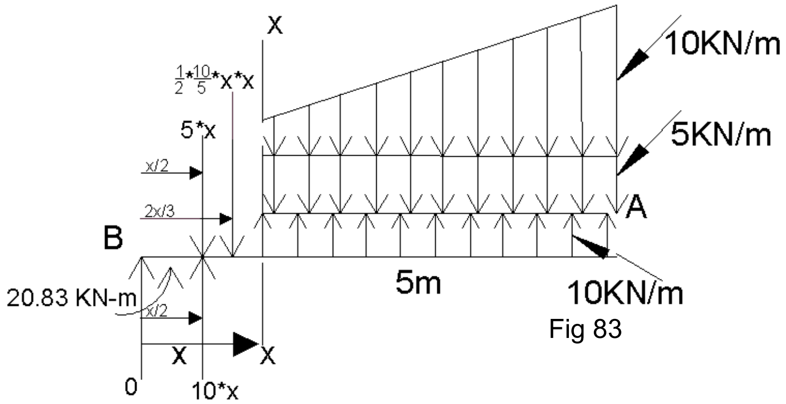


Fig 83

Table 13

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variation of B.M
B-A	0-5	$-x^2-5*x+10*x$	0,0	2° Parabola	$-20.83-x^2*\frac{x}{3}-5*x*\frac{x}{2}+10*\frac{x^2}{2}$	0,-20.83	3° Parabola

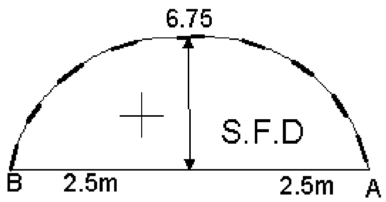


Fig 84

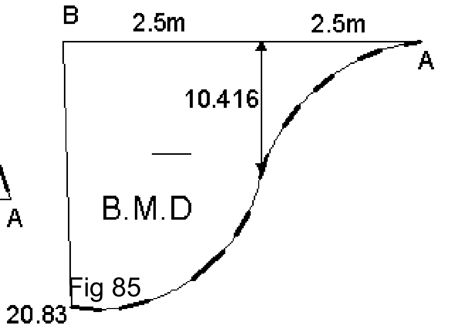
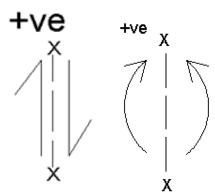


Fig 85



SHEAR FORCE & BENDING MOMENT DIAGRAM

THRUST DIAGRAM

Thrust force is the algebraic sum of all the longitudinal forces acting on either side of the section. (Remember, after finding the reaction, treat reaction as a externally applied load).

Sign convention :-

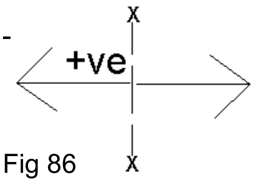


Fig 86

x-x is a section

The left hand force on left hand side of section is +ve

The right hand force on Right hand side of section is +ve.

E

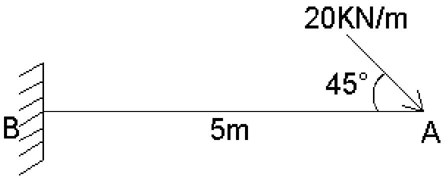


Fig 87

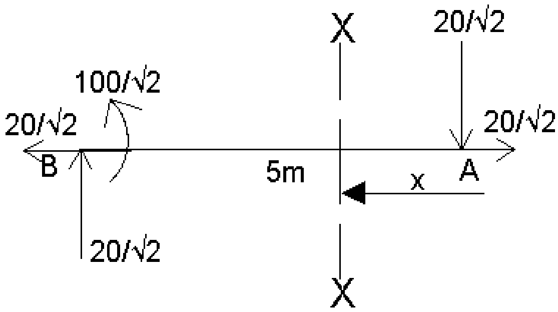


Fig 88

Table 14

Region	Limit of X	Eq. of thrust	Boundary values of Thrust	Variation of Thrust
B-A	0-5	14.14	(0,14.14) (14.14,0)	Constant

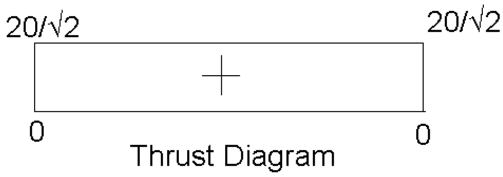


Fig 89

3

Example 13 Draw the shear force and bending moment and thrust diagram for.

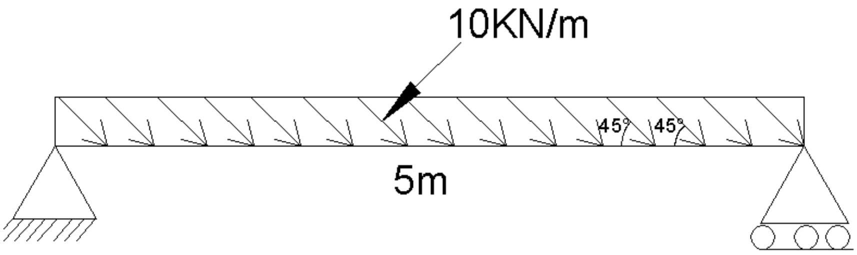


Fig 90

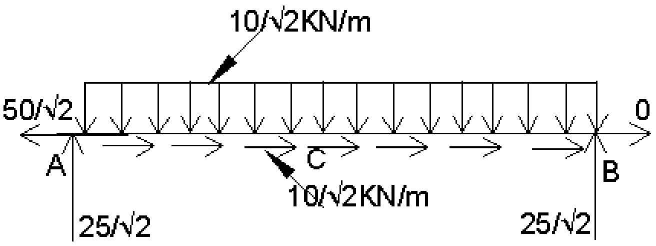


Fig 90

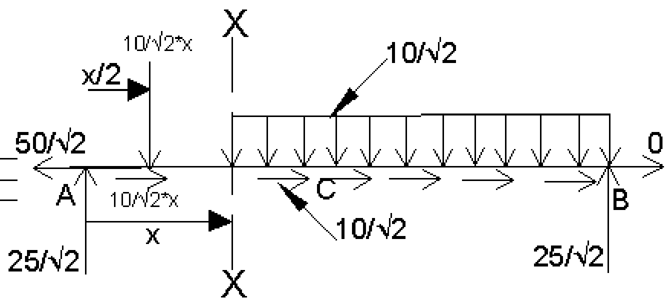


Fig 91

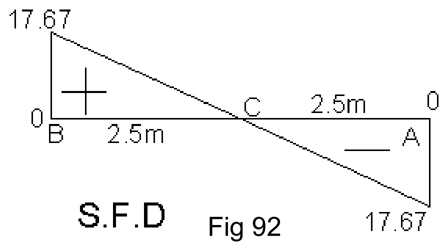
Table 15

Region	Limit of X	Eq. of S.F	Boundary values of S.F	Variation of S.F	Eq. of BM	Boundary values of B.M	Variartion of B.M
A-B	0-5	17.67-7.07*x	(0,17.67), (-17.67,0)	Linear	$17.67*x-7.07*x^2$	0,0	2^o Parabola

SHEAR FORCE & BENDING MOMENT DIAGRAM

Table 16

Region	Limit of X	Eq. of thrust	Boundary values of S.F	Variation of thrust
A-B	0-5	35.35-7.07*x	(0,35.35) (0)	linear



S.F.D Fig 92

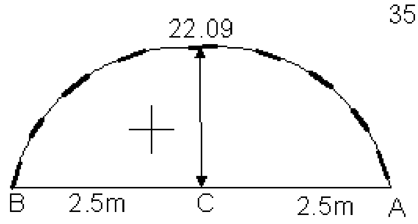


Fig 93 B.M.D

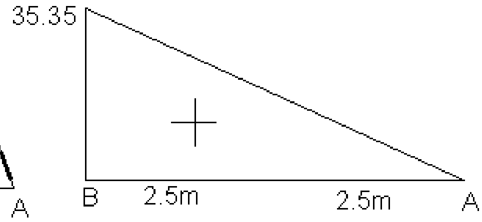


Fig 94

Thrust

- At point C, Shear force is zero hence slope of BMD is zero and bending moment is maximum at point C.

How to draw the SFD and BMD without writing the equation

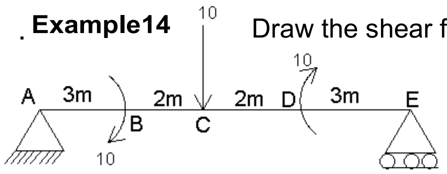
Remember the table and directly draw the SFD & BMD.

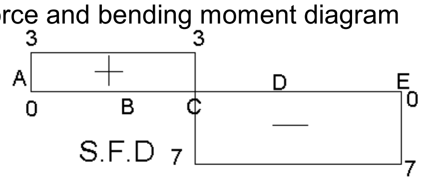
Table 17

Loading	S.F. diagram	B.M. diagram
Point load (downward)	Vertical ordinate of value equal to point load	Change in slope at that point
Rectangular loading (downward)	Incline line, slope clockwise	2^{nd} degree parabola, slope anticlockwise
Triangular loading (downward)	2^{nd} degree parabola, slope clockwise	3^{rd} degree parabola, slope anticlockwise
Concentrate Moment	No effect	Vertical ordinate of value equal to point load.
No loading (in between the other loading)	Horizontal line	Incline line
Symmetrical Beam	Skew – symmetrical diagram (the diagram is exact opposite about central line.)	Symmetrical diagram

- if you move from left to right and load is acting downward than shear force will be decreasing and vice-versa and if you move from right to left and load is acting act downward shear force will be increasing and vice-vera.
- if you move from right to left and moment generated by loading act anticlockwise than bending moment will be increasing and vice-versa and if you move from left to right and moment generated by loading act clockwise than bending moment will be increasing and vice-versa
- At any point,if S.F is having 2 values (due to point load) than at that point B.M will have 2 slopes.
- Thrust diagram works same as shear force diagram.

Example14 Draw the shear force and bending moment diagram





S.F.D

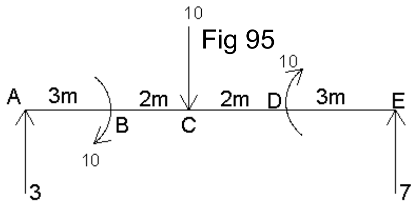
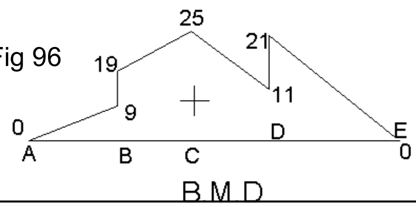


Fig 95

Fig 96



B.M.D

Find the reactions and treat them as loading as shown in Fig 97
A,C & E are having point loads so vertical ordinates and moving from A to C no loading hence shear force is constant. Moving just left of C to just right of C the 10KN is downward hence shear force will decrease. Similarly for A & E.

SHEAR FORCE & BENDING MOMENT DIAGRAM

Concentrated moment at B and D so vertical ordinate at B and D. moving from A to B the moment is clockwise so bending moment is increasing. At B. 3*3=9, , At C 3*5+10=25 At D 7*3=21

Example 15   Draw the shear force and bending moment diagram

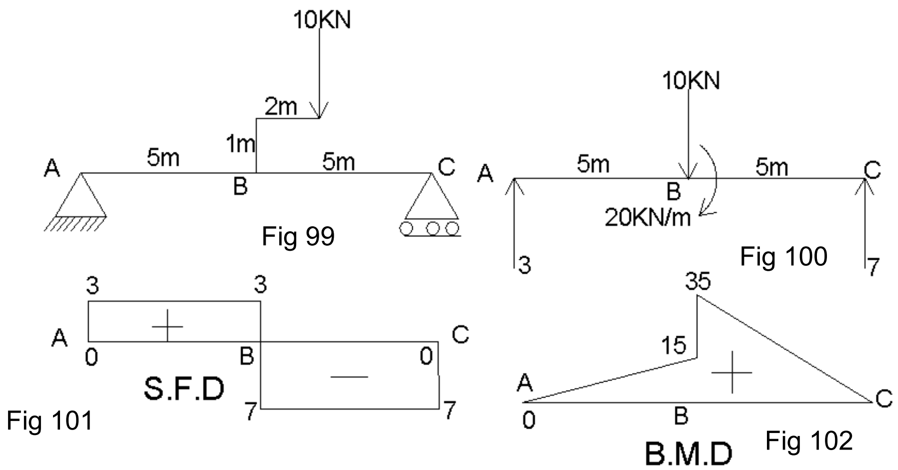


Fig 99 Fig 100 Fig 101 Fig 102

Using the concept of free body diagram shift the loading to the beam.and find reaction.

For SFD, from Fig 100 the beam carries a point load at A, B, and C (treating reaction as loading) hence we will be having three ordinate equal to point load. In span AB & BC no loading hence, constant shear force.

For BMD, from Fig 100 moving from left to right the moment generated by 3 is clockwise, hence bending moment is increasing up to B (5*3=15) then carries a concentrated moment clockwise at B hence, having a increasing ordinate of 20 (15+20=35). For span BC no loading hence inclined line joined to C at which bending moment is 0.

Example 16   Draw the shear force and bending moment diagram

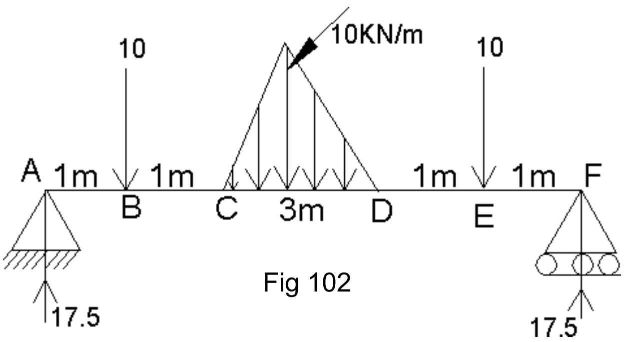


Fig 102

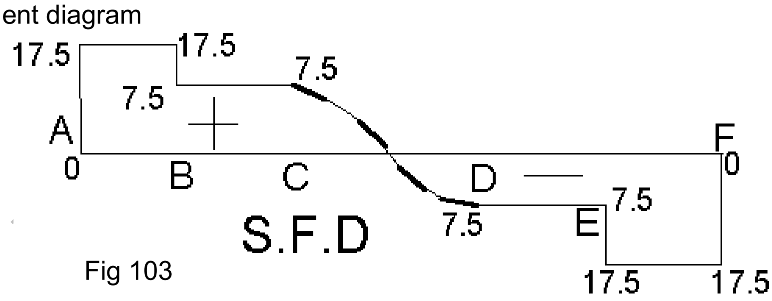


Fig 103

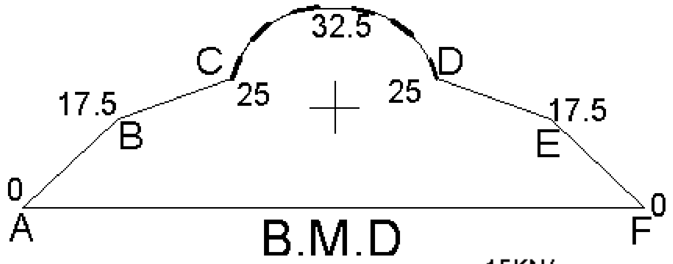


Fig 104

for SFD, 4 point load, 4 ordinate. At point C, S.F=+7.5, but at D, S.F= -7.5, it means in between C and D shear force is 0.

$(S.F)_{CD}$ = 17.5-10-(10/3)*x*x/2=0 ⇒x=1.5m

At center shear force is 0, hence bending moment will be maximum. $(B.M)_{CD}$=17.5*3.5-10*2.5-(10*3)*3/(2*3)=32.5

Example 17 Draw the shear force and bending moment diagram

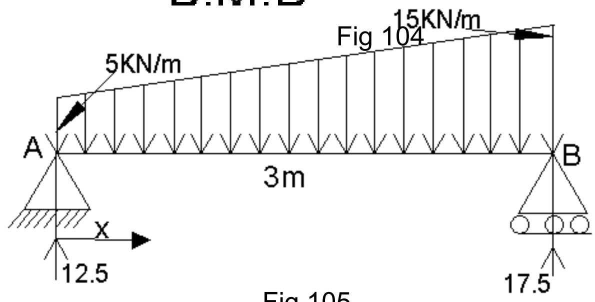


Fig 105

For SFD, at A vertical ordinate of 12.5 and at point B the vertical ordinate is of 17.5 downward. Join both the points by 2nd degree parabola (as loading is trapezoid)

$(S.F)_X$= 12.5-5*x-(0.5)*x^2*(10/3)

$(S.F)_X$=0⇒3.33x^2+5x-12.5=0, x=1.32m

$(B.M)_{1.32}$=12.5*1.32-5*1.32*1.32/2-0.5*(10/3)*1.32^2*1.32/3

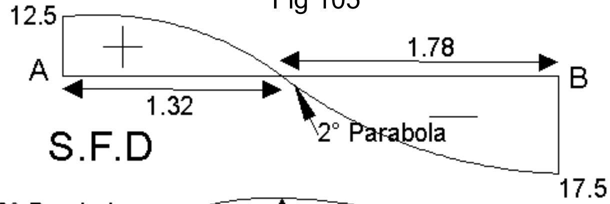


Fig 106

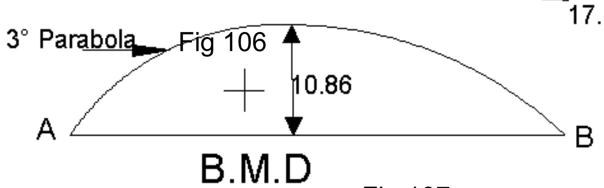


Fig 107

SHEAR FORCE & BENDING MOMENT DIAGRAM

$(B.M)_{1.32}=(B.M)_{MAX}=10.86$ KN-m

The SFD will be 2nd degree parabola and BMD will be 3rd degree parabola.

Example 18 Draw the shear force and bending moment diagram

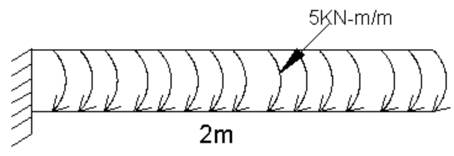


Fig 108

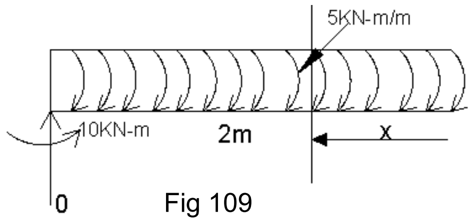


Fig 109

No vertical loading, hence shear force is 0.

Bending moment at section x , from free end is equal to area under the loading = 5*x

The variation of BMD is inclined line as shown in Fig.110A

Fig 110

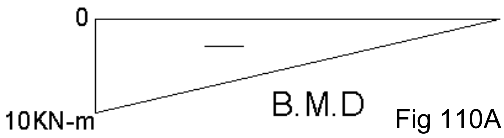


Fig 110A

Example 19 Draw the shear force and bending moment diagram

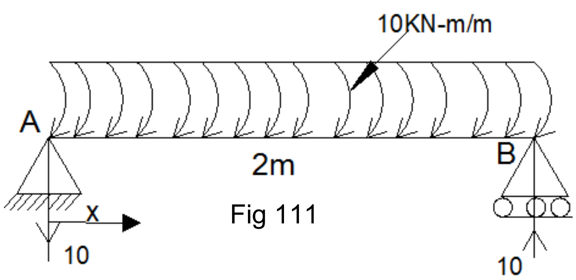


Fig 111

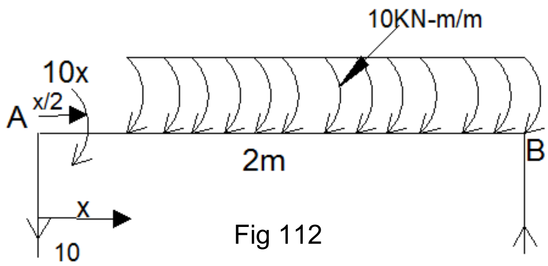


Fig 112

2 vertical loads at both the ends, hence 2 ordinate in shear force diagram.

$(B.M)_X=10*x-10*x=0$

The bending moment at each point is zero

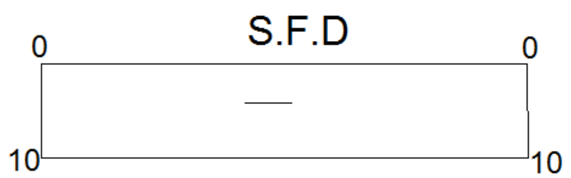


Fig 113

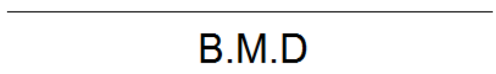


Fig 114

Example 20 Draw the shear force and bending moment diagram

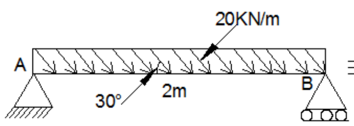


Fig 115

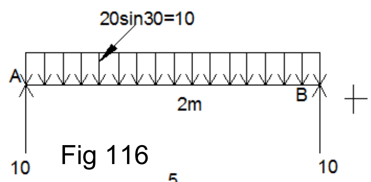


Fig 116

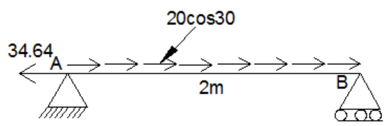


Fig 117

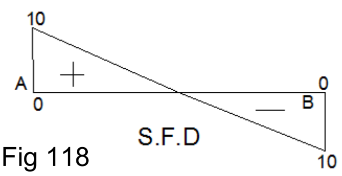


Fig 118

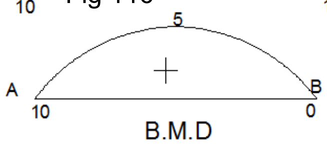


Fig 119

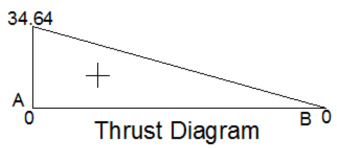


Fig 120

A longitudinal force in form of udl of intensity 17.32KN/m is acted on beam hence the thrust diagram is linear.

SHEAR FORCE & BENDING MOMENT DIAGRAM

Example 20 Draw the shear force and bending moment diagram.

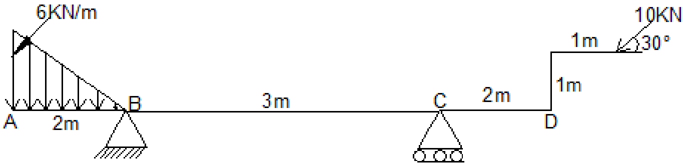


Fig 121

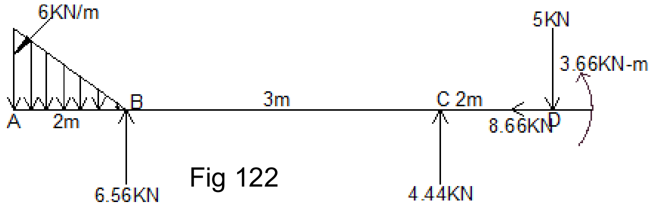


Fig 122

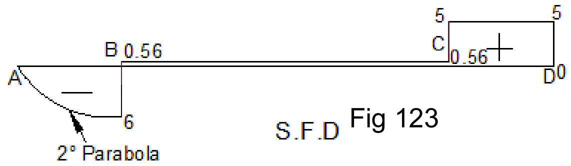


Fig 123

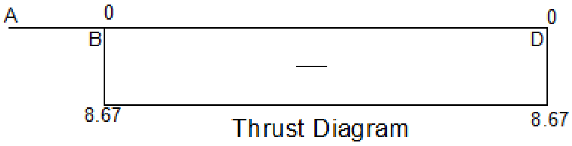


Fig 124

Fig 125

Example 21 Draw the shear force and bending moment diagram

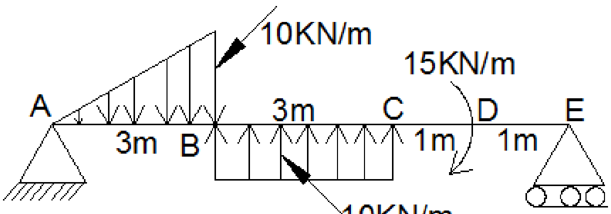


Fig 126

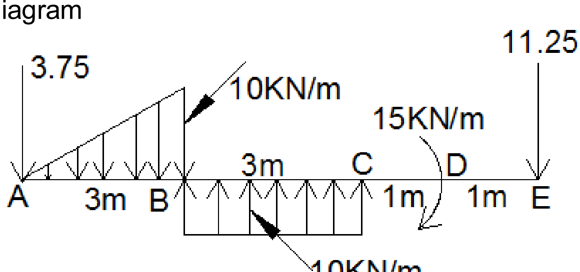


Fig 127

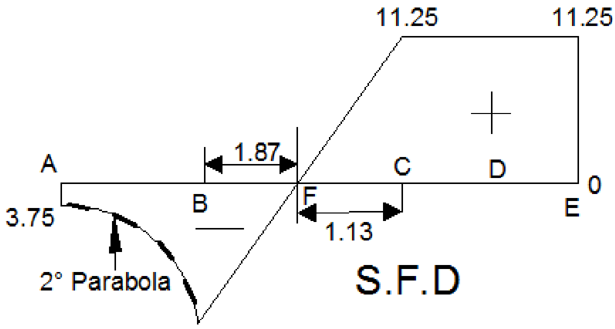


Fig 128

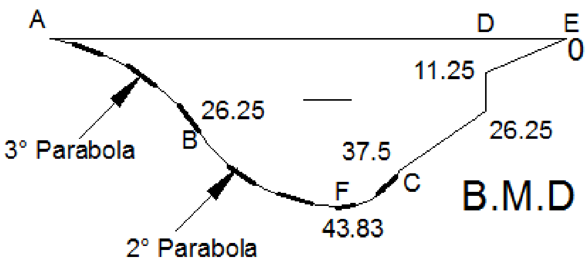


Fig 128

Maximum B.M will occur at point F(S.F=0), use similar triangle formula between B and C.

$$X = \frac{18.75 * 3}{18.74 + 11.25} = 1.87$$

$$(B.M)_F = -3.75 * (3 + 1.87) - \frac{1}{2} * 3 * 10\left(\frac{3}{3} + 1.87\right) + 10 * 1.87 * \frac{1.87}{2} = -43.83$$

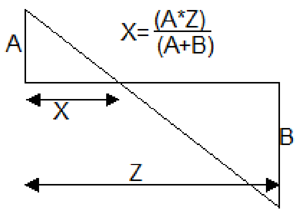


Fig 128

SHEAR FORCE & BENDING MOMENT DIAGRAM

Example 22 Draw the shear force and bending moment diagram and find the point of contraflexure.

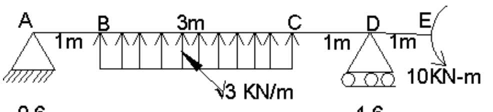


Fig 129

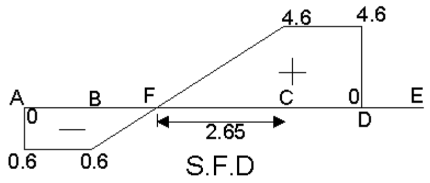


Fig 130

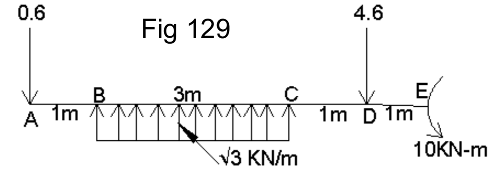


Fig 131

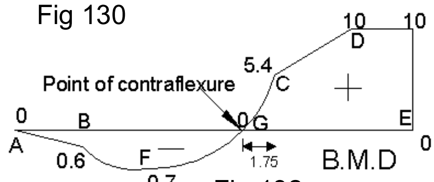


Fig 132

The shear force is 0 at point F, slope of B.M.D will be 0 at F.

Bending moment at B is -06*1=-0.6 and bending moment at D is +10, and bending moment in between B and C is parabola, point of contraflexure lies in between B and C. $(B.M)_{CB} = +4.6*(x+1)+\frac{1.732}{2}*x^2-10=0 \Longrightarrow$ x=1.75

Example 23 Draw the shear force and bending moment diagram and find the point of contraflexure

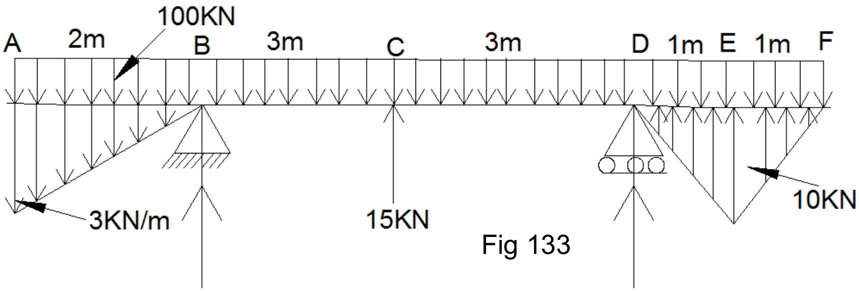


Fig 133

$R_B=50-7.5+10*\frac{1}{6}+\frac{1}{2}*3*2*(2*\frac{2}{3}+6)*\frac{1}{6}=47.83$

$R_B=78-47.83=30.17$

In this example, for UDL total load is given hence intensity of udl is 100KN/10m=10KN/m. same for D to F triangular loading the maximum intensity of loading is 10*2/2=10KN/m.

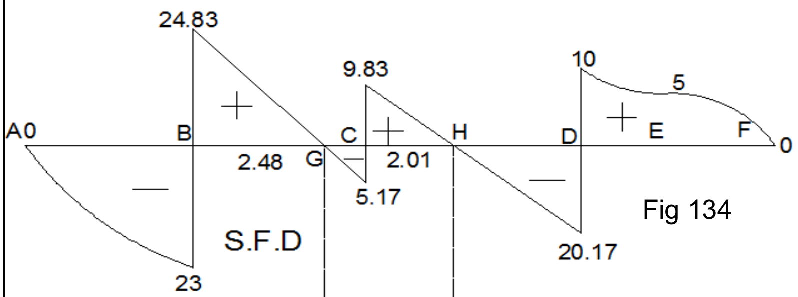
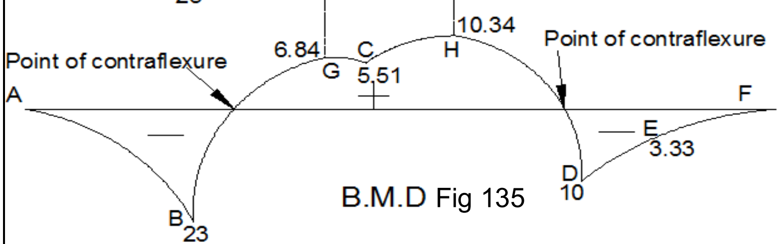


Fig 134



B.M.D Fig 135

S.F.D at point E, the rate of loading is zero hence slope of shear force diagram will be 0, the S.F is zero at point G and H as shown in Fig.134.or region AB and DF the variation of S.F is 2^0 Parabola.

For region AB and DF the B.M is 3^0 Parabola. For region BD the B.M is 2^0 Parabola.

Example24 Find the position of two supports such that maximum B.M should be minimum.

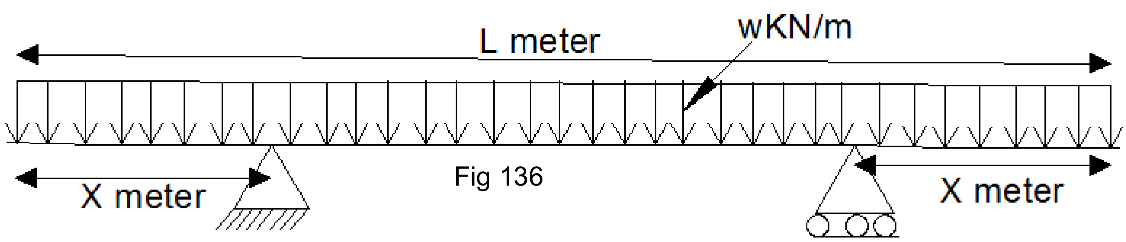


Fig 136

SHEAR FORCE & BENDING MOMENT DIAGRAM

Sol:-The structures are designed for maximum bending moment, so for a designer it is important to reduce the maximum bending moment, For understanding the problem, let us assume w=10KN/m and L=8m.

CASE 1

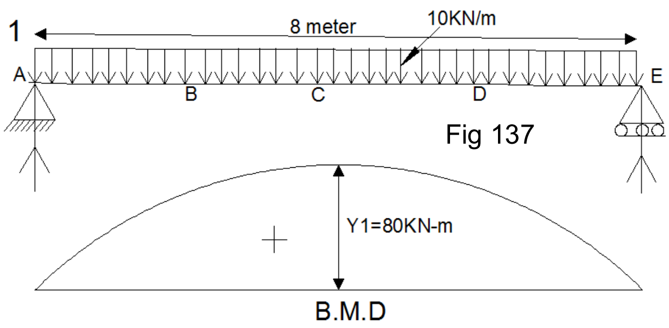


Fig 137

IF both the supports are placed at the end than the beam will become simply supported and BMD is as shown in Fig. the maximum +ve bending moment Y1=80KN-m and maximum –ve bending moment is 0

CASE 2

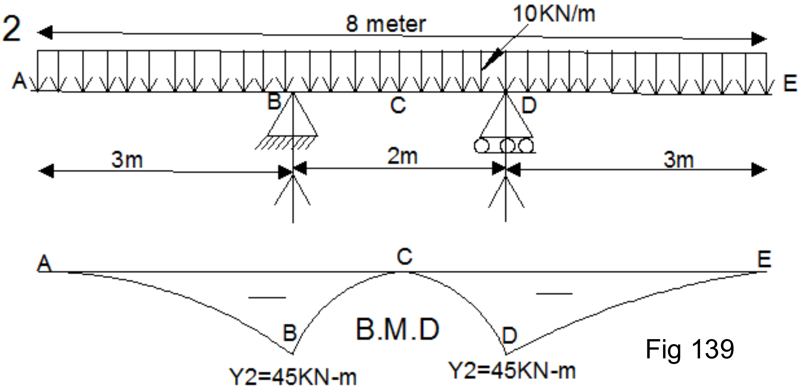


Fig 138

Fig 139

If,both the supports are placed at 3m from free end than the beam will become overhang and BMD is as shown in Fig. the maximum +ve bending moment is 0KN-m and maximum –ve bending moment Y2= 45.

CASE 3

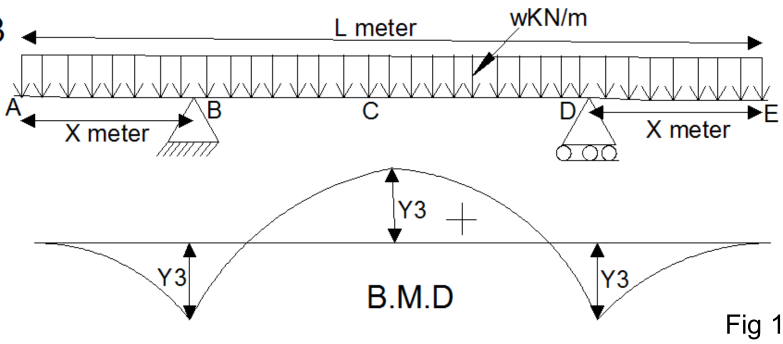


Fig 140

Fig 141

If both the supports are placed such that +ve bending moment is equal to –-ve moment than as shown in Fig $y_3 < y_2$ & $y_3 < y_1$ i.e max B.M (either +ve or –ve) is minimum

$(y_3)_B = \frac{wx^2}{2}$ (max –ve moment) & $(y_3)_c = \frac{wL}{2}\left(\frac{L}{2} - x\right) - \frac{wL^2}{2\times 4}$ (max +ve moment)

$$(y_3)_B = (y_3)_C \Rightarrow \frac{wx^2}{2} = \frac{wl}{2}\left(\frac{l}{2} - x\right) - \frac{w}{2} \times \frac{L^2}{8} \quad \Longrightarrow \quad \mathbf{x=0.207L}$$

If supports are positioned at x=0.207L for our example this is 1.65m, then the maximum moment for which beam is designed is 10.89KN-m which is less than 80KN-m and 45KN-m obtained in case 1 and case2. As shown in Fig 143

CASE 3

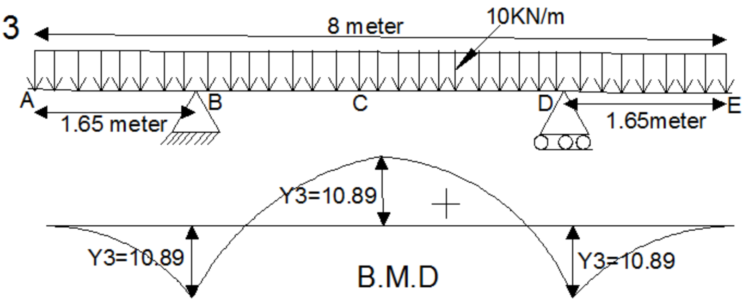


Fig 142

Fig 143

SHEAR FORCE & BENDING MOMENT DIAGRAM

Example 25 The left hand support hinged is fixed at end. Find the position of Right hand support Such that max. (B.M) is minimum. As shown in Fig. 144 .

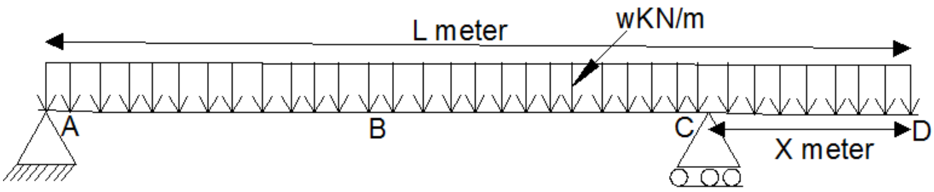


Fig 144

Sol- The given beam is not symmetrical, Maximum +ve bending moment will be in between A&C , maximum bending moment will not be at center but at a point (‘a’ distance from A) in between A & C, where shear force is 0 and Maximum –ve bending moment will be at right support i.e point C.

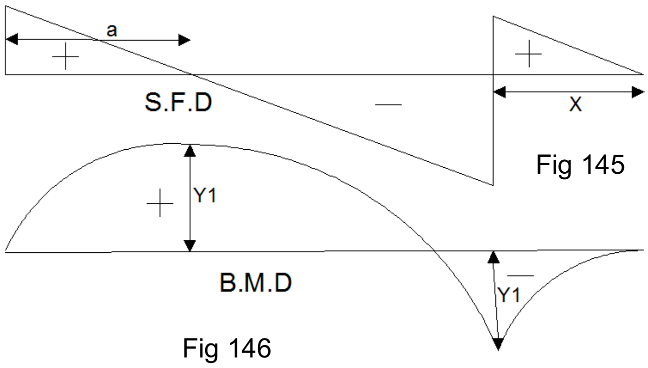


Fig 145

Fig 146

$$R_C = \frac{wL^2}{2(L-x)}, \quad R_A = \frac{wL(L-2x)}{2(L-x)}$$

$$(S.F)_{AC} = \frac{wL(L-2x)}{2(L-x)} - wa$$

$$(S.F)_{AC} = 0 \Rightarrow (B.M)_{max} + ve$$

$$(S.F)_{AC} = 0 \Longrightarrow \frac{wL(L-2x)}{2(L-x)} - wa = 0 \qquad \Rightarrow a = \frac{\frac{L}{2}(L-2x)}{L-x}$$

4

$$(B.M)_{max} + ve = \frac{WL^2}{8}\left[\frac{L-2x}{L-x}\right]^2 \; \& \; (B.M)_{max} - ve = \frac{wx^2}{2}$$

Equating the $(B.M)_{max}$ sagging $= (B.M)_{max}$Hogging and solving for x, **x=0.293L**

Example26 Analyse the beam as shown in Fig.147

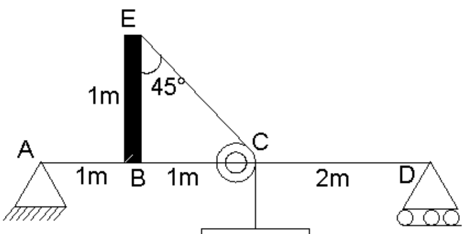


Fig 147

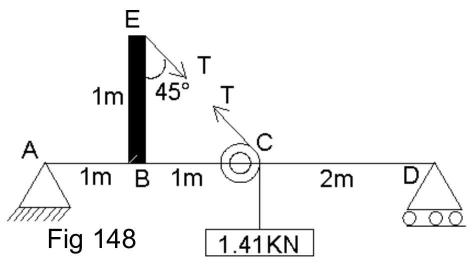
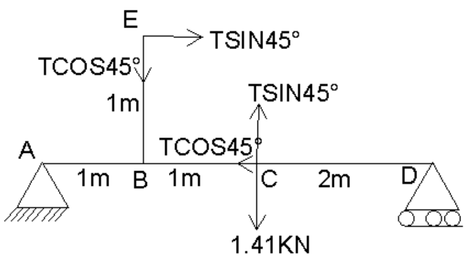
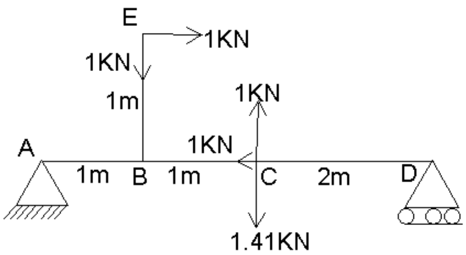


Fig 148





the rope EC will be in tension. The free body diagram of the rope is shown in Fig148

Resolving the forces in horizontal and vertical direction at point C & E. as shown in Fig149, considering the vertical equilibrium of forces at point C $T\sin 45^0 = 1.41 \Rightarrow T = 1$

Substituting the value of T, as shown in Fig151

SHEAR FORCE & BENDING MOMENT DIAGRAM

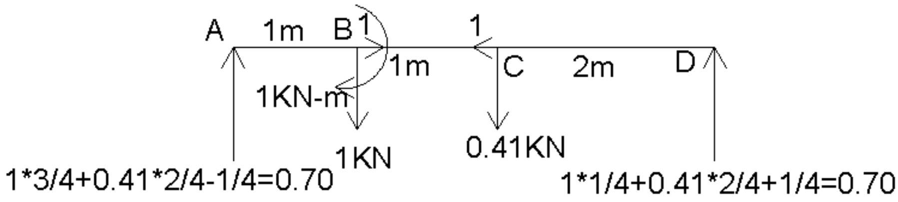
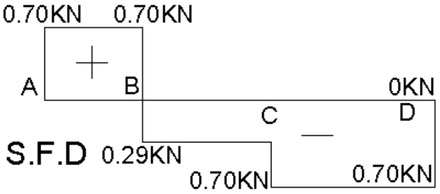


Fig 151



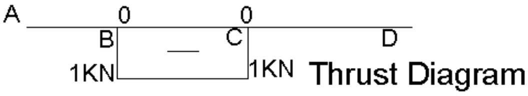


Fig 152

Fig 153

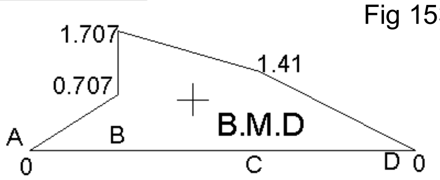


Fig 154

How to draw Loading Diagram from Shear Force and Bending Moment Diagram

For the given SFD or BMD or both we can draw the loading diagram by using the Table17 (page number 15).

- if you move from left to right and shear force is increasing than load act upward and vice-versa and if you move from right to left and shear force is increasing than load act downward and vice-vera. 5
- If shear force is inclined line than slop of inclined line will give you intensity of loading.
- if you move from right to left and the bending moment is increasing than loading is upward (moment generated by loading act anticlockwise) and vice-versa and if you move from left to right and bending moment is increasing than loading is upward (moment generated by loading act clockwise) and vice-versa.

Example 27 For the given SFD, draw the loading diagram.

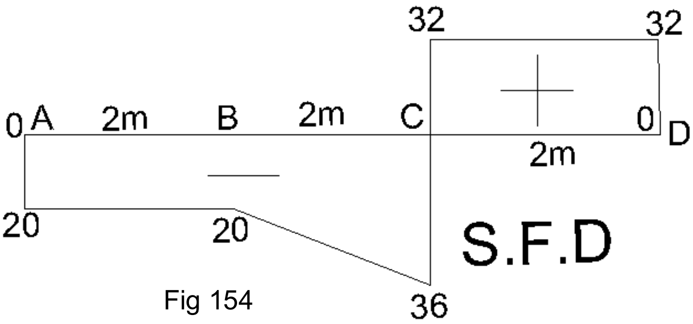


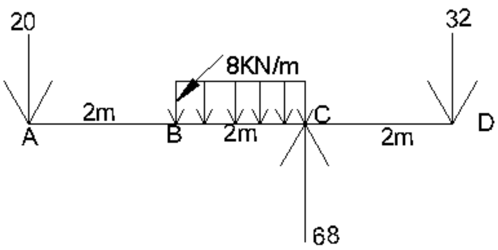
Fig 154

1. The first view is at point A,,20KN point load at point C 68KN point load and at D 32KN point loads. In between AB and CD no loading. In between BC UDL.

2. Moving from just left of A to just right of A, shear force decreases it means loading act downward, hence 20KN act downward.

3. Moving from just left of C to just right of D, shear force increases it means loading act upward, hence 68KN act upward.

4. Moving from just right of D to just left of D, shear force increases it means loading act downward, hence 20KN act downward.



5. Moving from C to B (right to left), shear force increases (-36 to -20) hence udl act downward.

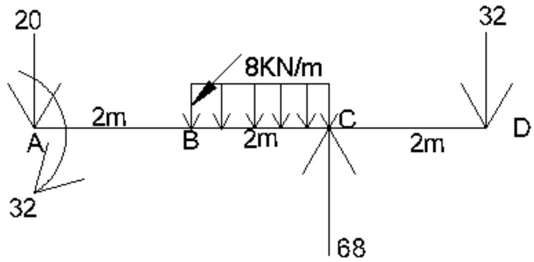
6. Intensity of UDL is equal to slope of inclined line ⇒(36-20)/2=8KN/m

SHEAR FORCE & BENDING MOMENT DIAGRAM

Important:- Check for $\Sigma V = 0\ and\ \Sigma M = 0$ for complete loading

$$\Sigma V = 0 \Rightarrow +68 - 20 - 32 - 8*2 = 0 \qquad OK$$

$(\Sigma M)_A = 32*6 + 8*2*3 - 68*4 = -32$, to make $(\Sigma M)_A$=0 there must be a concentrated moment some where in AD of magnitude 32 clockwise. You can place the concentrated moment anywhere as it will not hamper the SFD. Its always better to place at end as shown in Fig155



Loading Diagram

Example 28 For the given BMD, draw the loading diagram.

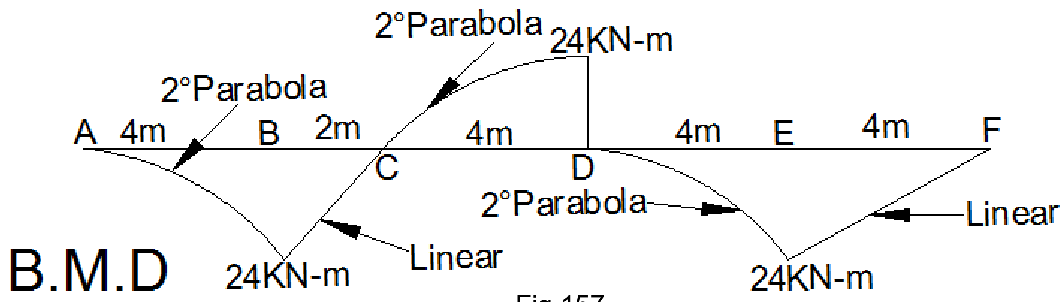


Fig 157

1. Starting from A and moving towards B, the variation of BMD is 2degree parabola, no point load at A, (slope at A is 0) only udl in between AB, assuming intensity of udl as w acting downward. as moving from left to right the value of bending moment is decreasing, the bending moment at B will be $-w*4^2/2=-24 \Longrightarrow$ w=3KN/m.

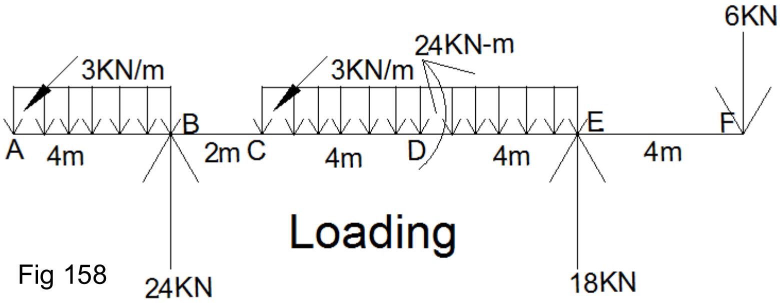


Fig 158

2. From B to C the variation of BMD is inclined line, hence as per table no loading in between BC but 2 slopes at B so point load at B. The bending moment at C without considering point load is M'_C= -3*4*(2+2)= -48 but the value of .the bending moment at C =0 it means this point load will be giving clockwise moment to make moment at C =0,assuming W_B as point load and taking moment at point C from left hand side. M_C= -3*4*(2+2)+W_B*2=0$\Longrightarrow$ W_B=24KN.

3. From C to D, the variation of BMD is 2degree parabola, no point load at C (only one slope at C) hence only udl in between CD, The bending moment at D without considering UDL load is M'_D= -3*4*(2+6)+24*6= +48 but the value of .the bending moment at D =24 it means this UDL load will be giving anti clockwise moment to make moment at D =24KN-m assuming intensity of udl as w acting downward, the bending moment at D from left hand side will be M_D=3*4(2+8)-24*6-w*4*2=24$\Longrightarrow$ w=3KN/m

4. At point D, there is a vertical ordinate in BMD, hence a concentrated moment at D, the intensity of moment is 24, this is moment is decreasing the bending moment value while moving from left to right hence the direction of moment is clockwise.

5. From D to E, the variation of BMD is 2degree parabola, no point load at D (slope of bending moment atD=0) hence only udl in between DE, The bending moment at E without considering UDL load is M'_E= -3*4*(2+10)+24*10-

3*4(2+4)-24= 0 but the value of .the bending moment at E = -24 it means this UDL load will be giving anti clockwise moment to make moment at E = -24KN-m assuming intensity of udl as w acting downward, the bending moment at E from left hand side will be M_D=3*4(2+10)-24*10-3*4*(2+4)-w*4*2= -24$\Longrightarrow$ w=3KN/m

5. Moving from F to E (from left to right) the variation of bending moment is linear. Hence point load at F, since the value of bending moment at E is negative, hence the point load will generate clockwise moment at D i.e point load act downward. Let us assume the value of point load W_F, taking moment at E from right hand side, M_D=W_F*4=24$\Longrightarrow$ W_F=6KN.

6. At point E, bending moment is having 2 slopes, hence point load at E. Assuming point load P_E to act upward and satisfying the $\Sigma V = 0 \Longrightarrow$ 24+P_E-3*4-3*8-6=0$\Longrightarrow$ P_E=18KN.

Check for equilibrium equations.

$$(\Sigma M)_A = 3*4*2-24*4+3*8*(4+6)-18*14-6*18=0 \text{ O.K}$$

Example 29 For the given BMD, draw the loading diagram

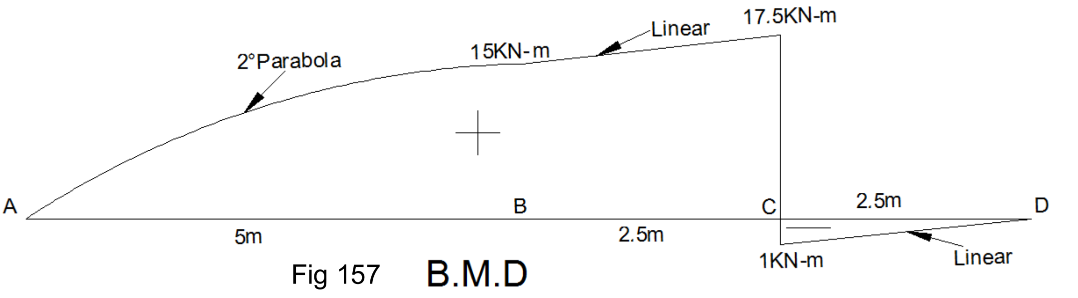


Fig 157

1. Starting from A and moving towards B, the variation of BMD is 2^0 parabola, but the slope at A is not zero, it means that there is a point load at A. if we start from A than we have 2 unknowns one intensity of udl and other point load so in this example we will start from D.

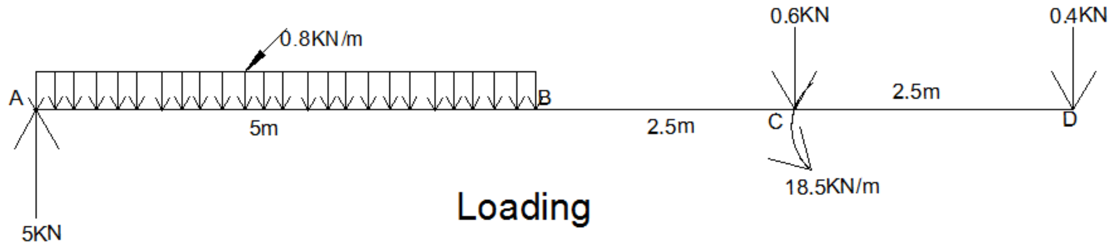
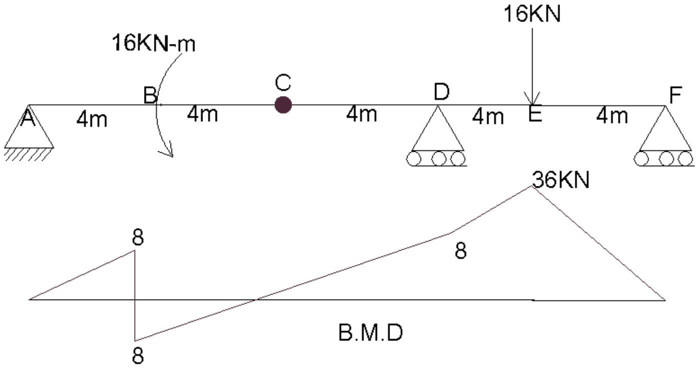


Fig 158

2. From D to C the bending moment is linear i.e no loading in between C to D, only point load at D. since bending moment sign is –ve hence point load act downward. The value of point load is 1/(2.5)=0.4KN.
3. At point C, we have a vertical ordinate of (17.5-(-1))=18.5, hence a concentrated moment is applied at C having magnitude of 18.5 and anticlockwise.
4. At point C, we have a point load since the slope of linear line between B&C and D&C is not same. Bending moment at B without considering point load at B is -0.4*5+18.5=16.5, but actual moment at B is 15 hence point load will act downward of value 1.5/2.5=0.6KN
5. From C to B, again variation of BMD is linear hence no loading.
6. From B to C, the variation of BMD is parabolic hence udl is acted in between B to A, since only one slope at B, no point load is acted at B. Bending Moment at A from left side without considering udl is 18.5-0.4*10-0.6*7.5=10, but bending moment at A is 0 udl act downward the intensity of udl $\Longrightarrow$ w*5*2.5=10$\Longrightarrow$ 0.8KN-m.
7. At point A, bending moment is having slope, hence point load at A. Assuming point load P_A to act upward and satisfying the $\Sigma V = 0 \Longrightarrow$ P_A-0.8*5-0.6-0.4=0$\Longrightarrow$ P_A=5KN

SHEAR FORCE & BENDING MOMENT DIAGRAM

Eample-30 Find the reaction and draw the bending moment diagram for the beam having internal hinges.



Taking moment about at left side of the internal hinge.

Σ Mc=0 $R_A * 8 - 16 = 0 \Longrightarrow R_A = 2KN$

Taking moment about right side of the internal hinge

Σ Mc=0 $R_D * 4 + R_F * 12 - 16 * 8 = 0$

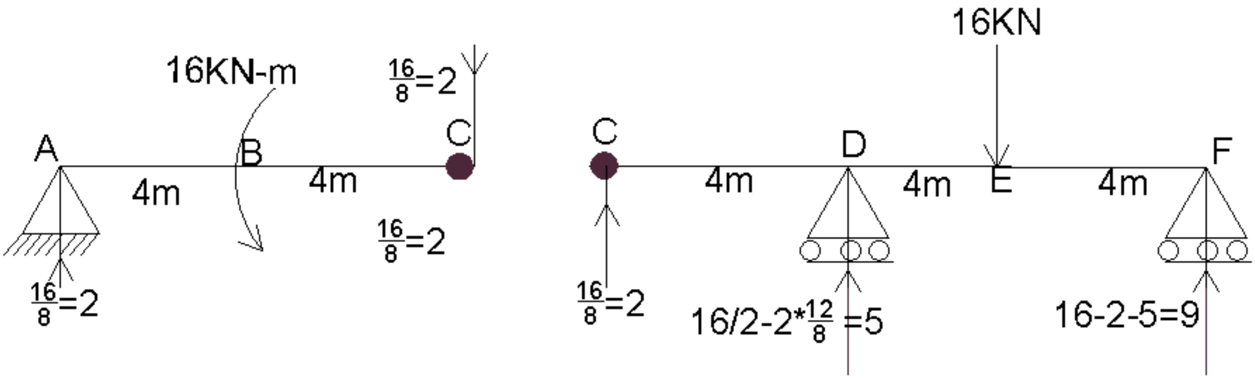
$$R_D + 3R_F - 32 = 0 \ldots\ldots\ldots..(1)$$

Taking moment about A ΣM_A=0

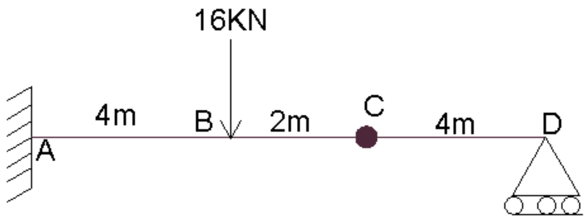
$$R_F * 20 + R_D * 16 - 16 * 16 - 16 = 0 \ldots\ldots\ldots(2)$$

Solving 1 and 2 $R_F = 9KN\ and\ R_D = 5KN$. The BMD is shown in the above Fig.

The free body diagram method to solve the above problem is as follows.



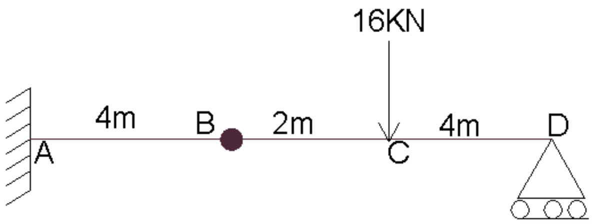
Example-31 Find the reaction for the beams shown in below



Since 'C' is a hinge, hence bending moment at 'C' is zero on either side of the 'C'.

Taking right side of the 'C' $\Sigma M = 0 \Rightarrow R_D * 4 = 0 \Rightarrow R_D = 0$

$\Sigma V = 0 \Rightarrow R_A = 16KN \quad M_A = 16 * 4 = 64KN - m\ (A.C.W)$



Since 'C' is a hinge, hence bending moment at 'C' is zero on either side of the 'C'.

Taking right side of the 'C' $\Sigma M = 0 \Rightarrow R_D * 6 - 16 * 2 = 0$

$$\Rightarrow R_D = 5.33KN$$

$$\Sigma V = 0 \Rightarrow R_A + R_D = 16KN \quad \Rightarrow R_A = 16 - 5.33 = 10.67KN$$

$$M_A = 16 * 6 - 5.33 * 10 = 42.7KN - m\ (A.C.W)$$

BENDING STRESSES

- **Assumptions**
  1. The material is isotropic, homogenous and obey hook's low (stress ∝ strain, and stress ≤ elastic limit)
  2. Plane transverse section remain plane and normal before and after the bending (as shown in Fig.1), hence effect of shear is ignored.
  3. Every layer of material is free to expand or contract longitudinally and laterally under stress and do not exert pressure upon each other.

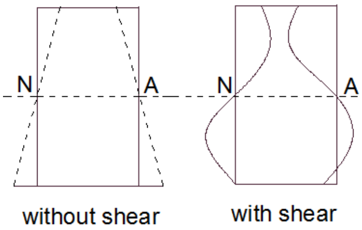


Fig.1

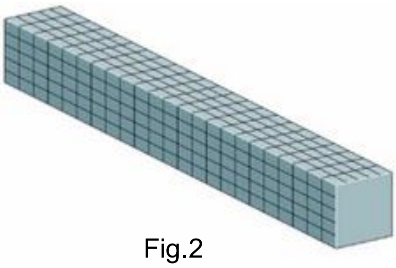


Fig.2

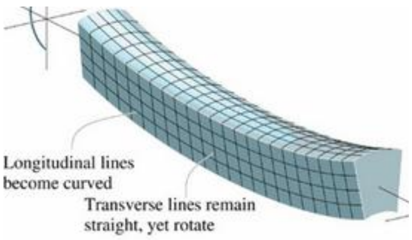


Fig.3

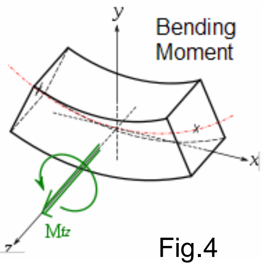


Fig.4

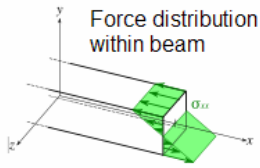


Fig.5

- When any member is acted by load which results in bending as shown in Fig.3 than due to bending the cross-section changes
- if the bending is Hogging than fiber above neutral axis is under tension and fiber below the neutral axis is under compression.
- If the bending is Sagging than fiber above neutral axis is under compression and fiber below the neutral axis is under tension as shown in Fig.4
- As shown in Fig.6 is a member having cross-section is acted by sagging moment than the extreme fiber will experience maximum change while as we move toward the neutral axis the change will be less and at neutral axis there will be no change.
- Hence, Neutral axis imply the location of zero stresses in the member subjected to pure bending.
- **Neutral axis always passes through Center of gravity.**
- As shown in Fig.7 the maximum strain in compression is e_c, corresponding to maximum stress f_c, and maximum strain in tension is e_t and corresponding to maximum stress f_t
- If the section having a cross section area A, is having a moment of resistance as M_r, than
- $M_r = f_C * A_c * (y_1 + y_2)$ or $M_r = f_t * A_t * (y_1 + y_2)$or $M_r = f_c * A_c * y_1 + f_t * A_t * y_2$
- $f_t = f_c$ (since no external force is applied in transverse direction)
- If at any distance y from N.A, the stress is 'f' than $\frac{M_r}{I} = \frac{f}{y} = \frac{E}{R}$

Where, M_r=moment of resistance, I=moment of intarsia at N.A E=Young Modulus, f= stress at a distance of y from neutral axis, R=Radius of curvature. $\frac{1}{R} = \rho$ (curvature)

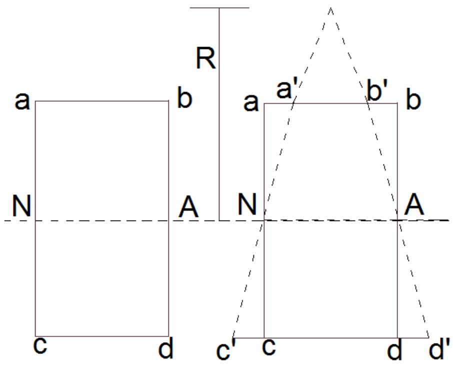


Fig.6

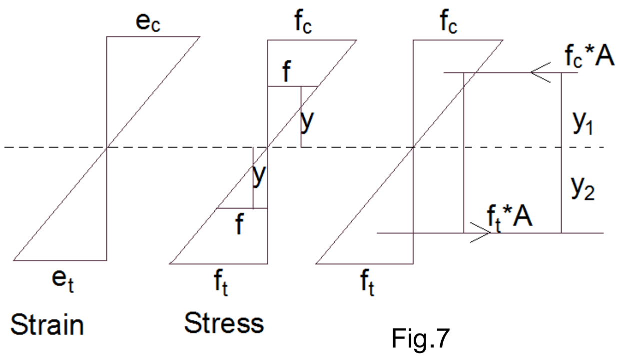


Fig.7

- $\frac{M_r}{I} = \frac{f_{max}}{y_{max}} \Longrightarrow M = \frac{f_{max}}{y_{max}} * I \Longrightarrow M_r = Z * f_{max}$

Where, Z is section modulus

- Greater the section modulus Z, greater will be the strength of the section.
- Z is a property of cross-section only i.e it only depend on geometry of the section.
- Z does not depend on loading, material.

Example1. Compare the flexural strength of three beam of equal weight.

(a) I-section 100mmX200mm having 10mm flange thickness and 8mm thickness.
(b) A rectangular section having depth equal to twice the width.
(c) Solid circular section.

Solution All the three beams having equal weight implies that all is having equal cross-sectional area.

(a) I section $A_1 = 2 * 100 * 10 + 180 * 8 = 3440\ \mathrm{mm}^2$

$I = \frac{1}{12}[BD^3 - bd^3] = \frac{1}{12} * (100 * 200^3 - 92 * 180^3) = 2195 * 10^4 \mathrm{mm}^4$

$Z_1 = 2195 * \frac{10^4}{100} = 219.5 * 10^3 \mathrm{mm}^3$

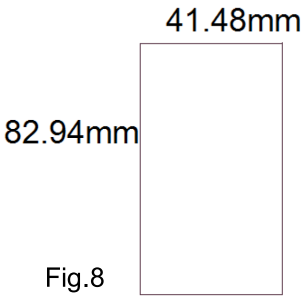


Fig.8

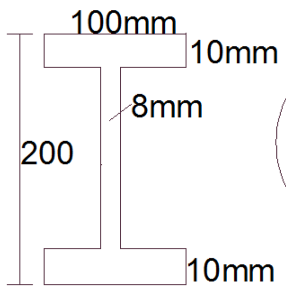


Fig.9

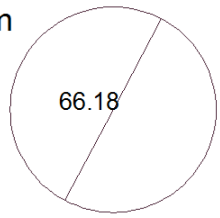


Fig.10

(b) Rectangular section $A_2 = bd = b * 2b = 2b^2 = A_1 = 3440 \Longrightarrow b = 41.473\mathrm{mm}\ \&\ d = 82.94\mathrm{mm}$

$$Z_2 = \frac{1}{6}bd^2 = \frac{1}{6} * 41.473 * 82.94^2 = 47.6 * 10^3\ \mathrm{mm}^3$$

(c) Circular cross-section $A_3 = \frac{\pi}{4} * d^2 = 3440 \Longrightarrow d = 66.18\mathrm{mm}$

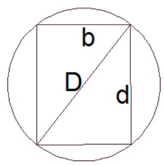
$$Z_3 = \frac{\pi}{32}d^3 = \frac{\pi}{32} * 66.18^3 = 28.5 * 10^3 \mathrm{mm}^3$$

$Z_1 > Z_2. > Z_3$, Hence, of equal weight I-section is strongest than rectangular and least is circular.

Example2 Find the width and depth of the strongest beam that can be cut out of a cylindrical log of wood, whose diameter is 'D'

Solution $b = \frac{D}{\sqrt{3}}, \quad \&\ d = D * \sqrt{\frac{2}{3}}$

Fig.11



Example3. A long rod of length 'L' 't' thickness is bent into a circular arc with central deflection 'd'. Find the longitudinal strain.

Solution when we bend the rod, this becomes a case of pure bending, let R be radius of curvature than

$\frac{f}{y} = \frac{E}{R} = \frac{M}{I} \Rightarrow \frac{f}{E} = \frac{y}{R} \quad \frac{f}{E} = e\ (\mathrm{Strain}\ ) = y/R,$ 'y'=t/2, $\quad e = \frac{t}{2R}$

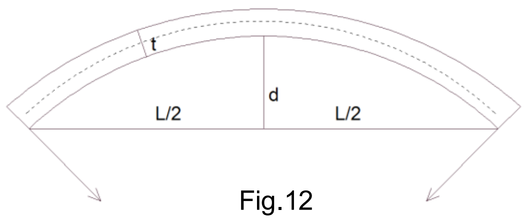


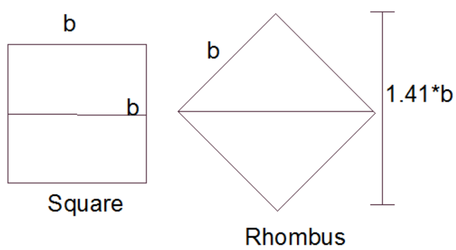
Fig.12

Using the property of chord $R = \frac{L^2}{8d}$ hence $e(\mathrm{strain}) = \frac{4td}{L^2}$

Example4. Compare the moment of resistance of a beam of square section made of same material but placed (a) with two sides horizontal (b) with a diagonal horizontal.

Case1. It is a square having $Z_1 = \frac{1}{12} * \frac{b^4}{\frac{b}{2}} = \frac{1}{6}b^3$

Fig.13



Case2. It a rhombus with one diagonal horizontal

$$I_{xx} = 2 * [\frac{1}{12} * b\sqrt{2} * \left(\frac{1}{2} * b\sqrt{2}\right)^3 = \frac{b^4}{12} \Rightarrow Z_2 = \frac{b^4}{12} * \frac{1}{\frac{b}{\sqrt{2}}} = \frac{b^3}{6\sqrt{2}}$$

$M_1 = f_{1max} * Z_1$ and $M_2 = f_{2max} * Z_2$ since both the material is same hence $f_{1max} = f_{2max}$

$\frac{M_1}{M_2} = \frac{Z_1}{Z_2} = \sqrt{2} = 1.414$ Hence, Z_1 is 41.4% more stronger than Z_2.

Example5. A simply supported beam of span 2m, subjected to U.D.L of 10KN/m. Find the maximum tensile stress and maximum compressive stress.

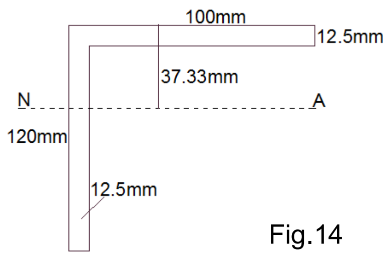


Fig.14

Solution $I_{xx} = 364.3 * 10^4 mm^4$ $(y_c)_{max} = 37.33mm$ and $(y_t)_{max} = 82.67mm$

Maximum bending moment $M = \frac{wl^2}{8} = 5 * 10^6$ N − mm

Maximum compressive stress $f_{cmax} = \frac{M}{I} * y_c = \frac{5*10^6}{364.33*10^4} * 37.33 = 51.24N/mm^2$

Maximum tensile stress $f_{tmax} = \frac{M}{I} * y_t = \frac{5*10^6}{364.33*10^4} * 82.67 = 113.46\ N/mm^2$

Example6. A steel wire of 10mm diameter is bent in a circular arc of 20m radius. Determine the maximum stress induced in it if E=$2 * 10^5 N/mm^2$.

Solution Diameter of wire 10mm ⇒ $y_{max} = 5mm$, R=20m, E= $2 * 10^5 N/mm^2$

$$\frac{f_{max}}{y_{max}} = \frac{E}{R} \Rightarrow f_{max} = \frac{5 * 2 * 10^5}{20000} = 50N/mm^2$$

- We can increase the moment of resistance by cutting upper and lower part of square section with the diagonal in the plane of bending as shown in Fig.15.
- If we cut the top and bottom edge by B/18 than section modulus will increase , the reason is $Z = \frac{I}{y}$, the distribution of mass is less as we move away from the N.A. i.e the 'I' decrease less but y decreases more hence Z increases.
- But after B/18, moment of inertia decreases more than y.
- The sectional modulus before cut is $Z_1 = \frac{B^3}{24}$ and after cut is $Z_2 = \frac{32}{729}B^3$ 5.35% more
- Similarly for circle as shown in Fig.16

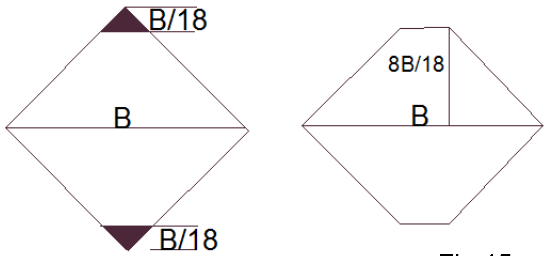


Fig.15

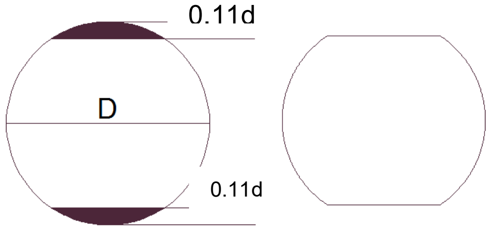


Fig.16

Beam of Uniform Strength

- A beam of uniform strength has the same maximum strength stress all along the length i.e the section will change along the length in such a way that the extreme fiber stress is constant all along the length.

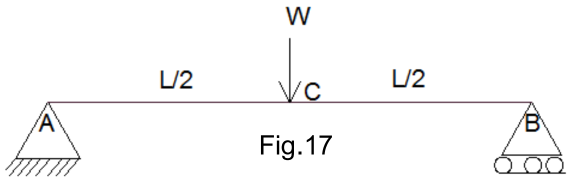


Fig.17

Case1. A simply supported beam with point load at the center. Find the section of the beam so that uniform strength is achieved along the span. (find the depth assume width to be constant=b).

- Let the top fiber stress is 'f' along the length and Z_x is section modulus at any distance 'x' from support A.

$$M_{rx} = fZ_x = f * \frac{1}{6} bd_x^2 = M_x = \frac{Wx}{2}$$

$\Rightarrow \frac{Wx}{2} = f * \frac{1}{6} bd_x^2 \Rightarrow d_x^2 = \frac{3Wx}{fb} \Rightarrow d_x = \sqrt{\frac{3W}{fb} * x}$ since $\sqrt{\frac{3W}{fb}}$ is constant for the given condition. Hence d_x varies square root of 'x'

Case2. For the same beam as in case1, find the section of the beam so that uniform strength is achieved along the span (find width assuming width to be constant 'd')

$M_{rx} = fZ_x = f * \frac{1}{6} b_x d^2 = M_x = \frac{Wx}{2}$ $\Rightarrow \frac{Wx}{2} = f * \frac{1}{6} b_x d^2 \Rightarrow b = \frac{3Wx}{fd^2} \Rightarrow b_x = \frac{3W}{fd^2} * x$ since $\frac{3W}{fd^2}$ is constant. Hence 'b' varies linearly with 'x'

- The section of uniform strength depend upon (1) type of loading, (2) amount of stress.

Example7. Find the force in 'ABCD' and 'EFGH' and their moment produced about N.A as shown in Fig.

4

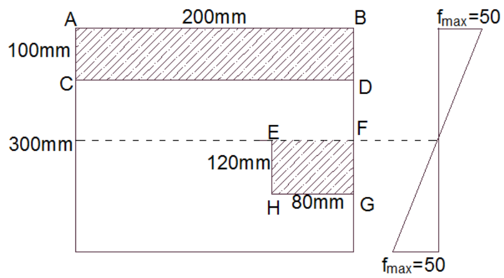


Fig.18

Solution

Force produced in any section is =Average stress x area of section

Stress at AB=50, at CD=50*150/50=16.67, EF=0, HG=50*120/150=40

$$F_{ABCD} = \frac{50+16.67}{2} * 200 * 100 = 666.67KN$$

$$F_{EFGH} = \frac{0+40}{2} * 80 * 120 = 192KN$$

Moment produced by section about N.A= Average stress x distance of centroid of the stress diagram of the section from N.A

$$M_{ABCD} = 666.67 * 108.33 = 72.22KN - m \qquad M_{EFGH} = 192 * \frac{2}{3} * 120 = 15.36KN - m$$

Example8 The cross-section of a beam is shown in Fig.19 the permissible stress in compression is 100N/mm^2 and in tension is 140N/mm^2. Calculate the moment of resistance of the section subjected to a sagging moment.

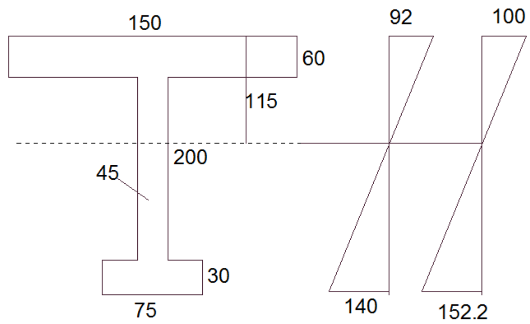


Fig.19

Solution $I_{xx} = 19372 * 10^4 mm^4$. $y_c = 115mm$ & $y_t = 175mm$.

If stress in compression reaches 100N/mm^2 than stress in tension will be 100*175*115=152.2>140 (permissible stress in tension) hence not possible.

If stress in tension reaches 140N/mm^2 than stress in compression will be 140*115/175=92N/mm^2 < 100 ok

Moment of resistance of the section is $\frac{140}{175} * 17372 * 10^4 = 139 * 10^6 \text{N} - \text{mm}$

Composite Section

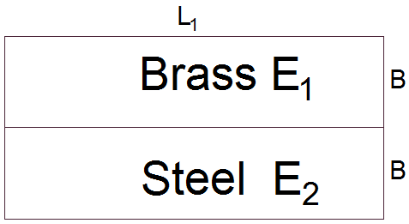


Fig.20

- When two different material having young modulus E_1, E_2 $(E_1 < E_2)$ and permissible stress (σ_1, σ_2) joined to make a composite section, the two members can't bend about their own axis but will bend about a common axis.
- Convert the two material into one material by changing the length dimension of one of the section. Mostly we change the dimension of material having more 'E'.
- The new dimension will be $L_2 = m * L_1$ where m is known as modular ratio, $m = E_1/E_2$ as shown in Fig.
- Next step, is find the C.G and moment of interial of the equivalent section.
- Check the permissible stresses at the respective points and find moment of resistance.

Example9 Two rectangular bars, one of brass and other of steel are placed together and firmly secured as shown in Fig. find the moment of resistance of the section if

Fig.21

$E_s = 2 * \frac{10^5 \text{N}}{\text{mm}^2}, E_b = 0.8 * \frac{10^5 N}{\text{mm}^2}$

$f_s, \max = \frac{120\text{N}}{\text{mm}^2}$ $f_b, \max = 75\text{N}/\text{mm}^2$

Solution since, both the material is firmly secured, both will bend about the common axis. In order to locate the N.A convert the section into equivalent section as shown in Fig.22

$m = \frac{E_s}{E_b} = 2.5$ the width of the equivalent section for steel will be 2.5*80=200mm as shown in Fig.22

Maximum permissible stress in brass with respect of steel is $\frac{f_b}{f_s} = \frac{E_b}{E_s} = \frac{1}{m} \Rightarrow f_b = \frac{120}{25} = \frac{48\text{N}}{\text{mm}^2} < \frac{75\text{N}}{\text{mm}^2}$ 5

$$I_{eqivalent} = 64.38 * 10^4 \text{mm}^4 \ \bar{y} = 15.71\text{mm}$$

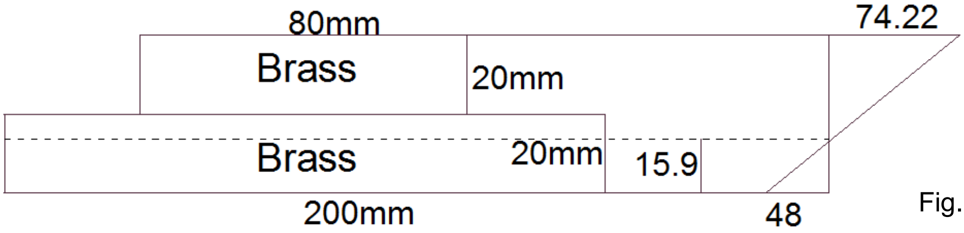


Fig.22

The stress diagram is shown in Fig.22 . the maximum stress in brass= $\frac{48}{15.71} * 24.29 = \frac{74.22\text{N}}{\text{mm}^2} < 75 \text{ OK}$

Moment of resistance of the section $M_r = 74.22 * 64.38 * \frac{10^4}{24.29} = 1.96 * 10^6 \text{N} - \text{mm}$

Unsymmetrical Bending

Till now we have considered only vertical loading (loading in Y-direction) but when beam is applied with loading in Y-direction and Z-direction as shown in Fig. 23

As shown in Fig.24 due to load F_y and F_z beam will bend in two planes and thus having two curvatures. As shown in Fig.24

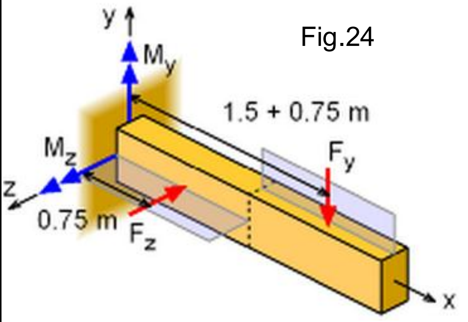
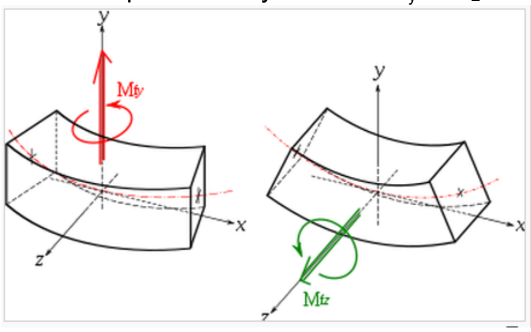


Fig.24

➢ Thus the total axial stresses at each end A,B,C,D will be algebraic sum of the stresses produced by moment M_y & M_z



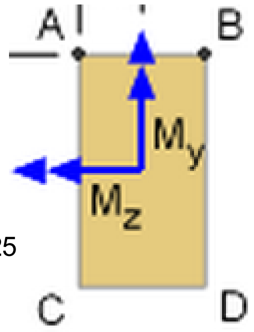


Fig.25

Equation of Neutral Axis

$$y = \left\{\frac{I_z}{I_y} tan\theta\right\} z$$

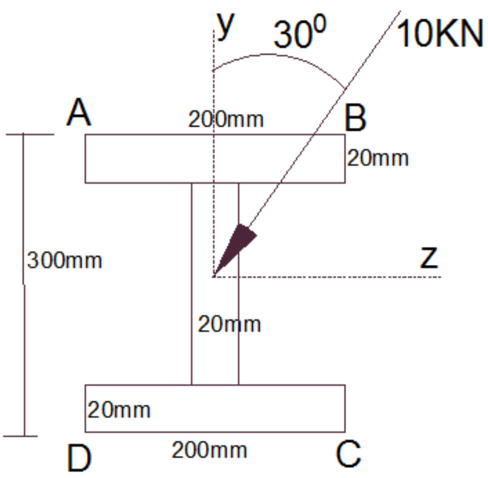
the angle of neutral axis with z-z $tan\emptyset = \frac{I_z}{I_y} tan\theta$

➢ If $I_z > I_y$ than $\emptyset > \theta$ and vice-versa

Example10 A cantilever beam of span 2m has inclined loading at the free end. The x-section of the beam is shown in Fig.25 Calculate the bending stress at the four corners A, B, C, D of the beam cross section at fixed end.

Solution at fixed end $M_z = 8.66 * 2 = 17.32KN - m$ & $M_y = 5 * 2 = 10KN - m$

$$I_z = \frac{200(300)^3}{12} - \frac{180(260)^3}{12} = 186.36 * 10^6 mm^4 \quad \& \quad I_y = 2 * \frac{20(200)^3}{12} + \frac{260(20)^3}{12} = 26.84 * 10^6 mm^4$$



Due to F_y , A & B point will be in tension and C & D will be in compression . Due to F_z B & C point will be in tension and A and D will be compression .

$$\sigma_x = \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad \frac{M_z y}{I_z} = \frac{17.32 * 10^6 * 150}{186.36 * 10^6} = \frac{13.94N}{mm^2}$$

$$\frac{M_y z}{I_y} = \frac{10 * 10^6 * 100}{26.84 * 10^6} = 37.25 N/mm^2$$

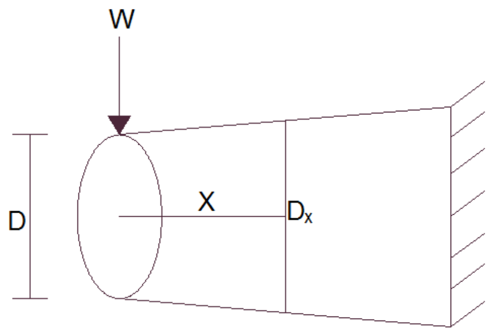
$$\sigma_{xA} = 13.94 - 37.25 = -\frac{23.32N}{mm^2} \qquad \sigma_{xB} = +13.94 + 37.25 = \frac{51.19N}{mm^2}$$

$$\sigma_{xC} = -13.94 + 37.25 = \frac{23.32N}{mm^2} \quad \sigma_{xD} = -13.94 - 37.25 = -\frac{51.9N}{mm^2}$$

Take Home Test

Question1. A rectangular steel beam 180x260mm in cross section is used as simply supported beam over a span of 8m and is subjected to a UDL of 1KN/m and also concentrated load of 5KN at 2m from each support inclined at 30^0 to the vertical axis. Determine the bending stress at the four corners of the beam and location of N.A of cross section.

Question2. A cantilever beam has a circular cross section of diameter 'D' at free end. Calculate the location at which maximum bending stress will act.



Column

- Any structural member carrying load axially in vertical direction are known as columns.
- Columns are classified by their mode of failure.
- Short Column fails by crushing, Long Columns fails by buckling, Intermediate column fails by combination of crushing and buckling.
- P_{cr} = Critical load, it is defined as smallest load at which buckled shape is possible is Critical load.

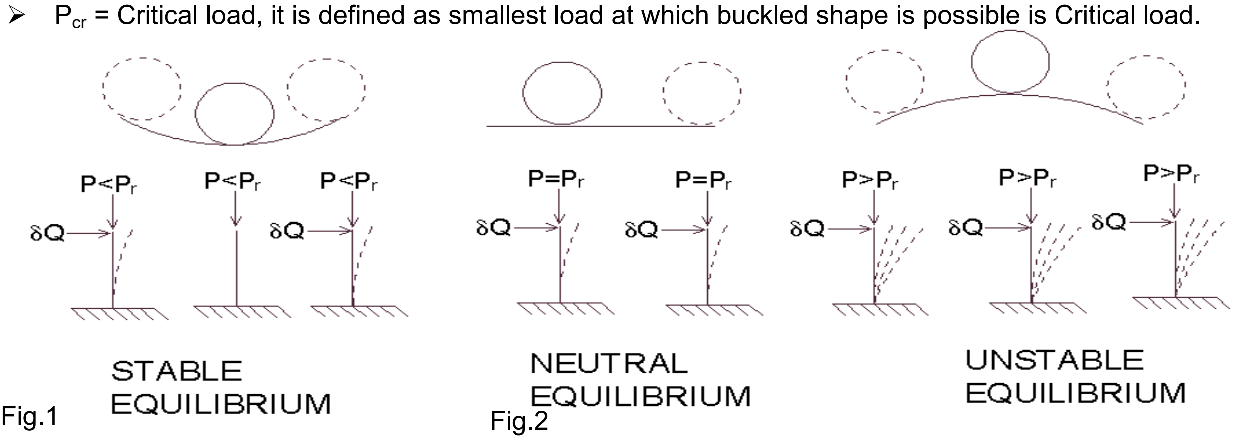
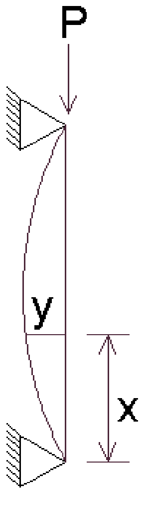


Fig.1 Fig.2 Fig.3

- $\text{Slenderness Ratio } (\lambda) = \frac{\text{Effective Length}}{\text{Least Lateral Dimension}} \text{ OR } \frac{\text{Effective Length}}{\sqrt{\frac{I}{A}}} \text{ OR } \frac{\text{Effective Lenght}}{\text{Radius of Gyration}}$
- $K \text{ or } r = \sqrt{\frac{I}{A}}$ is known as radius of gyration.
- If slenderness ratio is large, column will fail in buckling otherwise by crushing or yielding.

Euler's Theory



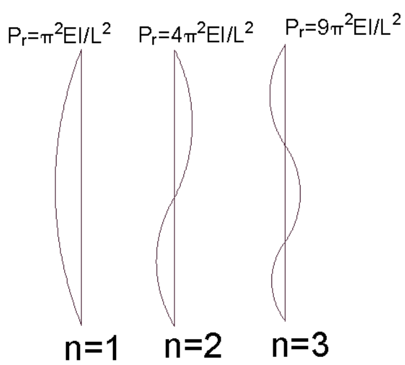
$$\frac{EId^2y}{dx^2} = M = +Py \Rightarrow \frac{d^2y}{dx^2} - \frac{Py}{EI} = 0$$

Solving above differential equation $y = C_1 \cos x\sqrt{\frac{P}{EI}} + C_2 \sin x\sqrt{\frac{P}{EI}}$

- Using end condition we get at ($x=0, y=0 \Rightarrow C_1=0$) and
- $x=L, y=0 \Rightarrow \text{Sin}L\sqrt{\frac{P}{EI}}=0 \Rightarrow L\sqrt{\frac{P}{EI}}=0, \pi, 2\pi$ …….
- $P = \frac{n^2\pi^2EI}{L_{eff}^2}$
- for n=1 $P_{cr} = P_E = \frac{\pi^2EI}{L_{eff}^2}$ (the smallest load at which buckle shape is possible)

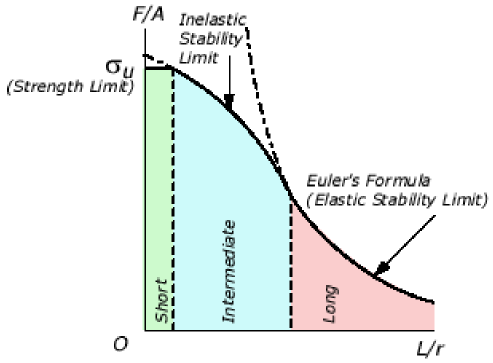
The critical load is also known as crippling load or Euler load $P_{cr} = P_E$

Fig.4



Limitation of Euler Formula

- The assumption that load is axial and no deflection of column before application of load is not practical.
- $\sigma_{critical} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L_{eff}^2 A} = \frac{\pi^2 E}{\left(\frac{L_{eff}}{r}\right)^2}$
- Euler formula can not be applied if $\lambda < \sqrt{\frac{\pi^2 E}{f_y}}$
- For mild steel having yield stress is 250N/mm^2 and corresponding $\lambda = 89.89$, Similarly for proportionality limit of 250N/mm^2 $\lambda = 99$ (approx)
- **Hence, Euler formula can only be applied for long column.**



- No factor of safety is taken into account.

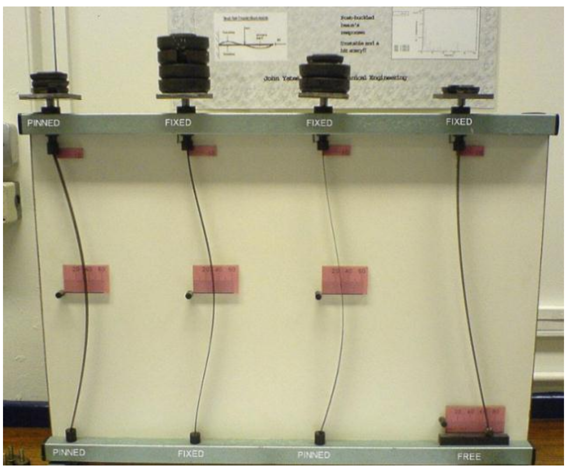
Effective length

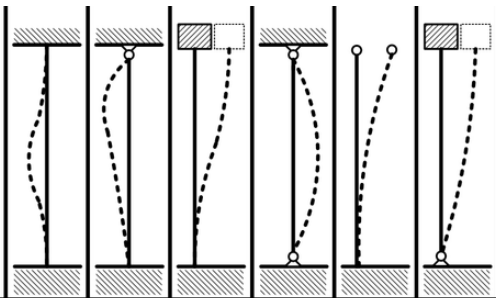
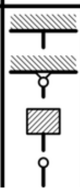
- **Effective length is the distance between two point of contra flexure on the buckling shape**.

$$\sigma_{critical} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L_{eff}^2 A} = \frac{\pi^2 E}{\left(\frac{L_{eff}}{r}\right)^2}$$

Where, $r_{min} = \sqrt{\frac{I_{min}}{A}}$ = radius of gyration

$\lambda = \frac{L_{eff}}{r_{min}} = \sqrt{\frac{\pi^2 E}{P_{cr}}}$ λ is known as slenderness ratio



Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0
End condition key		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

Rankine Formula

$$\frac{1}{P_r} = \frac{1}{P_c} + \frac{1}{P_E} \quad OR\ P_r = \frac{P_c * P_E}{P_E + P_c}$$

P_c is ultimate load which depend on ultimate crushing stress (constant for a material) $P_c = f_c * A$

P_E is crippling load by Euler and P_r is Rankine load.

I.S Code Method (Merchant Ranking Formula)

$\sigma_{ac} = \frac{0.6 f_{cc} f_y}{[f_{cc}^n + f_y^n]^{1/n}}$ where $f_{cc} = \frac{\pi^2 E}{\lambda^2}$ & f_y is yield stress

SELF BUCKLING

- A free standing vertical column with density ρ, Young Modulus 'E' and buckle under its own weight if its height exceed $\mathbf{H_{cr}} = \left(\frac{\mathbf{EI}}{\mathbf{4\rho g A}}\right)^{\mathbf{1/3}}$

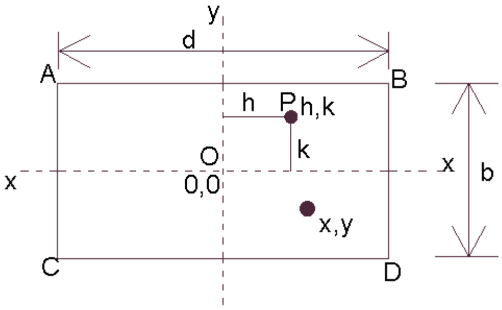
Euler postulated a theory for columns based on the following assumptions:

- Column is very long in proportion to its cross sectional dimensions
- Column is initially straight and the compressive load is applied axially
- Material of the column is elastic, homogeneous and isotropic
- Effect of direct stress is very small in comparison with bending stress.
- Column shall fail by buckling alone.
- Effect of self-weight of column is negligible

Combined Direct and Bending Stresses

Bending stress and direct stress changes the Neutral axis or sometime vanish the neutral axis.

A compressive member may be subjected to axial load which may not pass through its geometric axis, thus causes bending as well as direct stress.



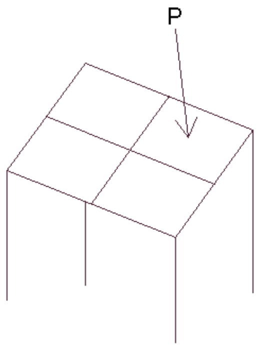


Fig.6

$$\sigma_{x,y} = \frac{P}{A} \pm \frac{(Ph)x}{I_y} \pm \frac{(Pk)y}{I_x}$$

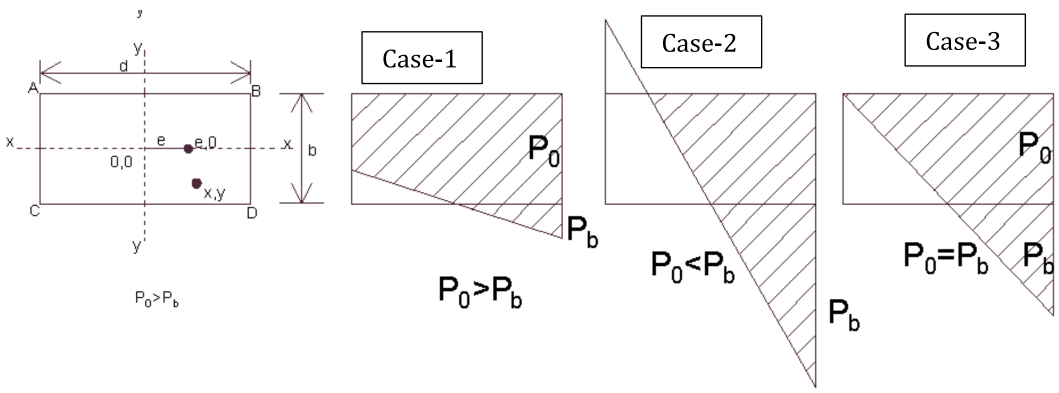
The value of x lies between +d/2 to –d/2

The value of 'y' lies between +b/2 to –b/2

- **If h=e and k=0**

$$\sigma_{x,y} = \frac{P}{A} \pm \frac{(Pe)x}{I_y} \quad \frac{P}{A} = P_o \text{ and } \frac{(Pe)x}{I_y} = P_b$$

x will be positive in OB and OD segment and negative in OA and OC, thus leading to reduce in P_o. as shown in Fig. the shaded portion shown the compression.



As shown in Fig. for case 1, the entire section is in compression but for case 3 the Point A is having neither tension nor compression and for case 2 the some part of section is in tension. Hence, the most favorable case is case1 when complete column will be in compression.

$$\frac{P}{A} > \frac{Pex}{I_y} \text{ for } x_{max} = -\frac{d}{2}, \frac{P}{A} > \frac{Ped}{2AK^2} \Rightarrow e_{max} \le \frac{2K^2}{d} \text{ or } e_{maz} \le \frac{Z}{A}$$

(A)For Rectangle

$$K = \sqrt{\frac{1*bd^3}{12bd}} = K = \frac{d^2}{12} \text{ and } e_{max} \le \frac{2K^2}{d} \Rightarrow e_{max} \le \frac{2d^2}{12d} \text{ or } e_{max} \le \frac{d}{6}$$

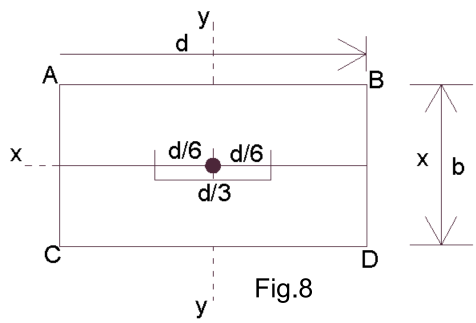


Fig.8

Middle Third Rule $e_{max} = \frac{d}{6}$, for no tension to develop in a rectangular section. Hence the stress will be of the same sign throughout the section if the load is within the middle third.

(B) For Circle $K = \frac{\pi d^4}{64} * \frac{d^4}{\frac{\pi}{4} * d^2} = \frac{d^2}{16}$ and $e_{max} \leq \frac{2d^2}{16d} \Rightarrow e \leq \frac{d}{8}$

Middle fourth Rule $e_{max} = \frac{d}{8}$ for no tension to develop in a rectangular section. Hence the stress will be of the same sign through the section if the load is within the middle fourth.

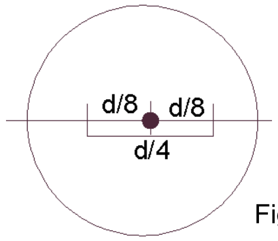


Fig.9

(C) For Hallow Circle $e \leq \frac{D^2+d^2}{8D}$

Core of Rectangle $\sigma_{x,y} = \frac{P}{A} \pm \frac{(Ph)x}{I_y} \pm \frac{(Pk)y}{I_x}$

$x_{max} - \frac{d}{2}$ & $y_{max} = -\frac{b}{2} \Rightarrow \sigma_{x,y} = P\left[\frac{1}{A} - \frac{6h}{bd^2} - \frac{6k}{db^2}\right] = 0$

the most favorable case is there is no tension in the section i.e $\sigma_{x,y} = 0$

which means $\left[\frac{1}{A} - \frac{6h}{bd^2} - \frac{6k}{db^2}\right] = 0$ for area A=bd

we get $\left[\frac{6h}{d} + \frac{6k}{b}\right] = 1$ for locus substitute (h,k)=(x,y) $\left[\frac{6x}{d} + \frac{6y}{b}\right] = 1$

this is a set of equation forming rhombus having length of diagonal b/6 and d/6 and side $=\sqrt{(b^2 + d^2)/6}$

This rhombus is called the core or kern. Hence stresses will be wholly compressive throughout the section, if the line of action of 'P' falls within the rhombus.

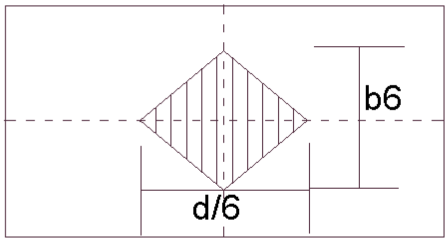


Fig.10

For circle the core is d/4 if 'd' is the diameter of circular column.

Example1. A short column of hallow circular section is attached with a projection bracket carrying a load of 100KN. The load line is off the axis of the column by 200mm. find the maximum and minimum stress intensity in the column, if its external diameter of 200mm and internal diameter of 150mm.

Solution $A = .7854 * (200^2 - 150^2) = 13744.5mm^2$ $Z = \frac{\pi}{32} * \frac{200^4 - 150^4}{200} = 536893mm^3$

$P_0 = \frac{P}{A} = \frac{100*10^3}{13744.5} = 7.28(comp)$ $P_b = \pm\frac{M}{Z} = \frac{100*10^3*200}{536893} = \pm 37.25N/mm^2$

$$f_{max} = 7.28 + 37.25 = 44.53\frac{N}{mm^2}\ (Compressive)\ \ \&\ f_{min} = 7.28 - 37.25 = -29.97\frac{N}{mm^2}\ (Tensile)$$

Example2. A load of 50KN act as shown in Fig.11 Find the stresses develop at each corners of the pier, what additional load should be placed at the center of the pier, so that there is no tension anywhere in the pier section.

Solution b=3m, d=2m, A=6m², $I_{xx} = \frac{1}{12} * 3 * 2^3 = 2m^4$ $I_{yy} = \frac{1}{12} * 2 * 3^3 = 4.5m^4$ $h = 0.4m$ $k = 0.8m$

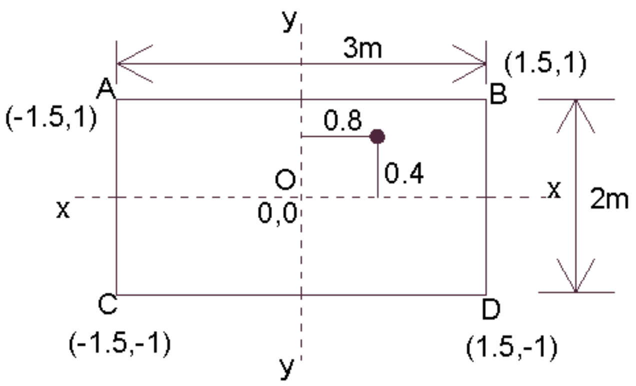


Fig.11

$\frac{(Ph)x}{I_y} = \frac{(50*0.8)*1.5}{4.5} = 13.33$ & $\frac{(Pk)y}{I_x} = \frac{(50*0.4)*1}{2} = 10$ & $\frac{P}{A} = \frac{50}{6} = 8.33$

$\sigma_{x,y} = \frac{P}{A} \pm \frac{(Ph)x}{I_y} \pm \frac{(Pk)y}{I_x}$

At A (-1.5,1) $\sigma_A = 8.33 + 10 - 13.35 = 5\text{KN/m}^2$ At B (1.5,1) $\sigma_B = 8.33 + 10 + 13.35 = 31.66\text{KN/m}^2$

At D (1.5,-1) $\sigma_D = 8.33 - 10 + 13.35 = 11.6\text{KN/m}^2$ At C (1.5,-1) $\sigma_C = 8.33 - 10 - 13.35 = -15\text{KN/m}^2$

Let 'W' be the additional load placed for no tension in the section than $\frac{W}{6} = 15 \Rightarrow W = 90\text{KN}$

Sectant Formula

$$\mathbf{M} = \frac{\mathbf{d^2y}}{\mathbf{dx^2}} = -\left(\frac{\mathbf{Pe}}{\mathbf{EI}} + \frac{\mathbf{Py}}{\mathbf{EI}}\right)$$

$$y_{max} = e * \sec\left(\frac{\pi}{2} * \sqrt{\frac{P}{Pcr}} - 1\right)$$

$$\sigma_{max} = \frac{P}{A} + \left(\frac{Pesec\frac{L_{eff}}{2}\sqrt{\frac{P}{EI}}}{Z}\right)$$