

LECTURE NOTES

ENGINEERING MECHANICS

(Th.4)

(Common to 1st & 2nd semester)

(DIPLOMA ENGINEERING)

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Th. 4. ENGINEERING MECHANICS

(2nd sem Common)

Theory: 4 Periods per Week
Total Periods: 60 Periods
Examination: 3 Hours

I.A : 20 Marks
End Sem Exam : 80 Marks
TOTAL MARKS : 100 Marks

Objective:

On completion of the subject, the student will be able to do:

1. Compute the force, moment & their application through solving of simple problems on coplanar forces.
2. Understand the concept of equilibrium of rigid bodies.
3. Know the existence of friction & its applications through solution of problems on above.
4. Locate the C.G. & find M.I. of different geometrical figures.
5. Know the application of simple lifting machines.
6. Understand the principles of dynamics.

Topic wise distribution of periods

Sl. No.	Topics	Periods
1	Fundamentals of Engineering Mechanics	14
2	Equilibrium	08
3	Friction	10
4	Centroid & moment of Inertia	14
5	Simple Machines	08
6	Dynamics	06
	TOTAL	60

1. FUNDAMENTALS OF ENGINEERING MECHANICS

1.1 Fundamentals.

Definitions of Mechanics, Statics, Dynamics, Rigid Bodies,

1.2 Force

Force System.

Definition, Classification of force system according to plane & line of action.

Characteristics of Force & effect of Force. Principles of Transmissibility & Principles of Superposition. Action & Reaction Forces & concept of Free Body Diagram.

1.3 Resolution of a Force.

Definition, Method of Resolution, Types of Component forces, Perpendicular components & non-perpendicular components.

1.4 Composition of Forces.

Definition, Resultant Force, Method of composition of forces, such as

1.4.1 Analytical Method such as Law of Parallelogram of forces & method of resolution.

1.4.2. Graphical Method.

Introduction, Space diagram, Vector diagram, Polygon law of forces.

1.4.3 Resultant of concurrent, non-concurrent & parallel force system by Analytical & Graphical Method.

1.5 Moment of Force.

Definition, Geometrical meaning of moment of a force, measurement of moment of a force & its S.I units. Classification of moments according to

direction of rotation, sign convention, Law of moments, Varignon's Theorem, Couple – Definition, S.I. units, measurement of couple, properties of couple.

2. EQUILIBRIUM

2.1 Definition, condition of equilibrium, Analytical & Graphical conditions of equilibrium for concurrent, non-concurrent & Free Body Diagram.

2.2 Lamia's Theorem – Statement, Application for solving various engineering problems.

3. FRICTION

3.1 Definition of friction, Frictional forces, Limiting frictional force, Coefficient of Friction.

Angle of Friction & Repose, Laws of Friction, Advantages & Disadvantages of Friction.

3.2 Equilibrium of bodies on level plane – Force applied on horizontal & inclined plane (up & down).

3.3 Ladder, Wedge Friction.

4. CENTROID & MOMENT OF INERTIA

4.1 Centroid – Definition, Moment of an area about an axis, centroid of geometrical figures such as squares, rectangles, triangles, circles, semicircles & quarter circles, centroid of composite figures.

4.2 Moment of Inertia – Definition, Parallel axis & Perpendicular axis Theorems. M.I. of plane lamina & different engineering sections.

5. SIMPLE MACHINES

5.1 Definition of simple machine, velocity ratio of simple and compound gear train, explain simple & compound lifting machine, define M.A, V.R. & Efficiency & State the relation between them, State Law of Machine, Reversibility of Machine, Self Locking Machine.

5.2 Study of simple machines – simple axle & wheel, single purchase crab winch & double purchase crab winch, Worm & Worm Wheel, Screw Jack.

5.3 Types of hoisting machine like derricks etc, Their use and working principle. No problems.

6. DYNAMICS

6.1 Kinematics & Kinetics, Principles of Dynamics, Newton's Laws of Motion, Motion of Particle acted upon by a constant force, Equations of motion, D'Alembert's Principle.

6.2 Work, Power, Energy & its Engineering Applications, Kinetic & Potential energy & its application.

6.3 Momentum & impulse, conservation of energy & linear momentum, collision of elastic bodies, and Coefficient of Restitution.

Syllabus coverage upto I.A

Chapter 1, 2 and 3.1

Books Recommended

1. Engineering Mechanics – by A.R. Basu (TMH Publication Delhi)
2. Engineering Machines – Basudev Bhattacharya (Oxford University Press).
3. Text Book of Engineering Mechanics – R.S Khurmi (S. Chand).
4. Applied Mechanics & Strength of Material – By I.B. Prasad.
5. Engineering Mechanics – By Timosheenko, Young & Rao.
6. Engineering Mechanics – Beer & Johnson (TMH Publication).

MECHANICS

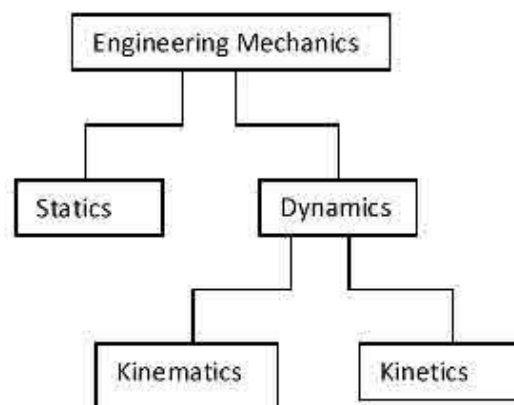
1. FUNDAMENTALS OF ENGINEERING MECHANICS

1.1 Fundamentals

Mechanics is that branch of science which deals with the forces and their effects on bodies on which they act and as a result the body may either be in rest or motion.

Mechanics is divided into two parts.

- (i) **Statics:-** Statics deals with the forces acting on a body under which the body is in rest.
- (ii) **Dynamics:-** Dynamics deals with the forces acting on a body under which it is in motion.



Further the Dynamics is divided into two parts

- (a) **Kinematics:-** Kinematics deals with the motion of bodies in which the agents responsible for motion are not considered.
- (b) **Kinetics:-** Kinetics deals with the motion of the bodies in which the agents responsible for motion are considered. It deals with the relationship between forces and the resulting motion of bodies on which they act.

RIGID BODY

A rigid body is one which does not change its shape and size under the effect of force acting over it. It differs from an elastic body in the sense that the latter undergoes deformation under the effect of forces acting on it and returns to its original shape and size on removal of the forces.

acting on the body. The rigidity of a body depends upon the fact that how far it undergoes deformation under the effect of forces acting on it.

In real sense no solid body is perfectly rigid because everybody changes its size and shape under the effect of forces acting on it. Actually the deformation in a rigid body is very small and is generally neglected.

DEFINATIONS OF SOME RELATED TERMS

Mass:- The amount of matter contained in a body is called its mass, and for most problems in mechanics, mass may be considered constant.

Weight:- The force with which a body is attracted towards the centre of earth by the gravitational pull (g) is called its weight.

The relation between Mass(M) and Weight(W) of a body is given by the equation $W=M \cdot g$

The value of $g = 9.81 \text{ m/s}^2$.

Length:- This term is applied to the linear dimensions of a straight or curved line. e.g. the diameter of a circle is the length of a straight line which divides the circle into two equal parts; the circumference is the length of its curved perimeter. Length is expressed in meter, cm, Km, feet etc.

Time:- The interval between two events is called time. It is expressed in Second, Minute and Hour.

Scalar:- Any physical quantity which has magnitude only is known as Scalar Quantity. Example:- Mass, distance, Volume, Time density etc.

Vector:- Any physical quantity which has magnitude as well as direction is known as Vector quantity. Example:- Force, Velocity, Displacement, Acceleration, Moment etc.

Fundamental units:- The basic quantities or fundamental quantities of mechanics are those quantities which cannot be expressed in terms of one another. Mass, Length, Time are usually considered as basic or fundamental quantities. The units of these quantities are called fundamental units and are denoted by M, L, T respectively.

Derived units:- The units of all other quantities except the fundamental Quantities are derived with the help of fundamental units and thus they are known as derived units. For Example units of Velocity, acceleration, Density etc are derived units and are as follows.

Velocity(V) = Displacement/Time = $L/T = LT^{-1}$.

Acceleration(a) = Velocity/Time = $LT^{-1}/T = LT^{-2}$.

$$\text{Density}(\rho) = \text{Mass/Volume} = M/L^3 = ML^{-3}.$$

SYSTEM OF UNITS:

Generally we use four system of units and they are as follows

1. Foot–Pound-Second(FPS) System:- In this system the units of fundamental quantities i.e Length, Mass and Time are expressed in Foot, Pound and Second respectively.
2. Centimeter- Gram-Second(CGS) System:- In this system the value of Length, Mass and Time are expressed as Centimeter, Gram and Second.
3. Meter-Kilogram-Second(MKS) System:- In this system units of Length, Mass and Time are expressed in Meter, Kilogram and second respectively.
4. International System of units(S I Unit):- This system considers three more fundamental units of Electric Current, Temperature and Luminous intensity in addition to the fundamental units of Mass, Length and Time.

FORCE

Defination of force and its units:-

Force:- Force is something which changes or tends to change the state of rest or of uniform motion of a body in a straight line. Force is the direct or indirect action of one body on another. It is a vector quantity.

There are different kinds of forces such as Gravitational, Frictional, Magnetic, Inertia or those caused by Mass and acceleration.

Units of Force:-

Absolute units:- Because the mass and acceleration are measured differently in different systems of units, so the units of force are also different in the various systems as below

FPS System:- FL/S^2 . (Poundal)

CGS System:- Cm/S^2 . (Dyne)

MKS or S I System :- M/S^2 . (Newton)

1 Newton = 10^5 Dynes.

Gravitational Units:- These are the units which are used by engineers for all practical purposes, these units depends upon the weight of a body. Now the weight of the body of mass (m) = mg, where g= Acceleration due to gravity.

So the gravitational units of force in the three systems of units i.e, FPS,CGS,and MKS are Pound weight, Gram weight and Kilogram weight.

The relationship of units of force is as follows

1 lb wt. = g Poundal = 32.2 Poundals

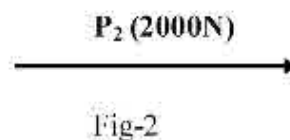
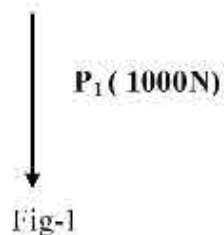
1 gm wt. = g Dynes = 981 Dynes

1 Kg wt. = g Netwon = 9.81 Newtons.

Gravitational units of force= 'g' times the corresponding absolute units of force.

Representation of Force by Vector:-

Vector Representation:- A force can be represented graphically by a vector as shown in Fig-1 and Fig-2.



CHARACTERISTICS OF FORCE:-

The characteristics or elements of the force are the quantities by which a force is fully represented. These are (i) Magnitude(i.e, 500 N, 1000N etc) (ii) Direction or line of action(Angle relative to a coordinate System) (iii) Sense or nature (Push or Pull) (iv) Point of application.

Effects of Force:-

When a force acts on a body, the effects produced in that body may be as follows:-

- (i) It may bring a change in the motion of the body i.e, the motion may be accelerated or retarded.
- (ii) It may balance the forces already acting on the body thus bringing the body to a state of rest or of equilibrium, and

- (iii) It may change the size or shape of the body i.e, the body may be twisted , bend, stretched, compressed or otherwise distorted by the action of the force.

FORCE SYSTEMS

A force system is a collection of forces acting on a body in one or more planes.

According to the relative positions of the lines of action of the forces, the forces may be classified as follows.

1. Coplanne Concurrent Colinear Force system:

It is the simplest force system and includes those forces whose vectore lie along the same straight line .



Fig-3

2. Coplanar Concurrent Non- Parallel Force System:-

Forces whose lines of action pass through a common point are called Concurrent forces. I

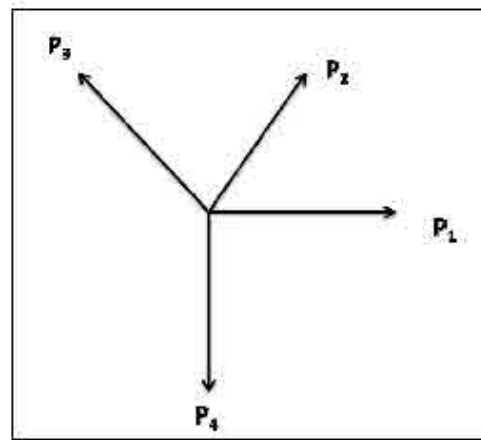


Fig-4

In this system lines of action of all the forces meet at a point but have different directions in the same plane are shown as in the figure.

3. Coplanar Non- Concurrent Parallel Force system.

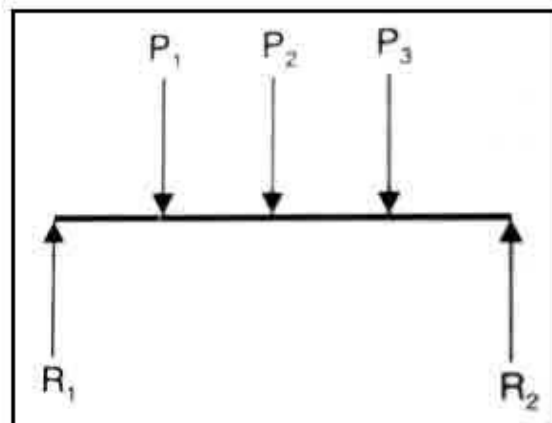


Fig-5

In this system, the lines of action of all the forces lie in the same plane and are parallel to each other but may not have same direction as shown in the figure.

4. Coplanar Non- Concurrent Non- Parallel Force system:-

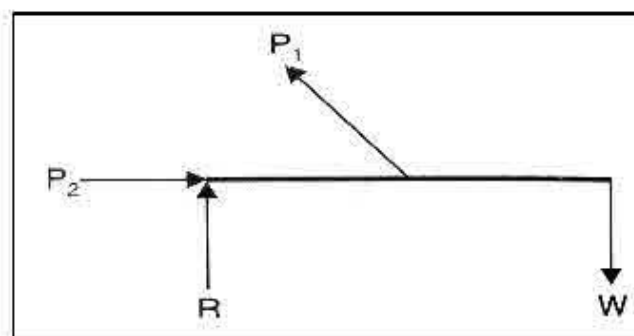


Fig-6

Such a system exists where the lines of action of all forces lie in the same plane but do not pass through a common point as shown in the figure.

5. Non- Coplaner Concurrent Force System:-

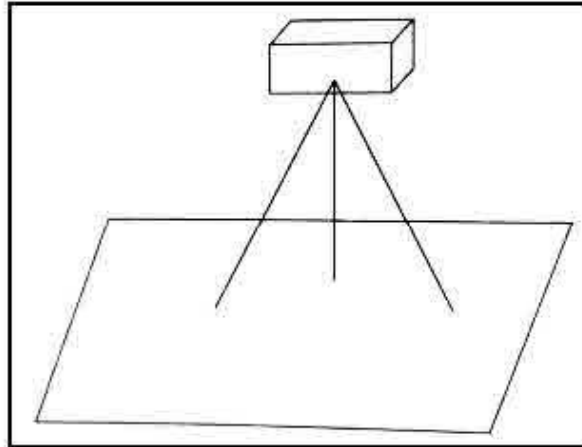


Fig-7

This system is evident where the lines of action of all forces do not lie in the same plane but do pass through a common point. An example of this force system is the forces in the legs of tripod support of a dumpy level.

6. Non- Coplaner Non- Concurrent Force system:-

Where the lines of action of all forces do not lie in the same plane and do not pass through a common point, a Non- Coplaner Non- Concurrent system is present.

Principle of transmissibility:- The Principle of Transmissibility of force states that when a force acts upon a body, its effect is the same whatever point in its line of action is taken as the point of application provided that the point is connected with the rest of the body in the same invariable manner.

Laws of Superposition:-

The action of a given system of force on a rigid body will in no way be changed if we add or subtract from them another system of forces in equilibrium.

Action & Reaction Forces:- Whenever there are two bodies in contact, each exerts a force on the other. Out of these forces one is called action and the other is called reaction. Action and reaction are equal and opposite.

FREE BODY DIAGRAM:-

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces. A free body diagram is a process of isolating a body from all of its supports and in the place of the support a force of required magnitude is provided so that the position of the body will not change.

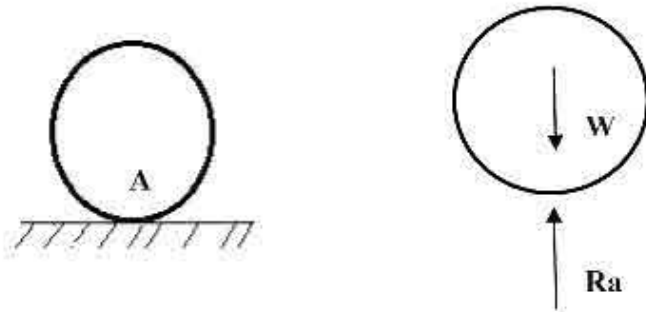


Fig-a

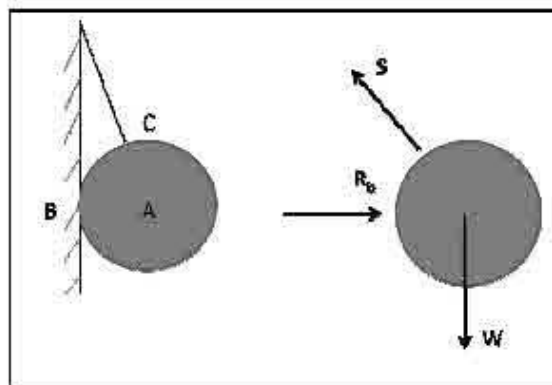


Fig-b

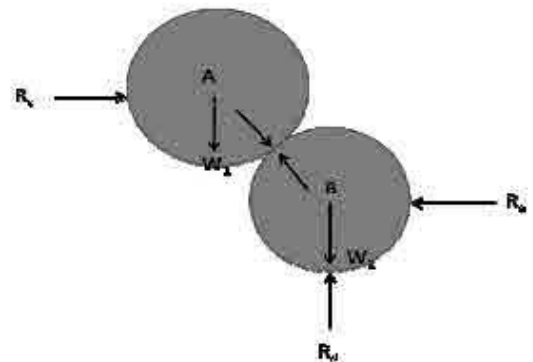
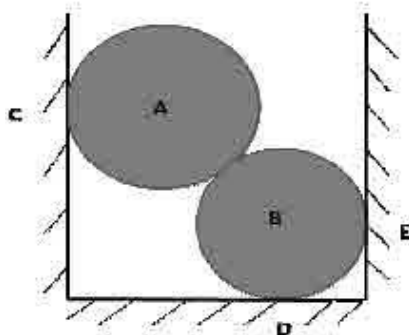


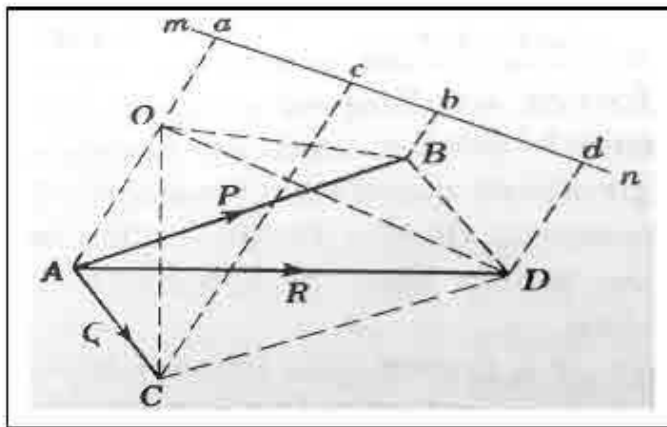
Fig-c

Theorem of Varignon:-

The moment of the resultant of two concurrent forces with respect to a center in their plane is equal to the algebraic sum of the moments of the components with respect to the same center.

Proof:-

Let us consider two forces 'P' and 'Q' with respect to the center 'O'.



In the plane of action of forces, we take any line mn perpendicular to the line OA joining the moment center with the point of concurrence of the forces and construct the perpendiculars Aa, Bb, Cc and Dd , as shown in figure.

Now the area of $\triangle OAB = \frac{1}{2} OA \cdot ab$, the area of $\triangle OAC = \frac{1}{2} OA \cdot ac$, and the area of $\triangle OAD = \frac{1}{2} OA \cdot ad$.

Since $ad = ab + bd = ab + ac$

We conclude that Area $\triangle OAD = \text{area } \triangle OAB + \text{area } \triangle OAC$.

This proves the theorem.

1.3 RESOLUTION OF A FORCE

As two forces acting simultaneously on a particle acting along directions inclined to each other can be replaced by a single force which produces the same effect as that of the given force similarly a single force can be replaced by two forces acting in directions which will produce the same effect as that of the given force.

This breaking up of a force into two parts is called the resolution of a force.

RESULTANT OF A FORCE

A resultant force is a single force which can replace two or more forces and produce the same effect on the body as that of the forces.

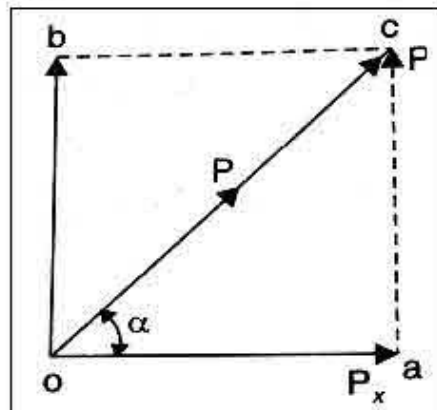
COMPONENT OF A FORCE

Generally a force is resolved into the following two types of components.

1. Mutually perpendicular components.
2. Non- Perpendicular components.

1. Mutually Perpendicular Components:-

Let us consider a force 'P' which is to be resolved is represented in both magnitude and direction by 'oc' in the figure below.



Let P_x is the component of force P in the direction oa making an angle ' α ' with the direction oc.

Complete the rectangle oacb.

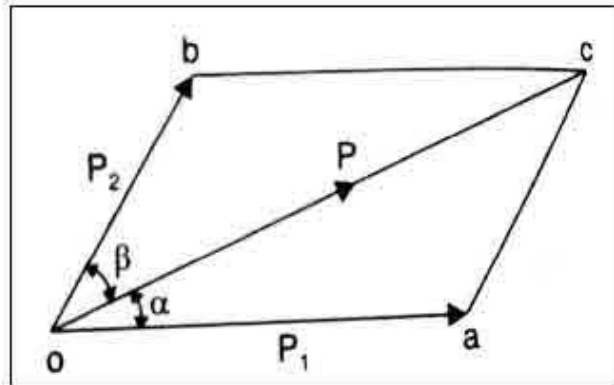
The other component P_y at right angle to P_x will be represented by 'ob' which is also equal to ac.

Now from the right angled triangle oac,

$$P_x = oa = P \cos \alpha \quad \text{and} \quad P_y = P \sin \alpha.$$

2. Non- Perpendicular Component.

Referring figure below Let oc represents the given force P in magnitude and direction to some scale.



Draw oa and ob making angle ' α ' and ' β ' with oc .

Through c draw ca parallel to ' ob ' and ' cb ' parallel to ' oa ' to complete the parallelogram ' $oacb$ '.

Now the vectors oa and ob represent in magnitude and direction as P_1 and P_2 respectively.

Now from the triangle ' oac ', by applying sine rule

$$\frac{oa}{\sin \beta} = \frac{oc}{\sin[180-(\alpha+\beta)]} = \frac{oc}{\sin \alpha}$$

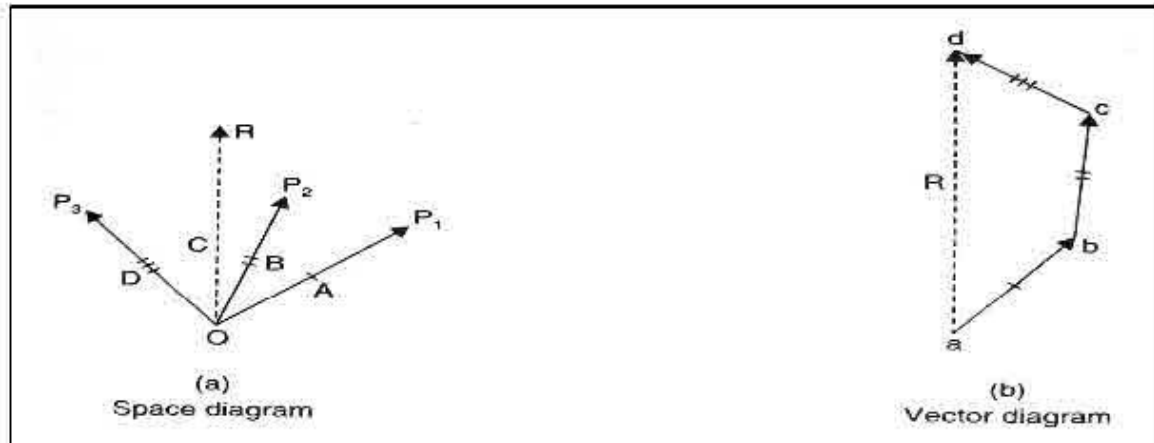
$$\text{or } \frac{P_1}{\sin \beta} = \frac{P}{\sin(\alpha+\beta)} = \frac{P_2}{\sin \alpha}$$

$$\text{so } P_1 = P \cdot \frac{\sin \beta}{\sin(\alpha+\beta)} \quad \text{and} \quad P_2 = P \cdot \frac{\sin \alpha}{\sin(\alpha+\beta)}$$

RESULTANT OF SEVERAL COPLANNER CONCURRENT FORCES:-

For the resultant of a number of concurrent forces any of the following two methods are used.

1. Graphical Method



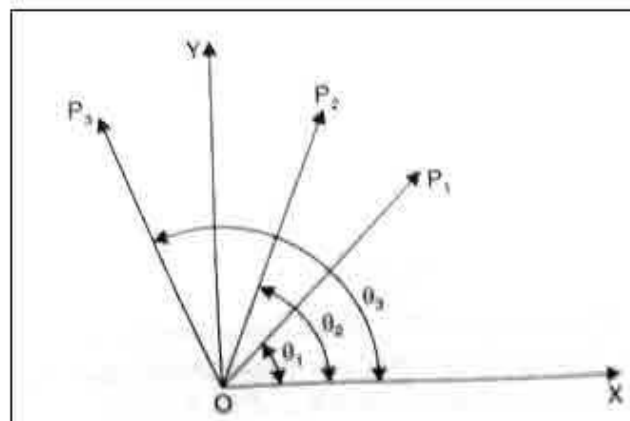
The figure above shows the forces P_1 , P_2 and P_3 simultaneously acting at a point O .

Draw vector ab equal to force P_1 to some scale and parallel to the line of action of P_1 .

From 'b' draw vector ' bc ' to represent force P_2 in magnitude and direction.

Now from 'c' draw vector ' cd ' equal and parallel to P_3 . Join ' ad ' which gives the required resultant in magnitude and direction as per fig-(b) the vector diagram above.

2. Analytical Method



The resolution of P_1 in the direction OX is $P_1 \cos \theta_1$, and in the direction OY is $P_1 \sin \theta_1$.

The resolution of P_2 in the direction OX is $P_2 \cos \theta_2$, and in the direction OY is $P_2 \sin \theta_2$.

And the resolution of P_3 in the direction OX is $P_3 \cos\theta_3$, and in the direction OY is $P_3 \sin\theta_3$.

If the resultant R makes an angle θ with OX, then as per resolution,

$$R \cos\theta = P_1 \cos\theta_1 + P_2 \cos\theta_2 + P_3 \cos\theta_3 = \sum H$$

$$\text{And } R \sin\theta = P_1 \sin\theta_1 + P_2 \sin\theta_2 + P_3 \sin\theta_3 = \sum V$$

$$\text{Now Resultant } R = \sqrt{(\sum H^2) + (\sum V^2)}$$

$$\text{Now } \frac{R \sin\theta}{R \cos\theta} = \frac{\sum V}{\sum H}$$

$$\text{So } \theta = \tan^{-1} \left(\frac{\sum V}{\sum H} \right)$$

Sign Convention for resolution:-

The upward forces ($\uparrow$) is considered - Positive (+)

The downward forces ($\downarrow$) is considered - Negative (-)

The Right Hand Side forces is - Positive (+)

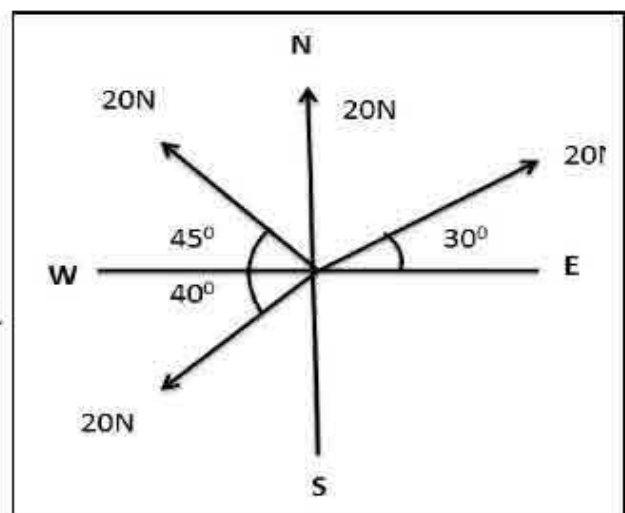
The left Handed forces is - Negative (-)

Example -1

The following forces act a point:

- (i). 20N inclined at 30° towards north of east.
- (ii). 25N towards north
- (iii). 30N towards north west and
- (iv). 35N inclined at 40° towards south of west.

Find the magnitude and direction of the resultant force.



Answer:-

Magnitude of the resultant force

Resolving all the forces horizontally i.e. along East-west line,

$$\begin{aligned}EH &= 20 \cos 30^\circ + 20 \cos 90^\circ + 30 \cos 135^\circ + 35 \cos 220^\circ \text{ N} \\&= (20 \times 0.866) + (25 \times 0) + 30(-0.707) + 35(-0.766) \text{ N} \\&= -30.7 \text{ N}\end{aligned}$$

And now resolving all the forces vertically i.e. along north-south line,

$$\begin{aligned}EV &= 20 \sin 30^\circ + 25 \sin 30^\circ + 30 \sin 135^\circ + 35 \sin 220^\circ \text{ N} \\&= (20 \times 0.5) + (25 \times 1.00) + (30 \times 0.707) + 35(-0.6428) \text{ N} \\&= 33.7 \text{ N}\end{aligned}$$

We know that the magnitude of the resultant force,

$$\begin{aligned}R &= \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} \text{ N} \\&= 45.6 \text{ N} \quad \dots\dots\dots (\text{Ans.})\end{aligned}$$

Direction of the resultant force

Let θ = Angle, Which the resultant force makes with the East.

$$\text{Thus, } \tan \theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098 \text{ or } \theta = 47^\circ 42'$$

Since $\sum H$ is - Ve and $\sum V$ is +Ve, therefore θ lies between 90° and 180° .

$$\text{Thus, Actual } \theta = 180^\circ - 47^\circ 42' = 132^\circ 18' \quad \dots\dots\dots (\text{Ans.})$$

Example-2

Determine the magnitude and direction of the resultant of the two forces of magnitude 12 N and 9 N acting at a point, if the angle between the two forces is 30° .

Given:-

$$F_1 = 12\text{ N} \quad F_2 = 9\text{ N} \quad \alpha = 30^\circ$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$$R = \sqrt{12^2 + 9^2 + 2 \times 12 \times 9 \times \cos 30^\circ}$$

$$R = 20.3\text{ N}$$

$$\theta = \tan^{-1} \left(\frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \right)$$

$$\theta = \tan^{-1} \left(\frac{9 \sin 30^\circ}{12 + 9 \cos 30^\circ} \right)$$

$$\theta = 12.81^\circ$$

Example-3

2. Find the magnitude of two equal forces acting at a point with an angle of 60° between them, if the resultant is equal to $30\sqrt{3}\text{ N}$
GIVEN:

$$F_1 = F_2 = F, \text{ say}$$

$$R = 30\sqrt{3}\text{ N}, \alpha = 60^\circ$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$$R = \sqrt{F^2 + F^2 + 2F \times F \times \cos 60^\circ}$$

$$R = \sqrt{F^2 + F^2 + F^2}$$

$$R = \sqrt{3}F$$

$$F = 30\text{ N}$$

Example-4

Two forces of magnitudes $3P$ and $2P$ respectively acting at a point have a resultant R . If the first force is doubled, the magnitude of the resultant is doubled. Find the angle between the two forces ($3P$ and $2P$).

Ans:- Let θ = angle between two forces $3P$ and $2P$.

$$\begin{aligned}\text{We Know that } R^2 &= (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta \\ &= 9P^2 + 4P^2 + 12P^2\cos\theta \\ &= 13P^2 + 12P^2\cos\theta \quad \dots\dots\dots(i)\end{aligned}$$

Now doubling the first force and the resultant both, we have

$$\begin{aligned}(2R)^2 &= (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta \\ \Rightarrow 4R^2 &= 36P^2 + 4P^2 + 24P^2\cos\theta = 40P^2 + 24P^2\cos\theta \quad \dots\dots\dots(ii)\end{aligned}$$

Substituting the value of R^2 from eqn (i) in eqn (ii), we get

$$\begin{aligned}4(13P^2 + 12P^2\cos\theta) &= 40P^2 + 24P^2\cos\theta & \Rightarrow 52P^2 + 48P^2\cos\theta &= 40P^2 + 24P^2\cos\theta \\ \Rightarrow 12P^2 &= -24P^2\cos\theta & \Rightarrow \cos\theta &= -1/2.\end{aligned}$$

Hence $\theta = \cos^{-1}(-1/2) = 120^\circ$. (Ans)

1.4 MOMENT OF FORCE

The moment of a force about a point is the product of the force and the perpendicular distance from the point of rotation. A moment is a measure of rotation about a point. A moment has both magnitude and direction.

GEOMETRICAL REPRESENTATION OF MOMENT

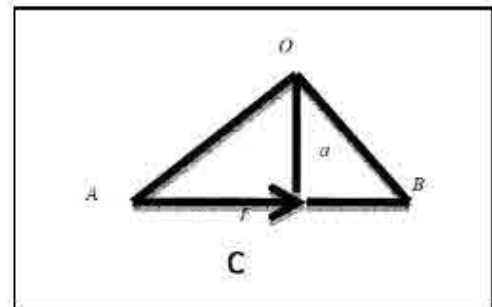
Consider a force F represented, in magnitude and direction by the line AB . Let O be a point about which the moment of the force F is required. Let OC be the perpendicular drawn. Join OA and OB

Moment of force F about $O = F \times a$

$= AB \times OC$

$= \text{twice the area of triangle } OAB$

Thus moment of F about $O = 2 \times \text{Area of triangle } OAB$



1. Moment Theory

The moment of a force about a point is a measure of the tendency of that force to rotate about that point.

For example, the moment of force F about point “O” in Figure 1(a) is a measure of the tendency of the force to rotate the body about line A-A. Line A-A is perpendicular to the plane containing force F and point “O”.

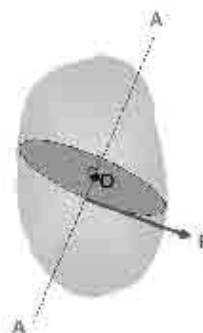


figure 1(a)

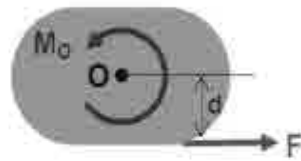


Figure 1(b)

Figure 1(b) shows the plane containing F and "O". Point "O" is called the moment centre, distance "d" is called the moment arm and line A-A is called the axis of the moment.

2. Moment Formula

The moment of a force is the product of that force by its perpendicular distance from the point of rotation.

Moment = Force $\times$ $\perp$ Distance

$$M = F \times \perp d$$

Note: The sum of moments about a point is equal to zero.

$$\sum M_O = 0$$

3. The Unit Of A Moment

The measurement unit of a moment is the Newton metre (Nm).

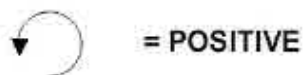
4. Moment sign convention

When dealing with moments it is important to apply a CONSISTENT sign convention with the direction of the moment.

Look at which way the moment is turning. Is it turning CLOCKWISE or ANTICLOCKWISE?

For the purposes of this activity, let us apply a sign convention:

If a moment is moving ANTICLOCKWISE it is considered a POSITIVE moment.



If a moment is moving **CLOCKWISE** it is considered a **NEGATIVE** moment.



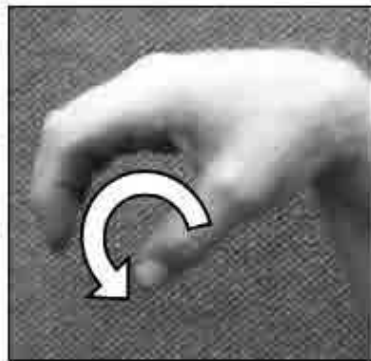
An easy way to remember this sign convention is by using your **RIGHT HAND** as a guide.

For a **POSITIVE** moment turning in an **ANTICLOCKWISE** direction:

Position your hand as shown in **Picture 1**.

Following the arrow with your fingertips and imagine that you are turning a doorknob in an **ANTICLOCKWISE** direction.

You will notice that your hand is turning in the direction where you have more fingers.



Picture 1

For a **NEGATIVE** moment turning in a **CLOCKWISE** direction, the same thinking applies:

Turn the doorknob in the opposite direction (i.e. the **CLOCKWISE** direction).

- Using your thumb as the guide, turn your hand to follow the arrow as shown in **picture2**.



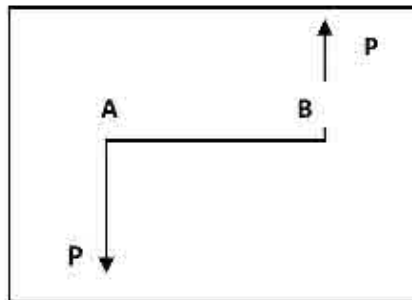
Picture 2

- Once again you should notice that you are turning your hand in the direction that has LESS fingers, that is, the NEGATIVE direction.

Also you can adopt the opposite sign convention but you have maintain it throughout the problem.

COUPLE:-

A couple is pair of two equal and opposite forces acting on a body in such a way that the line of action of the two forces are not in the same straight line.



The effect of couple acting on a rigid body is to rotate it without moving it as a whole.

The perpendicular distance between the line of action of two forces forming the couple is known as *arm of couple*.

The moment of a couple is known as torque which is equal to one of the forces forming the couple multiplied by arm of couple.

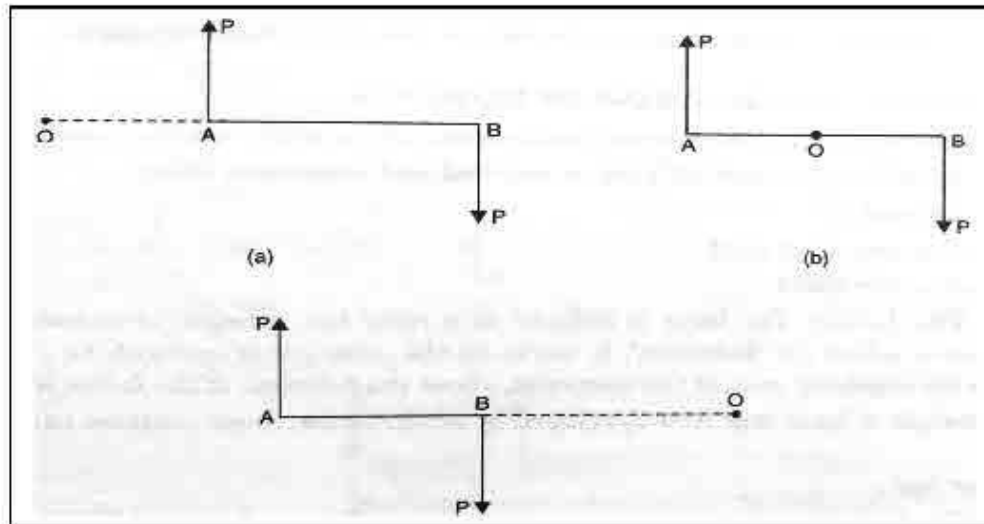
The following are the example of the couples in everyday life.

1. Opening or closing a water tap. The two forces constitute a couple as shown in Fig. 3.13.
2. Turning of the cap of a pen.
3. Unscrewing the cap of a Ink bottle.
4. Twisting of a screw driver.
5. Steering a motor-car (Fig. 3.14)
6. Winding a watch or clock with a key.

PROPERTIES OF A COUPLE:-

1. The algebraic sum of the moments of the forces forming a couple about any point in their plane is constant.

Let two parallel and unlike forces be of magnitude P is forming a couple $P \times AB$, where points A and B are the points where forces P and P act.



From Fig-(a)- Moments about $O = P \times OB - P \times OA = P(OB - OA) = P \times AB$

From Fig-(b)- Moments about $O = P \times OB + P \times OA = P(OB + OA) = P \times AB$

From Fig-(c)- Moments about $O = P \times OA - P \times OB = P(OA - OB) = P \times AB$

In all the three cases, we found that the sum of the moments in each case is independent of the position of the point O , and depends only on the constant arm of the couple, so the algebraic sum of the moments of the forces forming a couple about any point in their plane is constant.

2. Any two couples of equal moments and sense in the same plane are equivalent in their effect.
3. Two couples acting in one place upon a rigid body whose moments are equal but opposite in sense, balance each other.
4. A force acting on a rigid body can be replaced by an equal like force acting at any other point and a couple whose moments equals the moment of the force about the point where the equal like force is acting.
5. Any number of coplanar couples are equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the individual couples.

Example-1

Determine the sum of moments of the three forces about (a) Point A, (b) Point B and (c) Point C.

Answer:-

(a) The sum of the moments about A:

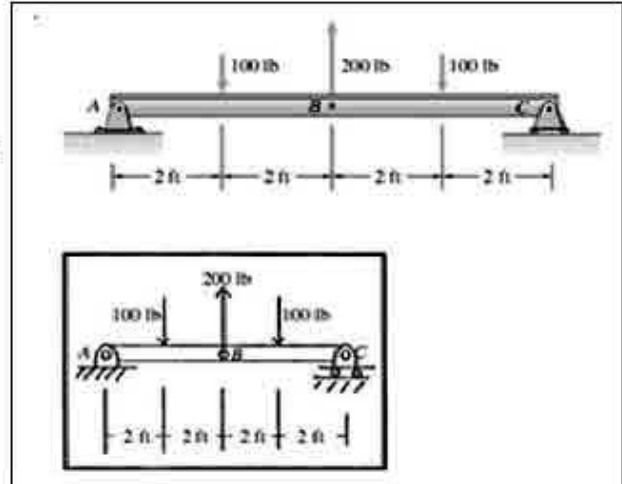
$$\sum M_A = -(2)(100) + (4)(200) - 6(100) = 0$$

(b) The sum of the moments about B:

$$\sum M_B = (2)(100) - (2)(100) = 0$$

(c) The sum of the moments about C:

$$\sum M_C = (6)(100) - (4)(200) + (2)(100) = 0$$



Example:-1

A Man and boy carry a weight of 300 N between them by means of a uniform pole 2m long and weighing 100N. Where must the weight be placed so that the man may bear twice as much of the weight as that of the boy.

As per the fig the weight of the pole acts at the centre G.

Let the boy bear the load W and the Man bears the load 2W.

Let the weight 300N act at a distance x meters from the man i.e. from the point B.

Now $\sum V = 0$

$$\Rightarrow W + 2W = 100 + 300 = 400$$

$$\Rightarrow 3W = 400$$

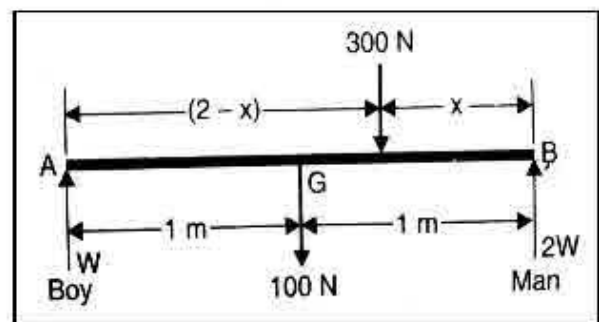
$$\text{So } W = 400/3 = 133.33 \text{ N}$$

Now taking moment at A (Boy's position)

$$2W \times 2 = 300(2 - x) + 100 \times 1$$

$$\Rightarrow 4W = 600 - 300x + 100 = 700 - 300x$$

$$\Rightarrow 4 \times 133.33 = 700 - 300x$$



$$\Rightarrow 300x = 166.8$$

$$x = 0.556\text{m}$$

Hence the weight will be placed 0.556 m from the position of man (point B) towards the boy (point A).

Example:-2

A rod of length 5m and weight 15N has its centre of gravity at a distance of 2m from A. It rests on two parallel smooth pegs at a distance 3m apart in the same horizontal plane so that equal portion of the rod project beyond the pegs. Find the reaction at the pegs.

Solution.

Refer to Fig. 3.26. Let G be the c.g of the rod AB, C and D be the pegs. R and S be the reactions at the pegs C and D respectively.

$$\text{Now } R + S = 15$$

$$S = 15 - R \quad \dots\dots\dots (i)$$

$$\text{Let } AC = x, DB = x$$

Taking moments about C, We get

$$15 \times CG = S \times 3$$

$$15 (2 - x) = S \times 3$$

$$10 - 5x = S$$

$$10 - 5x = 15 - R \quad \dots\dots\dots \text{From Eqns. (i)}$$

$$R - 5x = 5 \quad \dots\dots\dots (ii)$$

Taking moments about D, We get

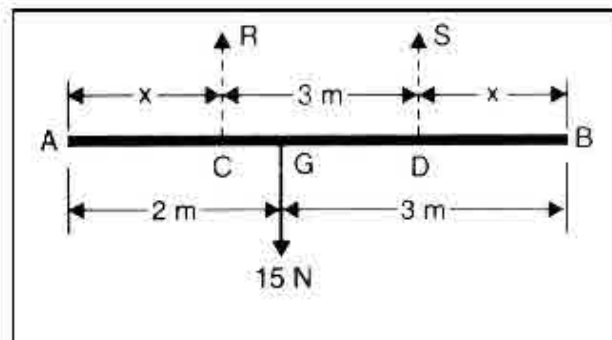
$$R \times CD = 15 \times GD$$

$$R \times 3 = 15 \times (3 - x)$$

$$R = 5 (3 - x) = 15 - 5x \quad \dots\dots\dots (iii)$$

On solving eqns. (ii) and (iii), we get

$$x = 1 \text{ m}, R = 10 \text{ N}, S = 5 \text{ N.} \quad (\text{Ans})$$



1.5 FORCE SYSTEM

Force:- Force is something which changes or tends to change the state of rest or of uniform motion of a body in a straight line. Force is the direct or indirect action of one body on another. It is a vector quantity.

Classification of force system:-

The classification of force system are as below.

1. According to the effect produced by a force:

- (i) **External Force:-** When a force is applied external to a body is called External Force.
- (ii) **Internal Force:-** The resistance to deformation, or change of shape exerted by the material of a body is called an internal force.
- (iii) **Active Force:-** An active force is one which causes a body to move or change its shape.
- (iv) **Passive Force:-** A force which prevents the motion, deformation of a body is called passive force.

2. According to the nature of the forces.

- (i) **Action and reaction:-** Whenever there are two bodies in contact, each exerts a force on other. Out of these force one is called action and the other is called reaction. Action and reaction are equal and opposite.
- (ii) **Attraction and repulsion:-** These are actually non contacting forces exerted by one body or another without any visible medium transmission such as magnetic forces.
- (iii) **Tension and Thrust.-** When a body is dragged with a string the force communicated to the body by the string is called the tension while, if we push the body with a rod the force exerted on the body is called a thrust.

3. According to whether the force acts at a point or is distributed over a large area.

- (i) **Concentrated force.-** The force whose point of application is so small that it may be considered as a point is called a Concentrated force.
- (ii) **Distributed force-** a distributed force is one whose place of application is area.

4. According to whether the force acts at a distance or by contact:-

- (i) **Non contacting forces or forces at a distance:-** Magnetic, Electrical and gravitational forces are examples of the Non contacting forces or forces at a distance.
- (ii) **contacting forces or forces by contact** – The pressure of steam in a cylinder and that of the wheels of a locomotive on the supporting rails are the examples of contacting forces.

LAWS OF FORCES.

The method of determination of the resultant of some forces acting simultaneously on a particle is called composition of forces.

The various laws used for the composition of forces are as follows.

1. Parallelogram law of forces
2. Triangle law of forces
3. Polygon law of forces.

Law of Parallelogram of Forces:-

It states that:

“If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.”

According to figure let two forces P and Q acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides oa and ob of a parallelogram $oacb$ drawn from a point o , their resultant R will be represented in magnitude and direction by the diagonal oc of the parallelogram.

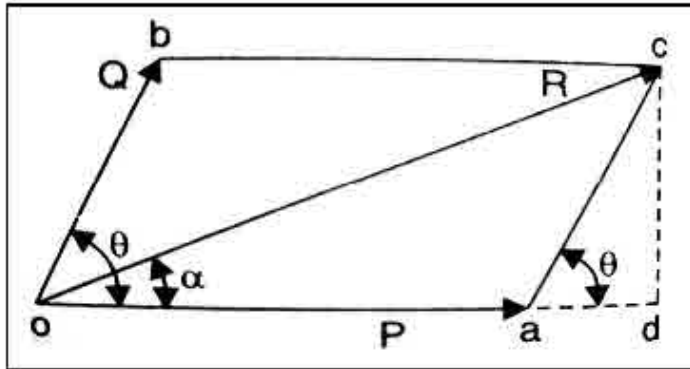
The value of R can be determined by either Graphically or analytically as explained below.

Graphical Method:-

Draw vectors ' oa ' and ' ob ' to represent two some convenient scale to the forces P and Q in magnitude and direction. Complete the parallelogram ' $oacb$ ' by drawing ' ac ' parallel to ' ob ' and ' bc ' parallel to ' oa '. The vector oc measured to the same scale will represent the resultant force R.

Analytical Method:-

As shown in the figure, in the parallelogram oacb from c drop a perpendicular 'cd' to 'oa' at 'd' when produced.



From the geometry of the figure $\angle cad = \theta$, $ac = Q$.

$\therefore cd = Q \sin \theta$ and $ad = Q \cos \theta$.

From right angle triangle, odc

$$oc = \sqrt{(od)^2 + (cd)^2} = \sqrt{(oa + ad)^2 + (cd)^2}$$

$$\text{or } R = \sqrt{(P + Q \cos \theta)^2 + (Q \sin \theta)^2} = \sqrt{P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta}$$

$$= \sqrt{P^2 + Q^2 (\sin^2 \theta + \cos^2 \theta) + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

i.e. when $\theta = 0^\circ$, $R = P + Q$, When $\theta = 90^\circ$, $R = \sqrt{P^2 + Q^2}$, When $\theta = 180^\circ$, $R = P - Q$

Let the resultant makes an angle α with P as shown in figure, Then

$$\tan \alpha = \frac{cd}{od} = \frac{cd}{oa + ad} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

Example-1

Find the Magnitude and direction of the resultant of the two forces P & Q 50 N and 60 N respectively acting at a point with an included angle of 30° between them. The force 80 N being horizontal.

Ans:- Given:- P = 50 N and Q = 60 N and $\theta = 30^\circ$

Applying the Parallelogram Law Formulae

$$\begin{aligned} R &= \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \\ &= \sqrt{50^2 + 60^2 + 2 \times 50 \times 60 \times \cos 30^\circ} \\ &= \sqrt{2500 + 3600 + 5196.15} \\ &= 106.28 \text{ N} \end{aligned}$$

Hence Magnitude of the resultant force is 106.28 N

$$\begin{aligned} \text{Now } \tan\alpha &= \frac{Q \sin\theta}{P + Q \cos\theta} = \frac{60 \sin 30^\circ}{50 + 60 \cos 30^\circ} \\ &= \frac{30}{50 + 51.96} = 0.294 \end{aligned}$$

Hence $\alpha = \tan^{-1}(0.294) = 16.38^\circ$.

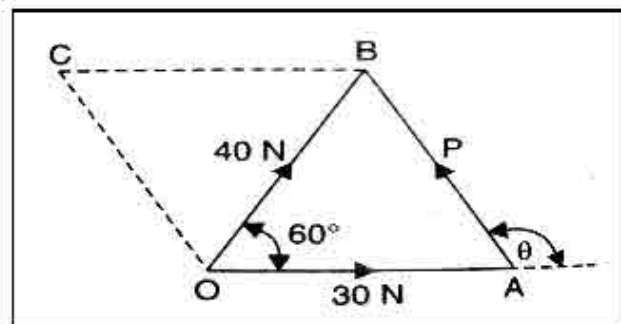
Example-2

The Resultant of two forces P and 30 N is 40 N and is inclined at 60° to the 30 N force. Find the magnitude and direction of P.

Ans:- Considering the ΔOAB ,

$$\begin{aligned} AB^2 &= OA^2 + OB^2 - 2.OA.OB.\cos 60^\circ \\ \Rightarrow P^2 &= 30^2 + 40^2 - 2 \times 30 \times 40 \times \cos 60^\circ \\ \Rightarrow P &= \sqrt{900 + 1600 - 2400 \times 0.5} \\ &= 36.06 \text{ N (Ans)} \end{aligned}$$

Now applying Sine rule,



$$\frac{P}{\sin 60^\circ} = \frac{40}{\sin(180^\circ - \theta)}$$

$$\Rightarrow \frac{36.06}{0.866} = \frac{40}{\sin(180^\circ - \theta)}$$

$$\Rightarrow \sin(180^\circ - \theta) = \frac{40 \times 0.866}{36.06} = 0.96$$

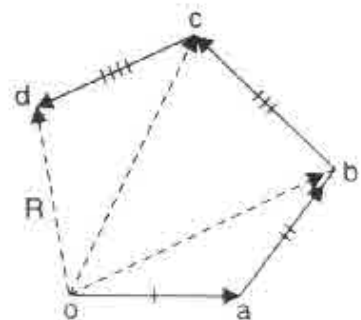
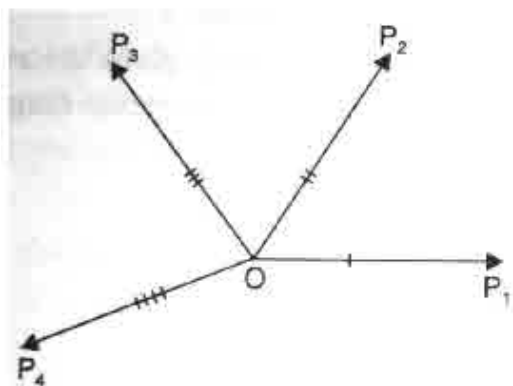
$$\Rightarrow 180 - \theta = \sin^{-1} 0.96 = 73.74^\circ = 73^\circ 44'$$

$$\Rightarrow \theta = 106^\circ 16'. \text{ (Ans)}$$

Polygon Law of Forces:-

It states that:

“If a number of coplanar concurrent forces, acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant may be represented in magnitude and direction by the side of a polygon, taken in the opposite order.”



If the forces P_1, P_2, P_3 and P_4 acting simultaneously on a particle be represented in magnitude and direction by the sides oa, ab, bc and cd of a polygon respectively, their resultant is represented by the closing side ' do ' in the opposite direction as shown in the figure.

The law is actually an extension of Triangle law of forces.

This is so because ob is the resultant of oa and ab and therefore oc which is resultant of ob and bc , is also the resultant of oa, ab and bc . Similarly od is the resultant of oc and cd .

So the resultant of the forces P_1, P_2, P_3 and P_4 is ' od ', which is the closing side of the polygon.

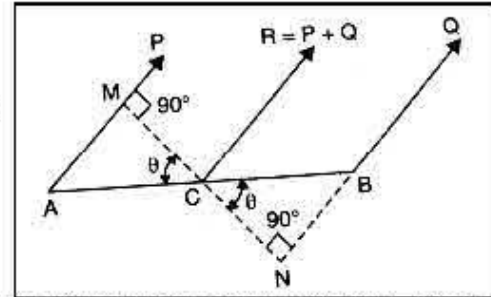
PARALLEL FORCES

Force whose line of action are parallel are called parallel forces. They are said to be like when they act in the same sense and are said to be unlike when act in the opposite sense.

(i) Resultant of two like parallel forces:-

As per the figure let the like parallel force P and Q act at the points A and B and their resultant R cut AB at C .

By resolving parallel to P or Q , we get $R = P + Q$,
and that R is parallel to P and Q .



The moment of R about C is zero, so that the algebraic sum of moments of P and Q about C must also be zero.

Through C draw MN perpendicular to P and Q .

Now, $CM = AC \cos \theta$

$$CN = BC \cos \theta$$

Taking moment about C ,

$$P \times CM = Q \times CN$$

$$\Rightarrow P \times AC \cos \theta = Q \times BC \cos \theta$$

$$\Rightarrow P \times AC = Q \times BC$$

$$\Rightarrow P/BC = Q/AC$$

$\therefore C$ divides AB internally in the inverse ratio of the forces.

(ii) Resultant of two unlike parallel forces:-

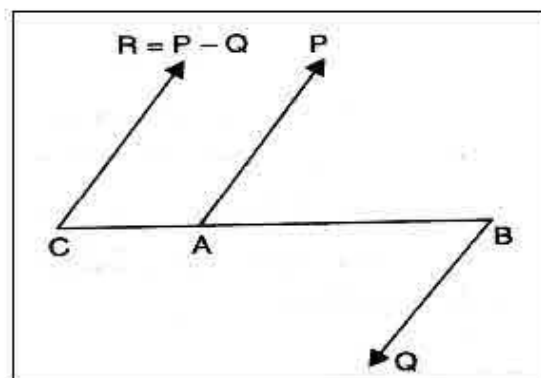
As per the figure Let two unlike parallel force

P and Q act at the points A and B .

Let their resultant R meet AB at C .

Let P be greater than Q .

By resolving parallel to P or Q , we get



$R = P - Q$, acting in the same sense as P .

The algebraic sum of the moments of P and Q about C must be Zero so that these moments must be equal and opposite. Hence C must lie outside AB , and it must be nearer to A than to B .

Taking moments about C , we get $P \times AC = Q \times BC$.

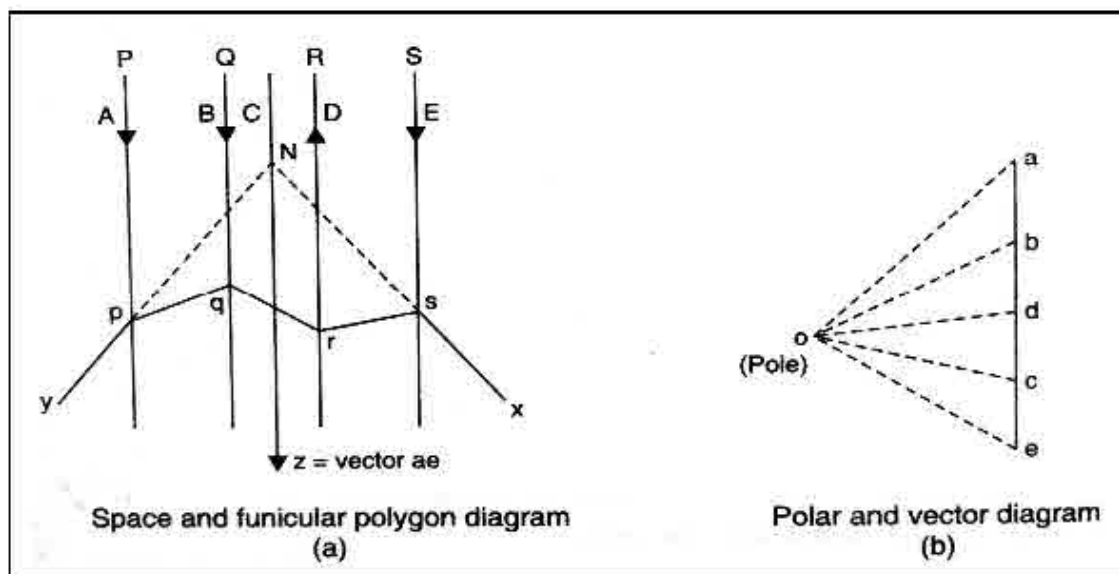
Or $P/BC = Q/AC$

$\therefore C$ divides AB externally in the inverse ratio of the forces.

The point C is called the Centre of the parallel forces. It is clear that the position of C is independent of the inclination of the forces.

GRAPHICAL METHOD FOR FINDING THE RESULTANT OF ANY NUMBER OF LIKE OR UNLIKE PARALLEL FORCES:-

Referring to fig-(a) P, Q, R, S are the parallel forces, where P, Q and S are in one direction and the force R in the opposite direction.



The various steps involved in finding the resultant by graphical method are given below.

- (i) Put letters A, B, C, D and E in the space about P, Q, R, S .
- (ii) To convenient scale mark off ab equal to force P , bc equal to force Q , cd equal to force R , in the opposite direction and de equal to force S .

Ae represents the resultant in magnitude and direction and its line of action is found as follows.

Referring Fig. b

- (i). Take any point O (called pole) outside the vector diagram abcde and join oa, ob, oc, od and oe.
- (ii). Take any point on Force S and the point s in the space E draw a line parallel to oe.
- (iii). Similarly in the spaces D, C, B and A lines sr, rq, and py parallel to od, oc, ob, and oa respectively.
- (iv). Produce lines xs and yp to meet each at N.

The resultant force Z which passes through point N is completely given by ae in the magnitude and direction.

Equilibrium of Forces

If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium.

The force, which brings the set of forces in equilibrium is called on equilibrant. The equilibrant is equal to the resultant force in magnitude, but opposite in direction.

Analytical conditions of equilibrium of a co-planner system of concurrent forces.

We know that the resultant of a system of co-planner concurrent forces is given by :

$$R = \sqrt{(\sum x)^2 + (\sum y)^2}, \text{ where } \therefore \sum y = 0$$

$\sum x$ = Algebraic sum of resolved parts of the forces along a horizontal direction.

$\sum y$ = Algebraic sum of resolved parts of the forces along a vertical direction.

$$R^2 = (\sum x)^2 + (\sum y)^2$$

If the forces are in equilibrium, $R=0$

$$0 = (\sum x)^2 + (\sum y)^2$$

Sum of the squared of two quantities is zero when each quantity is separately equal to zero.

$$\therefore \sum x = 0 \text{ and } \sum y = 0$$

Hence necessary and sufficient conditions of a system of co-plan concurrent forces are :

- i) The algebraic sum of the resolved parts of the forces in some assigned direction is equal to zero.
- ii) The algebraic sum of the resolved parts of the forces in a direction at right angles to the assigned direction is equal to zero.

Graphical conditions of equilibrium of a system of co-planner concurrent forces :

Let a number of forces acting at a point be in equilibrium. Then, it can be said that the resultant of these forces is nil. Hence the length of the closing line of the polygon drawn to represent the given forces taken in orders will be nil. In other words, the length of closing line of the vector diagram drawn with the given forces in orders, is nil. This means the end point of the vector diagram must coincide with the starting point of the diagram. Hence the vector diagram must be a closed figure.

So, graphical condition of equilibrium of a system of co-planner concurrent forces may be stated as follows :

If a system of co-planner concurrent forces be in equilibrium, the vector diagram drawn with the given forces taken in orders, must be a closed figure.

Lami's Theorem :

If three concurrent forces are acting on a body, kept in an equilibrium, then each force is proportional to the sine of the angle between the other two forces and the constant of proportionality is the same.

Consider forces, P, Q and R acting at a point 'O' mathematically, Lami's theorem is given by the following equilibrium :

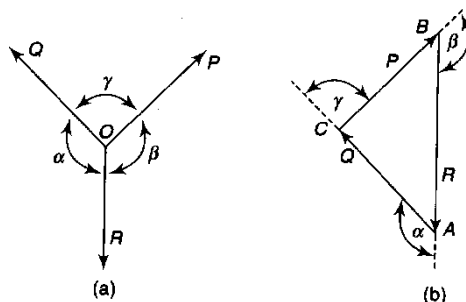
$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} = K$$

Since the forces are in equilibrium, the triangle of forces should close, corresponding to the forces, P, Q and R acting at a point 'O'. The angles of the triangle are

$$\angle A = (\lambda - \alpha)$$

$$\angle B = (\lambda - \beta)$$

$$\angle C = (\lambda - \gamma)$$



From the rule for the triangle we get.

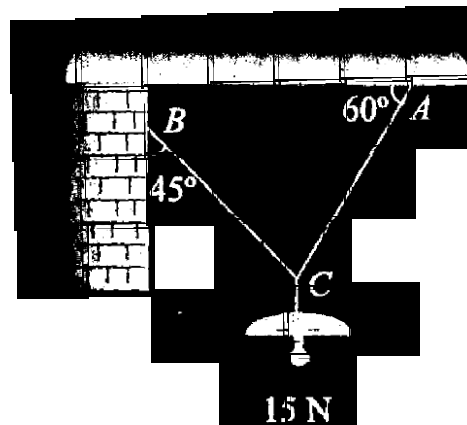
$$\frac{P}{\sin(\lambda - \alpha)} = \frac{Q}{\sin(\lambda - \beta)} = \frac{R}{\sin(\lambda - \gamma)}$$

Or we can write the equation according to Lami's Theorem i.e.

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Example :

An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the vertical as shown in figure, using lami's theorem determine the forces in the strings AC and BC.

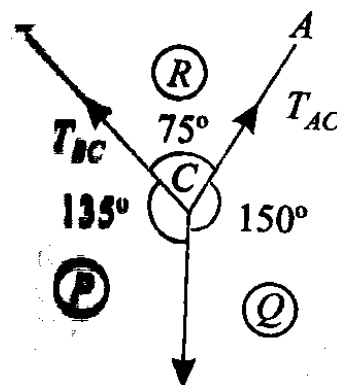


Solution :

Given weight at C = 15 N

T_{AC} = Force in the string AC.

T_{BC} = Force in the string BC.



According to the system of forces and the above figure, we find that angle between T_{AC} and ISN is 150° and angle between T_{BC} and 15 N is 135° .

$$\angle ACB = 180^\circ - (40^\circ + 60^\circ) = 75^\circ$$

Applying Lami's Theorem :

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\therefore T_{AC} = \frac{15 \sin 135^\circ}{\sin 75^\circ} = 10.98 \text{ N (Ans)}$$

$$\text{and } T_{BC} = \frac{15 \sin 150^\circ}{\sin 75^\circ} = 7.76 \text{ N (Ans)}$$

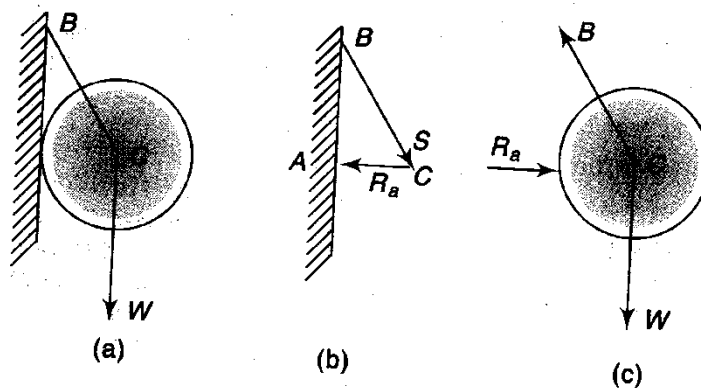
Free Body Diagram:

Free body diagram is a sketch of the isolated body, which shows the external forces on the body and the reaction extended on it by the removed elements.

The general procedure for constructing a free body diagram is as follows :

1. A sketch of the body is drawn, by removing the supporting surfaces.
2. Indicate on this sketch all the applied on active forces which send to set the body in motion, such as those caused by weight of the body on applied forces etc.
3. Also indicate on this sketch all the reactive forces, such as those caused by the constrains or supports that tend to prevent motion. (The sense of unknown reaction should be assumed. The correct sense will be determined by the solution of the problem. A positive result indicates that the assumed sense is correct A negative results indicates that the correct sense is opposite to the assumed sense).
4. All relevant dimensions and angles, reference axes are shown on the sketch.

Example :



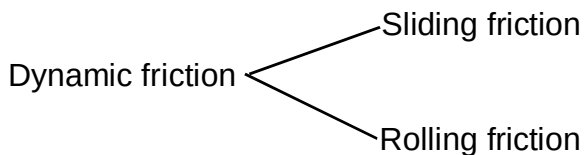
FRICTION

When a body moves or trends to move over another body, an opposing force develops at the contact surface. This force opposes the movement is called frictional force or friction.

Frictional force is the resistance offered when a body moves over another body assist the motion.

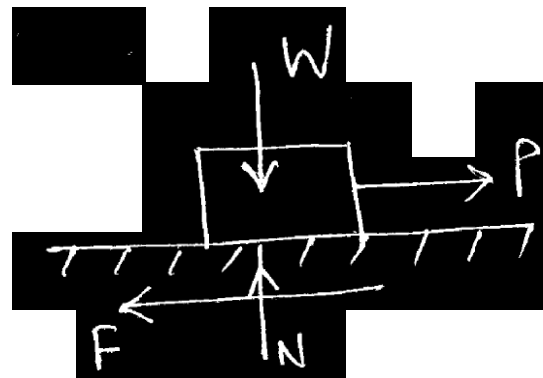
There is a limit beyond which the magnitude of this force cannot increase when applied force more than this limit there will be motion. When applied force less than this limit value, the body remains at rest and such frictional force is called static friction. When body moves (applied force more than limiting friction) the frictional resistance is known as Dynamic friction.

Dynamic friction is found less than limiting friction.



Coefficient of friction :

Experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces and this ratio is called coefficient of friction :



$$\text{Co-efficient of friction} = F/N = \mu$$

F → Limiting friction (Frictional force)

N → Normal reaction

Laws of friction:

- 1) Force of friction is directly proportional to the normal reaction and always opposite in the direction of motion.
- 2) Force of friction depends upon the roughness/ smoothness of the surface.

- 3) Force of friction is independent of the areas of the contact.
- 4) Force of friction is independent of the sliding velocity.

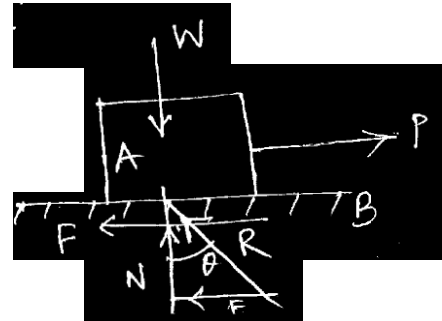
Angle of friction:

Consider the block 'A' resting on horizontal plane 'B'.

Let, $P \rightarrow$ horizontal force applied.

$F \rightarrow$ Frictional force

$N \rightarrow$ Normal reaction.



Let R be the resultant reaction between normal reaction and force of friction acts at angle θ to the normal Reaction. $\theta \rightarrow$ Angle of friction.

$$\tan \theta = F/N$$

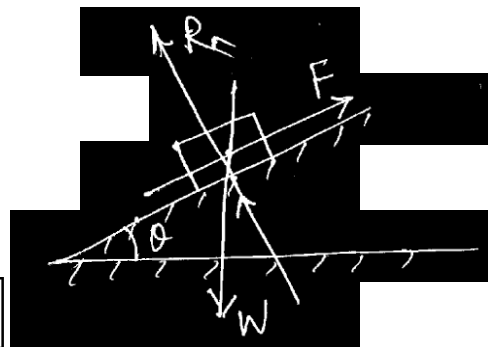
As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value

$$\tan \alpha = F/N = \mu$$

$\alpha \rightarrow$ Angle of friction. (Angle of limiting friction)

Angle of Repose :

It is the maximum inclined plane with the horizontal for which a body lying on the inclined plane will be on the point of sliding down.



Angle of Repose is equal to the angle of friction .

Consider a block of weight 'W' resting on a rough inclined plane (angle)

$R_n \rightarrow$ Normal reaction

$F \rightarrow$ Frictional force

$$F = W \sin \theta, R_n = W \cos \theta$$

$$\tan \theta = \frac{F}{R_n}$$

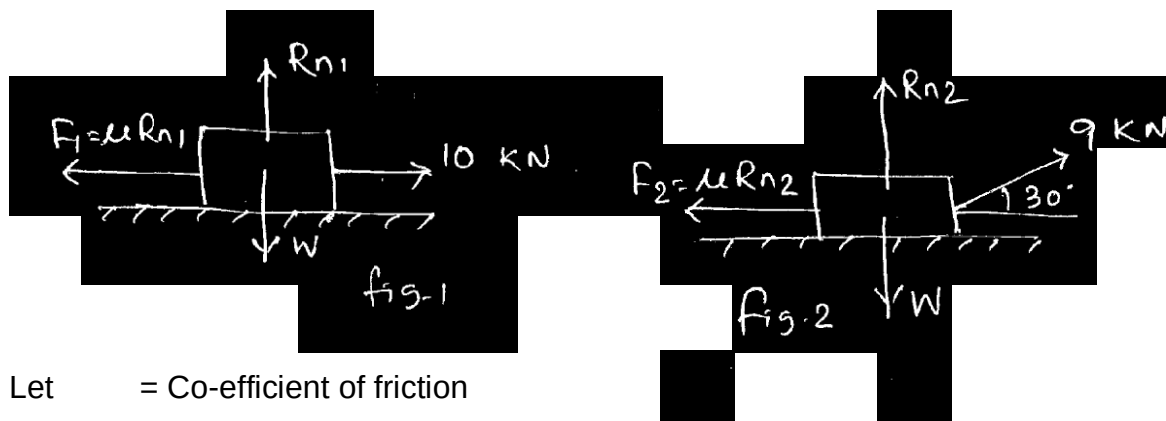
But we knew that $\tan \phi = \frac{\mu}{R_n}$

$\tan \theta = \tan \phi$ $\mu = \phi$

Ex - 1

A body resting on a horizontal plane can be moved slowly along the plane by a horizontal force of 10 KN. A force of a 9 KN inclined at 30° to the horizontal direction will suffice to move the block along the same direction. Determine the co-efficient of friction and weight of the body.

Solution :



Let μ = Co-efficient of friction
 W = Weight of the body.

From fig-1

$$F_1 = R_{n1} = 10 \text{-----} (1)$$

$$R_{n1} = W \text{-----} (2)$$

$$\text{So } W = 10 \text{-----} (3)$$

$$9 \cos 30^\circ = F_2 = R_{n2} \text{-----} (4)$$

$$R_{n2} + 9 \sin 30^\circ = W \text{-----} (5)$$

$$R_{n2} = W - 4.5 \text{-----} (6)$$

Putting the value of R_{n2} in equation(4)

$$[W - 4.5] = 9 \cos 30^\circ = 7.794$$

$$W - 4.5 = 7.794 \text{-----} (7)$$

Putting the value of W from (3)

$$10 - 4.5 = 7.794$$

$$4.5 = 10 - 7.794$$

$\mu = 0.49$

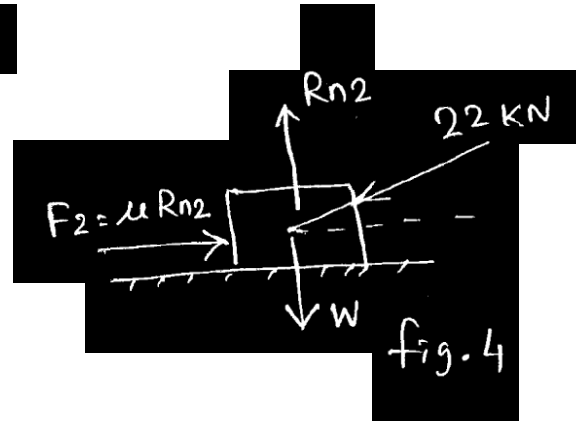
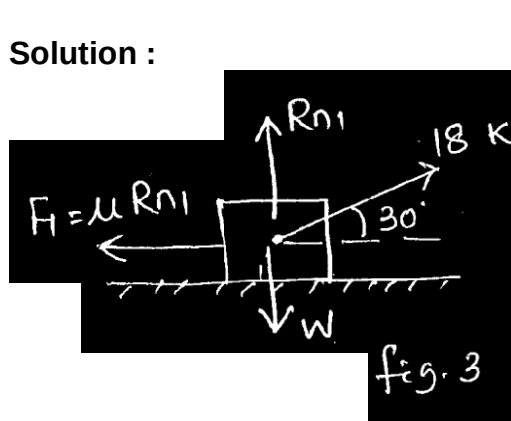
Putting the value of μ in equation (3)

$$.49 W = 10$$

$$W = 20.408 \text{ KN.}$$

A body resting on rough horizontal plane, required a pull of 18 KN at 30° to the plane just to move it. It was also found that a push of 22 KN inclined at 30° to the plane just moved the body. Determine the weight of the body and co-efficient of friction.

Solution :



Let $W \rightarrow$ Weight of the body

$\mu \rightarrow$ Co-efficient of friction

From fig - 3

$$F_1 = R_{n1} = 18 \cos 30^\circ = 15.59 \text{ KN} \text{ ----- (1)}$$

$$R_{n1} + 18 \sin 30^\circ = W$$

$$R_{n1} = W - 9 \text{ ----- (2)}$$

Putting the value of equation (2) in equation (1)

$$(W-9) = 15.59 \text{ ----- (3)}$$

From fig. 4.

$$F_2 = R_{n2} = 22 \cos 30^\circ = 19.05 \text{ ----- (4)}$$

$$R_{n2} = W + 22 \sin 30^\circ = W + 11 \text{ ----- (5)}$$

Putting the value of equation (5) in equation (4)

$$(W+11) = 19.05 \text{ ----- (6)}$$

From equation (3) and equation (6)

$$\mu = \frac{15.59}{W-9}, \mu = \frac{19.05}{W+11}$$

$$\frac{15.59}{19.05} = \frac{W-9}{W+11}$$

$$W = 99.12 \text{ KN}$$

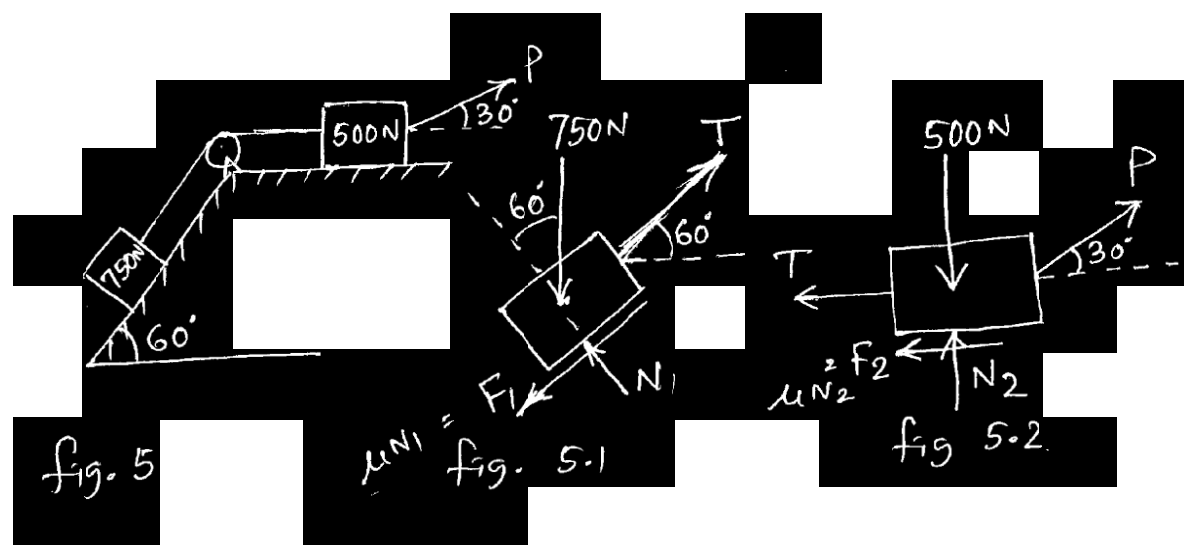
Putting the value of W in equation (3)

$$(99.12 - 9) = 15.59$$

$$\mu = \frac{15.59}{90.12} = 0.173$$

Ex - 3

What is the value of P in the system shown in fig- 5 to cause the motion of 500 N block to the right side? Assume the pulley is smooth and the co-efficient of friction between other contact surface is 0.20.



Solution :

Consider the FBD of fig. 5.1

$$N_1 = 750 \cos 60^\circ$$

$$N_1 = 375$$

$$750 \sin 60 + N_1 = T$$

$$750 \sin 60 + 0.2 \times 375 = T = 724.52 \text{ N} \text{ ----- (2)}$$

Consider the FBD of fig. 5.2

$$N_2 + P \sin 30^\circ = 500$$

$$N_2 = 500 - 0.5 P \text{ ----- (3)}$$

$$P \cos 30 = T + N_2 = 724.52 + 0.2 (500 - .5P)$$

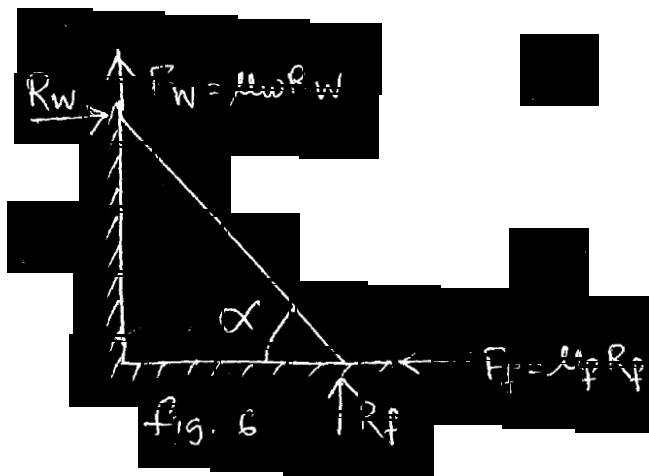
$$P \cos 30 + .1 P = 724.52 + 100$$

$$P (\cos 30 + 0.1) = 824.52$$

$$P = 853.52 \text{ N}$$

Ladder friction :

It is a device for climbing up or down.

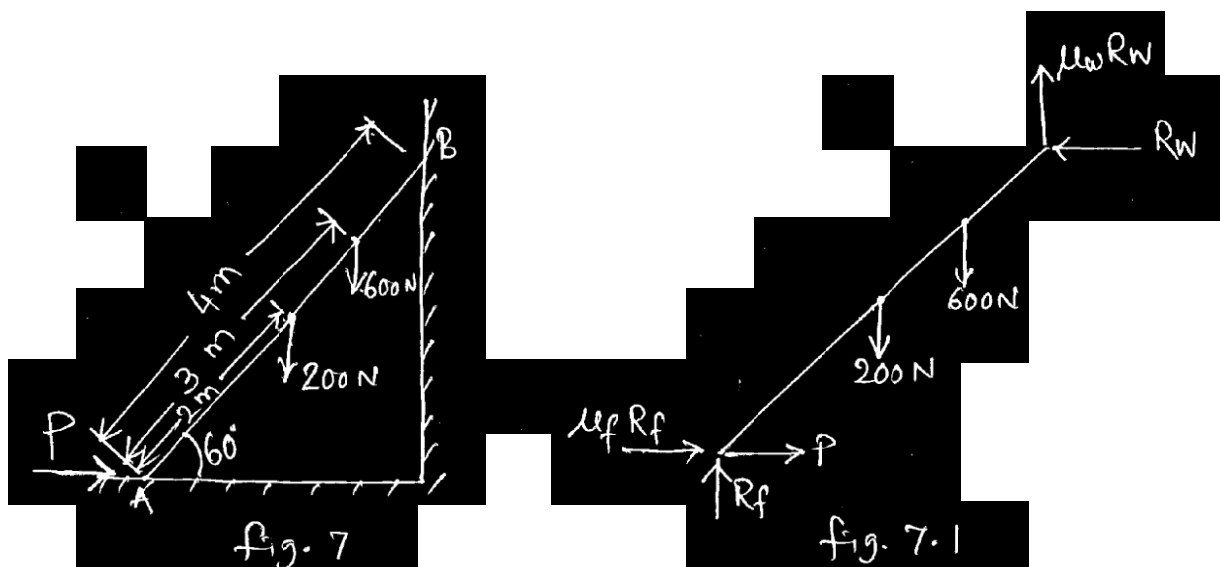


Since this system is in equilibrium, the algebraic sum of the horizontal and vertical component of the forces must be zero and the algebraic sum of the moment must also be zero.

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0$$

Ex – 5

A ladder of length 4m, weighing 200N is placed against a vertical wall as shown in fig. 7. The co-efficient of friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. In addition to self weight, the ladder has to support a man weighing 600N at a distance of 3 m. from A calculate the minimum horizontal force to be applied at A to prevent slipping.



Solution :

The FBD of the ladder is shown in fig. 7.1

Taking moment at A

$$R_w \times 4 \sin 60^\circ + w R_w 4 \cos 60^\circ = 600 \times 3 \cos 60^\circ + 200 \times 2 \cos 60^\circ$$

$$866 R_w + 0.2 \times 0.5 R_w = 275$$

$$0.966 R_w = 275$$

$$R_w = 284.68 \text{ (1)}$$

$$600 + 200 = R_f + w R_w$$

$$R_f = 800 - 0.2 \times 284.68$$

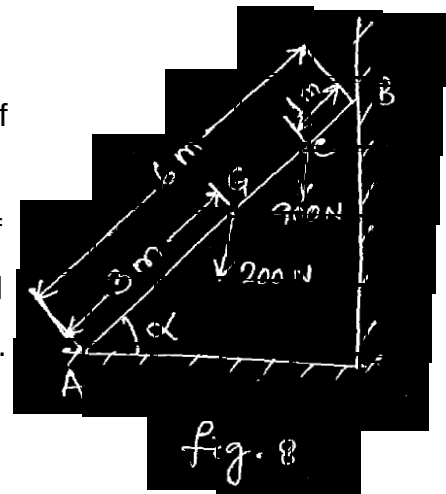
$$= 743.06 \text{ (2)}$$

$$P + f R_f = R_w$$

$$P = 284.68 - 0.3 \times 743.06 = 61.76$$

Ex – 6

The ladder shown in fig. 8 is 6m. long and is supported by a horizontal floor and vertical wall. The co-efficient of friction between the floor and the ladder is 0.25 and between the wall and the ladder is 0.4 The self weight of the ladder is 200 N. The ladder also supports a vertical load of 900 N at C which is at a distance of 1 m from B. Determine the least of value of α at which the ladder may be placed without slipping.



Solution :

w = Co-efficient of friction between wall and ladder = 0.4

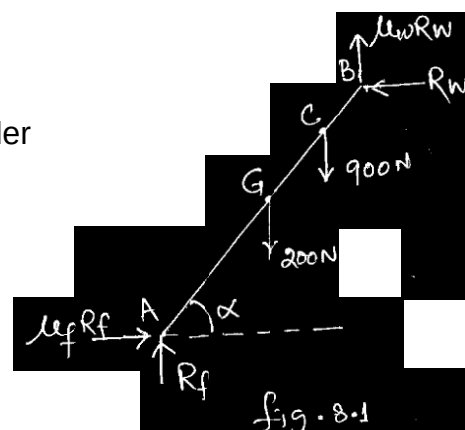
f = Co-efficient of friction between floor and ladder = 0.25

The FBD of the ladder is shown in fig. 8.1

$$R_w = f R_f = 0.25 R_f \text{ (1)}$$

$$R_f + w R_w = 900 + 200 = 1100$$

$$R_f + 0.4 \times 0.25 R_f = 1100$$



$$R_f = 100 \text{ N} \quad (2)$$

$$\text{So } R_w = 250 \text{ N} \quad (3)$$

Taking moment about A

$$R_w \times 6 \sin + w \times 6 \cos$$

$$= 200 \times 3 \cos + 900 \times 5 \cos$$

$$250 \times 6 \sin + 0.4 \times 250 \times 6 \cos = 600 \cos + 4500 \cos$$

$$1500 \sin = 4500 \cos$$

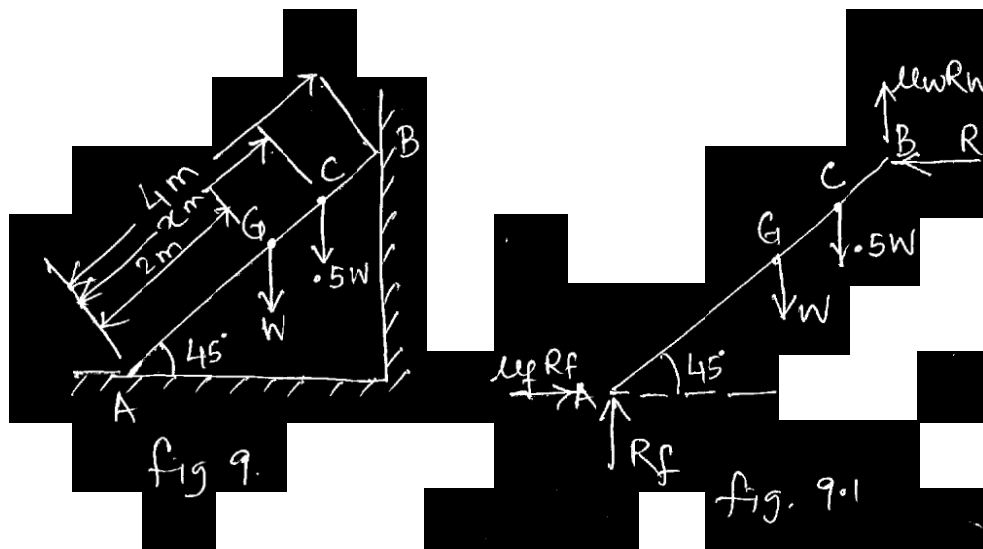
$$\tan = 3$$

$$= 71.563^\circ$$

Ex – 7

A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45° . The co-efficient of friction between the ladder and wall is 0.4 and that between ladder and the floor is 0.5. If a man, whose weight is half of that of the ladder ascends it, how height will it be when the ladder slips ?

Solution :



The FBD is shown in fig. 9.1

$$f = 0.5, \quad w = 0.4$$

$$f R_f = R_w$$

$$.5 R_f = R_w \quad (1)$$

$$R_f + w R_w = w + 0.5 w = 1.5w$$

$$R_f + .5 \times .4 R_f = 1.5 w$$

$$1.2 R_f = 1.5 w$$

$$R_f = 1.25 w \text{ -----(2)}$$

$$R_w = .5 \times 1.25 w = 0.625w \text{ -----(3)}$$

Taking moment about A

$$R_w \times 4 \sin 45^\circ + w \times 4 \cos 45^\circ = w \times 2 \cos 45^\circ + .5 w \times \cos 45^\circ$$

$$4 R_w + w \times 4 R_w = 2W + .5 w$$

$$4 \times .625 w + .4 \times 4 \times 0.625 w$$

$$= 2 w + .5 \times w$$

$$2.5 + 1 = 2 + .5x$$

$$.5 x = 1.5$$

$$x = 3 \text{ m.}$$

Wedge friction :

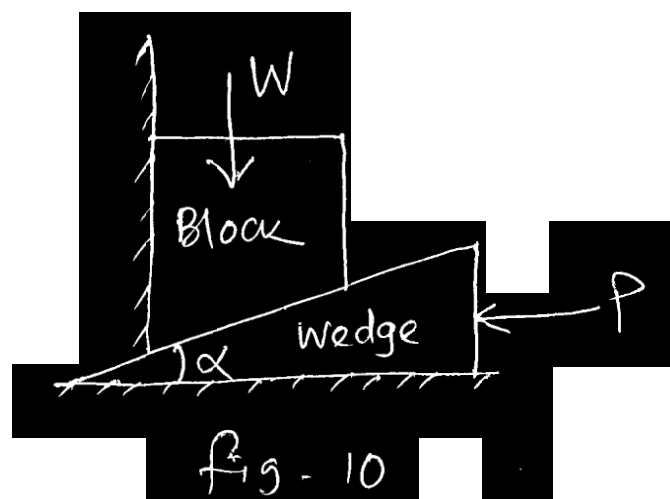
Wedges are generally triangular or trapezoidal in cross section. It is generally used for tightening keys or shaft. It is also used for lifting heavy weights. The weight of the wedge is very small compared to the weight lifted.

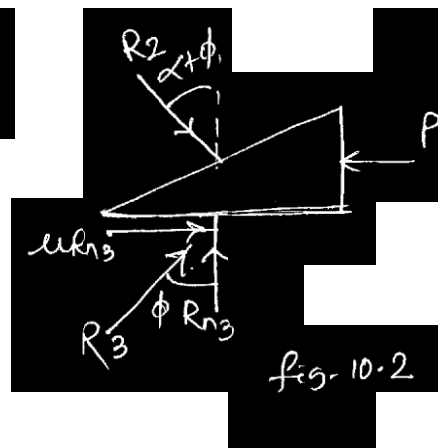
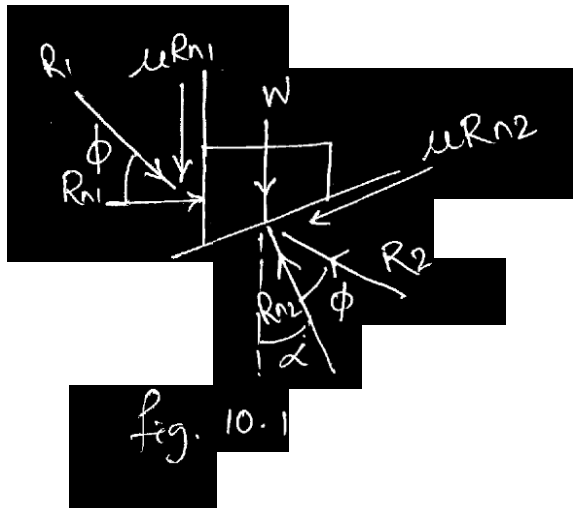
Let w = weight of the block

P = Force required to lift the load

= co-efficient of friction on all contact surface.

= Wedge angle.





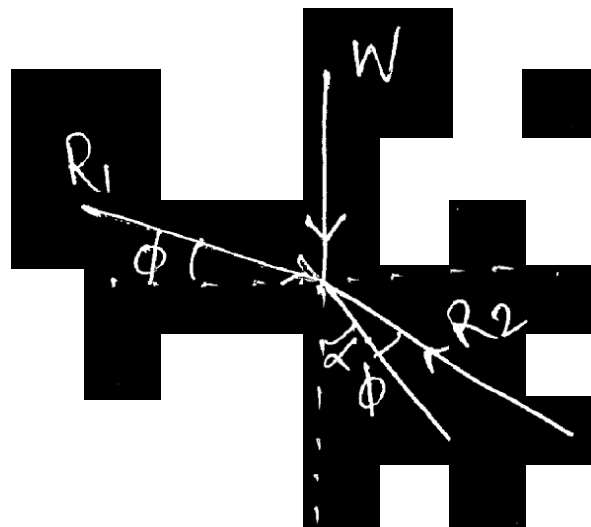
Considering the FBD of fig. 10.1 under the action of 3 forces the system is in equilibrium.

- 1) $R_1 \rightarrow$ Resultant friction of normal reaction and force of friction between block and wall.
- 2) $W \rightarrow$ Weight of the block.
- 3) $R_2 \rightarrow$ Resultant reaction of normal reaction and force of friction between contact surface of block and wedge.

Applying Lami's Theorem

$$\begin{aligned} \frac{W}{\sin[90 + (\alpha + \phi)]} &= \frac{R_1}{\sin[180 - (\alpha + \phi)]} \\ &= \frac{R_2}{\sin(90 - \phi)} \end{aligned}$$

$$\text{or } \frac{W}{\cos(\alpha - 2\phi)} = \frac{R_1}{\sin(\alpha + \phi)} = \frac{R}{\cos\phi}$$



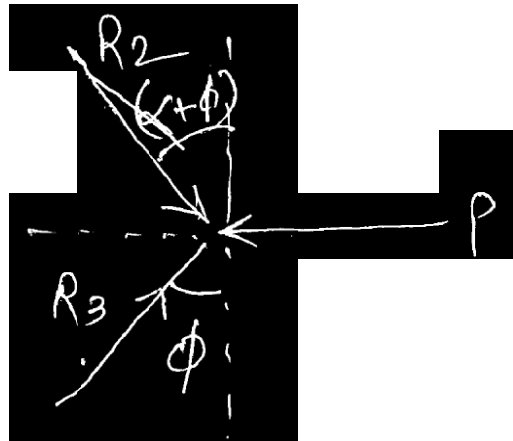
Again considering the FBD of Fig. 10.2 under the action of 3 forces the system is in equilibrium.

- 1) $R_2 \rightarrow$ Reaction given by the block.
- 2) $P \rightarrow$ Force applied on wedge.

3) $R_3 \rightarrow$ Resultant reaction of normal reaction and force of friction between wedge and floor.

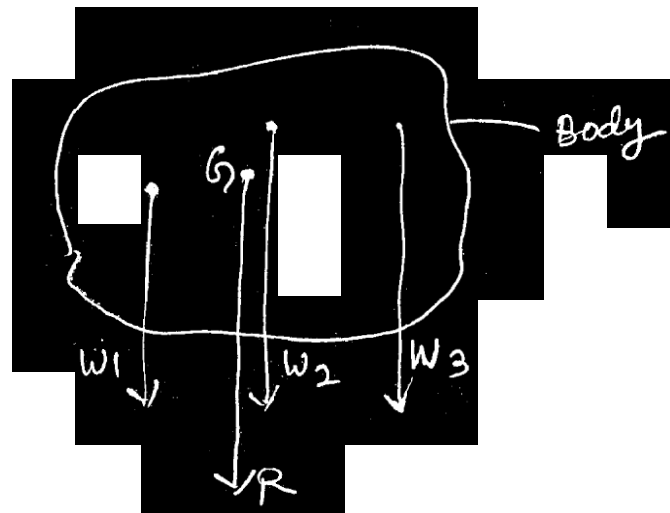
Applying Lami's theorem

$$\frac{P}{\sin(\alpha + 2\phi)} = \frac{R_2}{\cos\phi} = \frac{R_3}{\cos(\alpha + \phi)}$$



Centre of gravity (CG) – It is the point where whole of the mass of a body is supposed to act. In other words it is the point through which resultant of the parallel forces of attraction formed by the weight of the body, passes.

It is usually denoted by CG or simplify G.

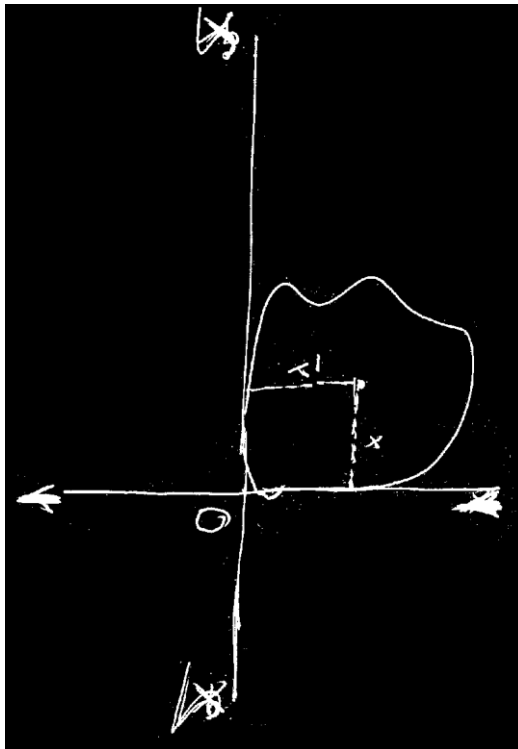


Centroid :

It is the point where the whole area of a body is assumed to be concentrated. Plane figures (known as laminas) have area only but no mass. The centre of gravity and centroid of such figures are the same point.

For plane areas, centroid is represented by two coordinates by selecting coordinate axes in the plane of the area itself.

The two co-ordinate axes are usually selected in the extreme left and bottom of the area.



Mathematically (continuous form)

$$\bar{x} = \frac{\int_A x \cdot dA}{A}$$

Similarly $\bar{y} = \frac{\int_A y \cdot dA}{A}$

$\int_A x dA$ and $\int_A y dA$ are known as first moment of area about x & y axes respectively.

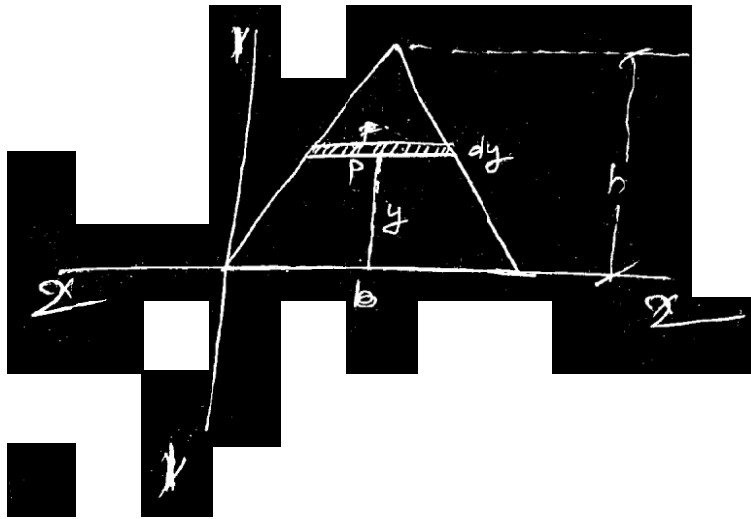
Mathematically – (Discrete form)

$$\bar{x} = \frac{\sum_{i=1}^n \Delta A_i \bar{x}_i}{\sum_{i=1}^n \Delta A_i}$$

$$\bar{y} = \frac{\sum_{i=1}^n \Delta A_i \bar{y}_i}{\sum_{i=1}^n \Delta A_i}$$

Ex - 1

Determine the y - co-ordinate of the centroid of a uniform triangular lamina.



From the concept of similar triangles.

$$\frac{p}{b} = \frac{h-y}{h}$$

$$\therefore p = \left(\frac{h-y}{h} \right) b$$

$$\therefore \text{The area of the shaded portion } dA = \left(\frac{h-y}{h} \right) b \cdot dy$$

$$\therefore \bar{y} = \frac{\int y \cdot dA}{A}$$

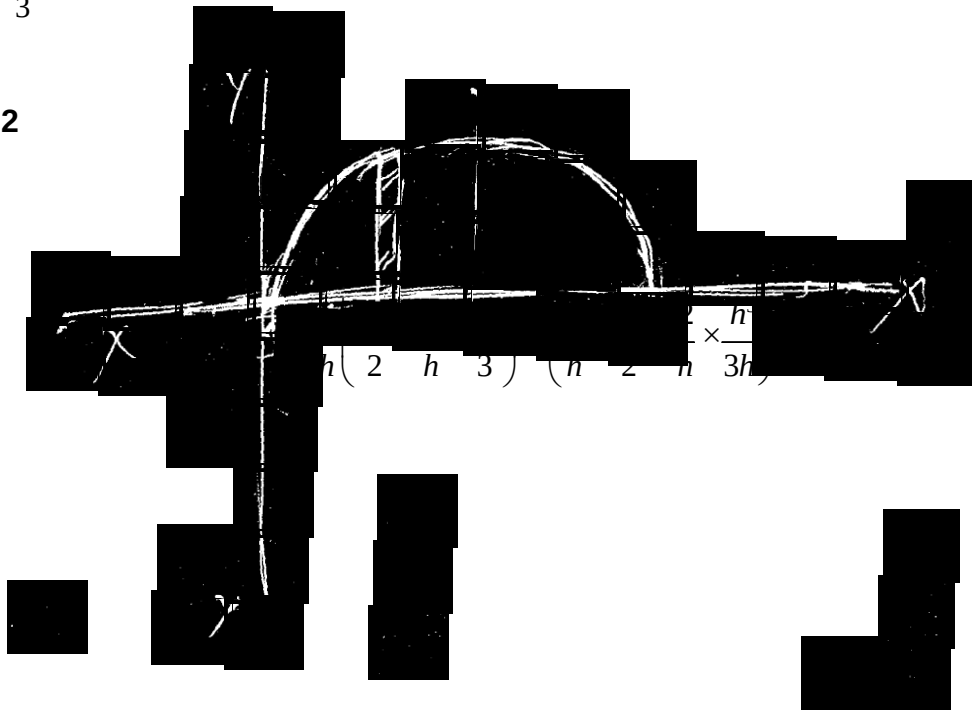
$$= \frac{\int_0^h y \left(\frac{h-y}{h} \right) b \cdot dy}{\frac{1}{2} \times b \cdot h}$$

$$= \frac{\int_0^h y \cdot b \left(1 - y/h \right) dy}{\frac{bh}{2}}$$

$$= \frac{b \times \int_0^h y \left(1 - \frac{y}{h} \right) dy}{\frac{bh}{2}}$$

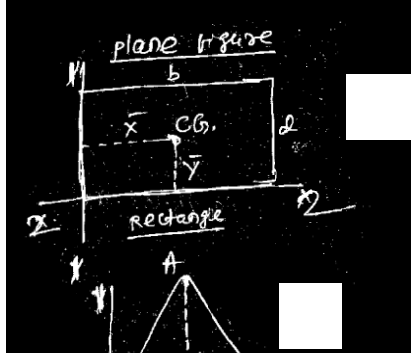
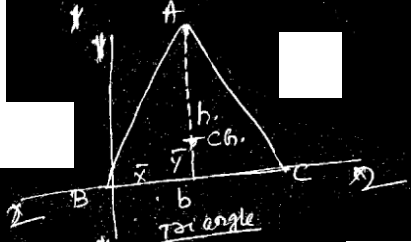
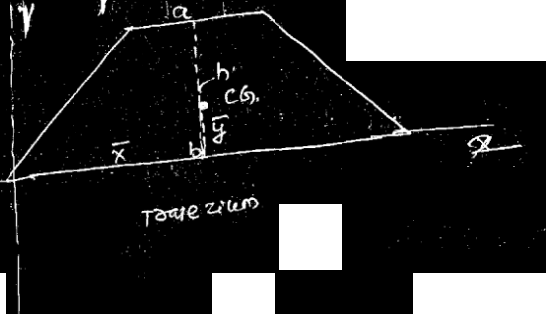
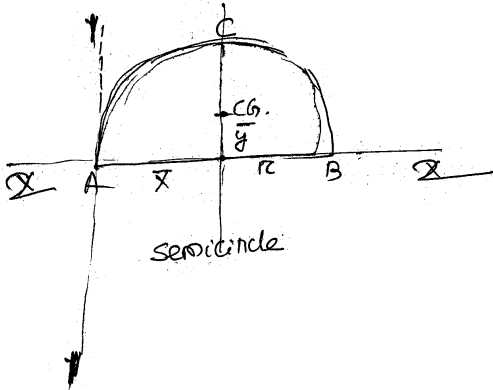
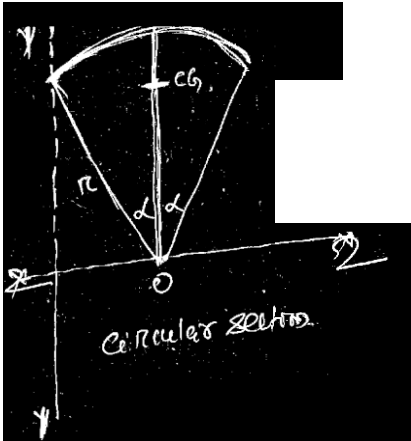
$$\begin{aligned}
&= \frac{2}{h} \left[y dy - \frac{y^2}{h} \cdot dy \right] \\
&= \frac{2}{h} \left[y^2 - \frac{1}{h} y^3 \right]_0^h \\
&= \frac{2}{h} \left(\frac{h^2}{2} - \frac{1}{h} \times \frac{h^3}{3} \right) = \left(\frac{2}{h} \times \frac{h^2}{2} - \frac{2}{h} \times \frac{h^3}{3h} \right) \\
&= h - \frac{2}{3}h \\
&= \frac{3h - 2h}{3} = h/3
\end{aligned}$$

Ex- 2



$$\begin{aligned}
\bar{x} &= \frac{\int x dA}{\int dA} \\
\bar{y} &= \frac{\int y dA}{\int dA}
\end{aligned}$$

A - Centre of gravity of some common figures :

<u>Sl. No.</u>	<u>Plane Figure</u>	<u>C.G. location</u>
1		$\bar{x} = \frac{b}{2}$ $\bar{y} = \frac{d}{2}$
2		$\bar{x} = \frac{b}{2}$ $\bar{y} = \frac{h}{3} \text{ (from base)}$
3		$\bar{x} = \frac{b}{2}$ $\bar{y} = \frac{h}{3} \left(\frac{b+2a}{b+a} \right)$
4		$\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$
5		$\bar{y} = \frac{2r}{3} \frac{\sin \alpha}{\alpha}$

B - Centre of gravity of built up symmetrical sections.

Built up sections are formed by combining plane sections. The centre of gravity of such built up sections are calculated using method of moments.

The entire area of the builtup section is divided into elementary plane areas i.e. a_1, a_2, a_3 etc. Two reference axes are selected and let the distance of c.g. of these areas be $x_1, x_2, x_3, \dots$ etc. respectively.

Let G be the centre of gravity of the entire area A and let the distances be $\bar{x}$ and $\bar{y}$ from y axis and x axis respectively.

Thus $A = a_1 + a_2 + a_3 + \dots$

Now sum of moment of all the individual areas about y-y axis.

$$= a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots$$

and the moment of entire area about y y axis = $A\bar{x}$

Now equalising the two moment

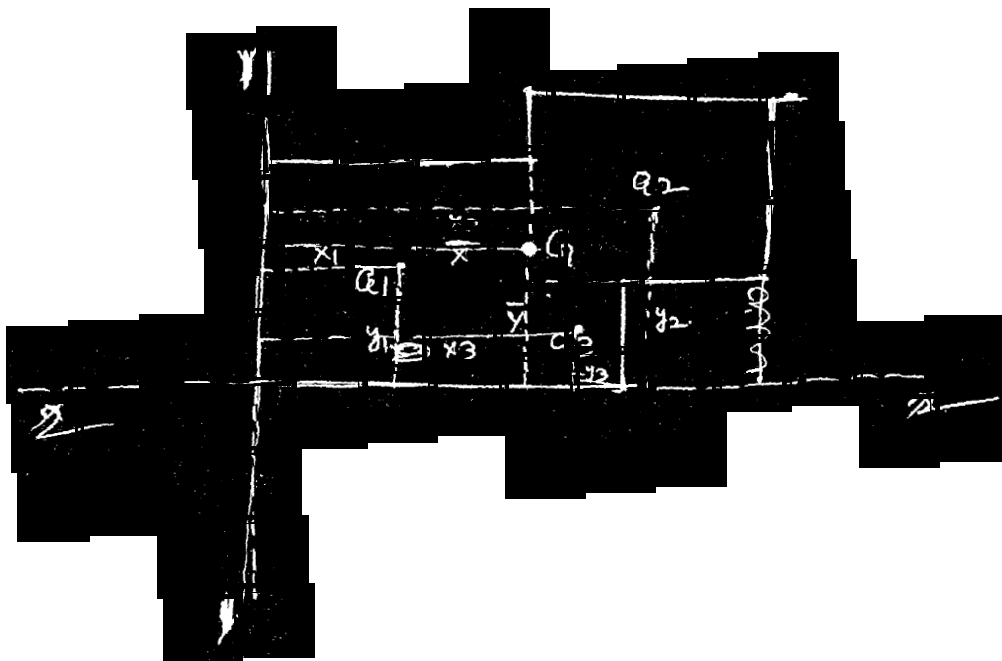
$$A\bar{x} = a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots$$

$$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots}{A}$$

$$\frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots}{a_1 + a_2 + a_3}$$

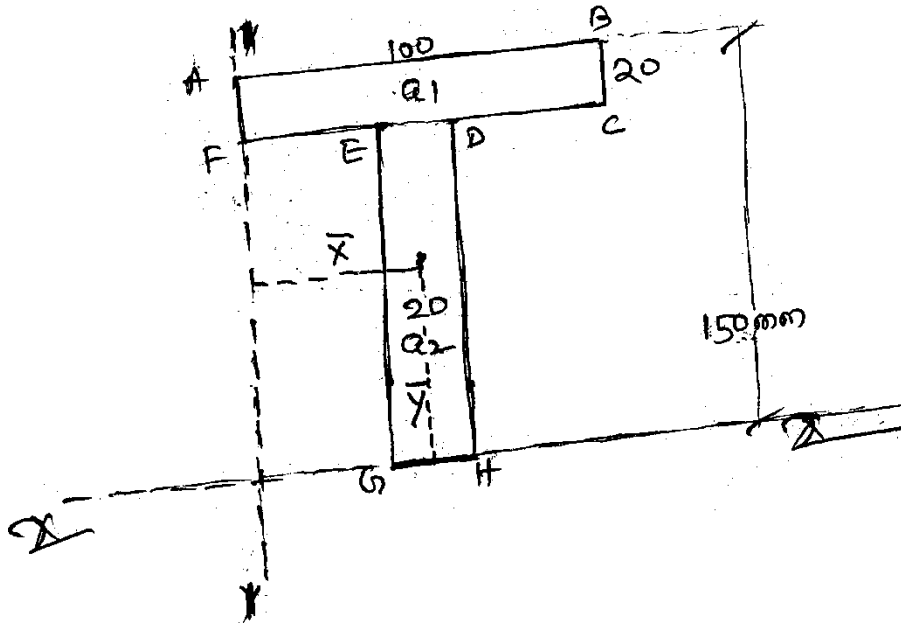
Similarly

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \dots}{a_1 + a_2 + a_3}$$



Example :

Locate the centre of gravity of a T-Section having dimension
100mm x 150mm x 20mm



Two reference axes are selected i.e. y-y and x-x at the extreme left and bottom of the given section.

The entire area of the T- Section is divided into two portions.

i.e.

$$\begin{aligned} a_1 &= \text{Area of the rectangle ABCD} \\ &= 100 \times 20 \\ &= 2000 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} y_1 &= \text{Distance of c.g. of the rectangle ABCD from x-x axes} \\ &= 140 \text{ mm} \end{aligned}$$

$$\begin{aligned} a_2 &= \text{Area of the rectangle EDHG} \\ &= 20 \times 130 \\ &= 2600 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} y_2 &= \text{distance of c.g. of the rectangle EDHG from x-x axes} \\ &= 65 \text{ mm} \end{aligned}$$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2000 \times 140 + 2600 \times 65}{2000 + 2600} = \frac{280000 + 169000}{4600}$$

$$= 97.61 \text{ mm}$$

As the given T-section is symmetrical about y-y axis,

$$\bar{x} = 50 \text{ mm}$$

Centre of gravity of section with cutout

Many a times some portions of a section are removed and thus the centre of gravity of the remaining portion is shifted to new locations.

The c.g. of such a section with cutout can be found out by considering the entire section first and then deducting the cutout section.

If the area of the main section and cut out are a_1 and a_2 respectively and c.g. distances are x_1, y_1 and x_2, y_2 from the reference lines, then the C.G. of the out section from the reference lines.

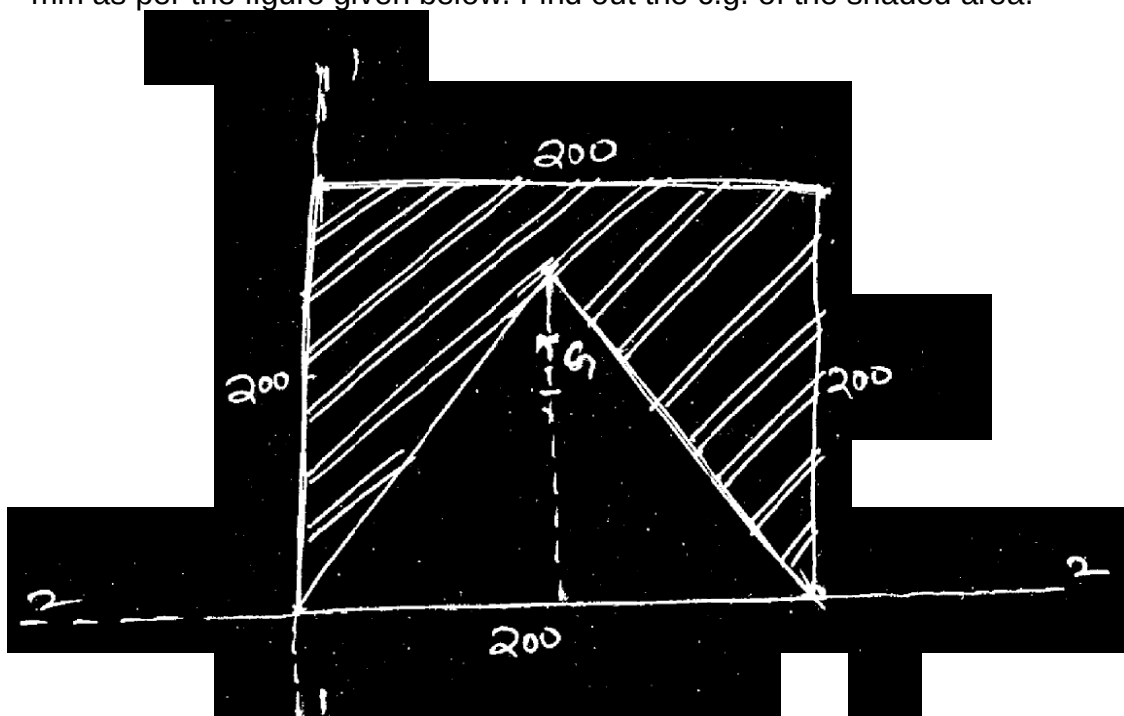
$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

&

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

Ex-

An isosceles triangle of side 200 mm has been cut from a square ABCD of side 200 mm as per the figure given below. Find out the c.g. of the shaded area.



Here,

$$a_1 = 200 \times 200 \\ = 40,000 \text{ sqmm}$$

$$x_1 = 100 \text{ mm}$$

$$y_1 = 100 \text{ mm}$$

$$a_2 = 1/2 \times 200 \times 173.2 \\ = 17,320$$

$$x_2 = 100$$

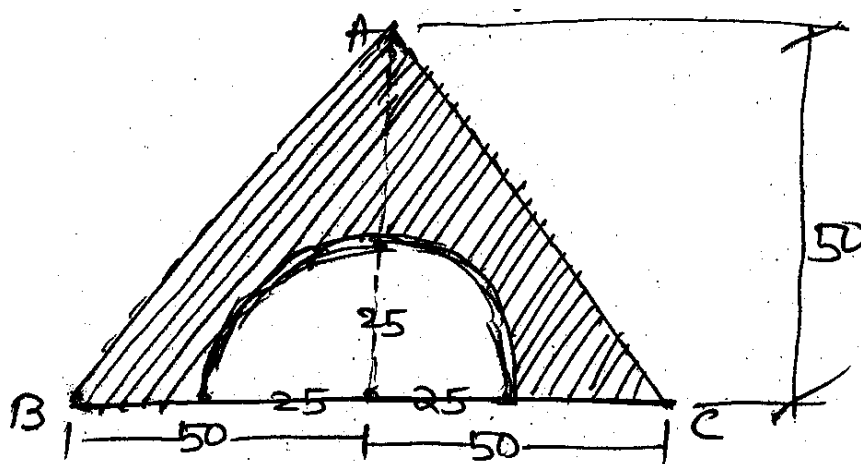
$$y_2 = 173.2 / 3 \\ = 57.73 \text{ mm.}$$

$$\therefore \bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} \\ = \frac{40,000 \times 100 - 17,320 \times 100}{40,000 - 17,320} \\ = 100 \text{ mm}$$

$$\bar{y} = \frac{40,000 \times 100 - 17,320 \times 57.73}{40,000 - 17,320} \\ = \frac{4000000 - 999883.60}{22,680} \\ = 132.28 \text{ mm}$$

Ex

Locate the centroid of the cut out section (Shaded area) as shown in the figure.



Here

$$a_1 = \frac{1}{2} \times 100 \times 50$$

$$= 2500 \text{ sqmm}$$

$$y_1 = \frac{h}{3}$$

$$= \frac{50}{3}$$

$$= 16.67 \text{ mm}$$

$$a_2 = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times 25^2}{2}$$

$$= 981.75 \text{ mm}^2$$

$$y_2 = \frac{4r}{3\pi}$$

$$= \frac{4 \times 25}{3 \times \pi}$$

$$= 10.61 \text{ mm}$$

$$\therefore y_1 = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

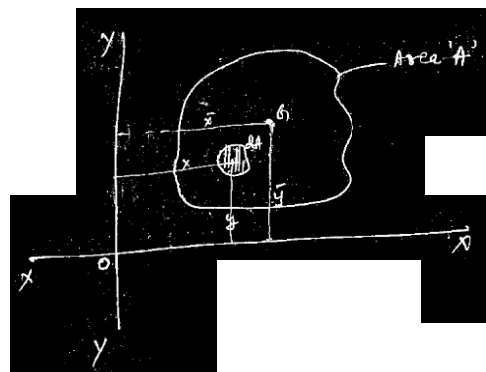
$$= \frac{2500 \times 16.67 - 981.75 \times 10.61}{2500 - 981.75}$$

$$= \frac{41675 - 10,416.37}{1518.25}$$

$$= 20.6 \text{ mm}$$

Moment of inertia (Second moment) of an area :

Moment of inertia of any plane area A is the second moment of all the small areas dA comprising the area A about any axis in the plane of area A.



Referring to the figure :

I_{yy} = moment of inertia about the axis y-y

$$= \sum x^2 dA$$

$$= \int x^2 dA$$

Similarly :

I_{xx} = Moment of inertia about the axis x-x

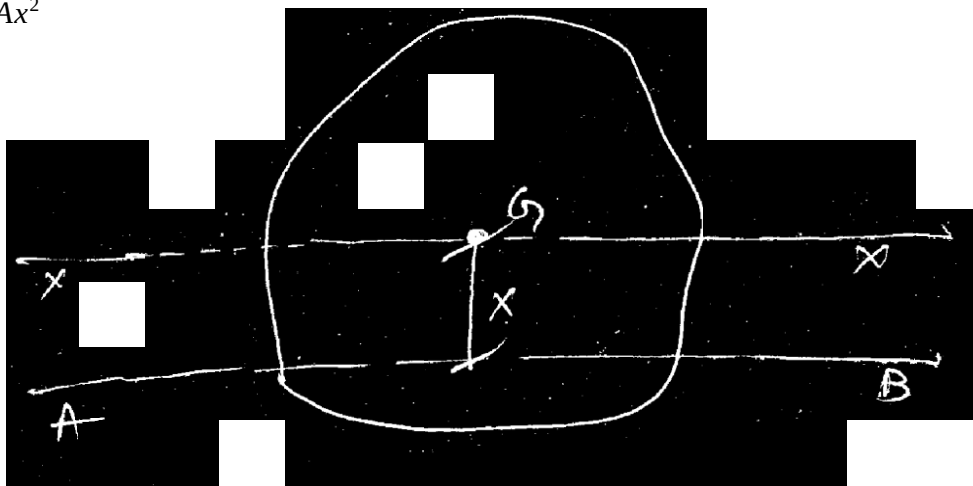
$$= \sum y^2 dA$$

$$= \int_A y^2 dA$$

Parallel axis theorem

If the moment of inertia of a plane area about an axis through the centroid is known, the moment of inertia about any other axis parallel to the given centroidal axis can be found out by parallel axis theorem. It states that if I_{cg} is the moment of inertia about the centroidal axis, then moment of inertia about any other parallel axis say AB at a distance of x from the centroidal axis is given by.

$$I_{AB} = I_{xx} + Ax^2$$



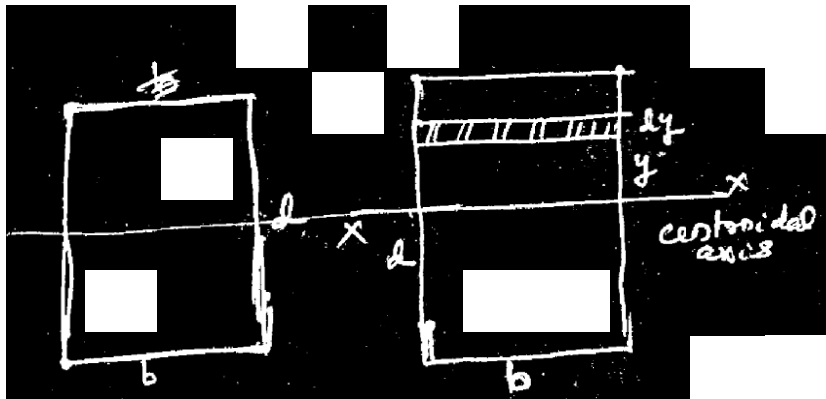
Perpendicular axis theorem

It states that second moment of an area about an axis perpendicular to the plane of the area through a point is equal to the sum of the second moment of areas about two mutually perpendicular axes through that point.

The second moment of area about an axis perpendicular to the plane of the area is known as polar moment of inertia.

$$I_{zz} = I_{xx} + I_{yy}$$

Moment of inertia of a rectangular section :



Let us consider as elementary strip of thickness dy at a distance of y from the centroidal axis $x-x$.

$\therefore$ Area of the strip of this elementary area = $b \cdot dy$

& MI about $x-x$ axis = $A \cdot y^2$

$$= b \cdot dy \cdot y^2$$

Now M.I. of the entire area.

$$= I_{xx}$$

$$= \int_{-d/2}^{d/2} b y^2 dy = b \cdot \frac{y^3}{3} \Big|_{-d/2}^{d/2}$$

$$= \frac{bd^3}{12}$$

Similarly it can be shown that $I_{yy} = \frac{db^3}{12}$

Let us consider a circle of radius r and an elementary ring of thickness dy at a distance of y from the centre 'O'.

Now area of this elemental ring = dA

$$= 2\pi y \cdot dy$$

M.I. of this ring about 'O' = $2\pi y \cdot dy \cdot y^2$

and M.I. of the entire area i.e. $I_{zz} = \int_0^r 2\pi y \cdot dy \cdot y^2$

$$= 2\pi \times \int_0^r y^3 dy$$

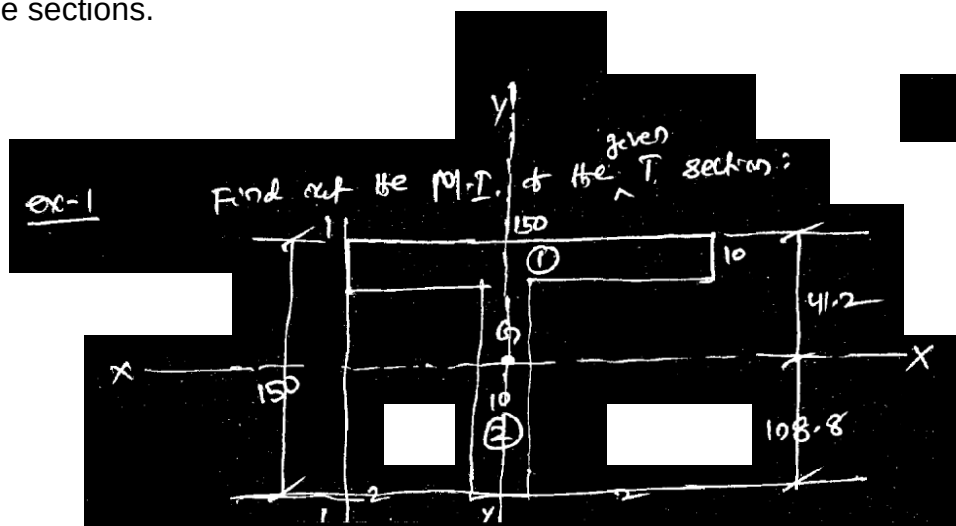
$$= 2\pi \times \frac{r^4}{4}$$

$$= \frac{\pi r^4}{2}$$

$$\begin{aligned} \therefore I_{xx} = I_{yy} &= \frac{I_{zz}}{2} \\ &= \frac{\pi r^4}{2} \times \frac{1}{2} \\ &= \frac{\pi r^4}{4} \\ &= \frac{\pi d^4}{64} \end{aligned}$$

Moment of inertia of built up section :

The second moment of area of a built up section which consists of a number of simple sections can be found out as the sum of the second moments of area of the simple sections.



$\bar{x} = 75\text{mm}$ (from the reference axis 1-1)

$$\begin{aligned}\bar{y} &= \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \quad (\text{from the reference axis 2-2}) \\ &= \frac{150 \times 10 \times 145 + 140 \times 10 \times 70}{150 \times 10 + 140 \times 10} \\ &= \frac{217500 + 98000}{1500 + 1400} \\ &= 108.8 \text{ mm}\end{aligned}$$

Moment of inertia of the T section can be found out by adding the M.I. of the two simple rectangular sections.

$$\begin{aligned}\text{M.I. of the 1st portion about the axis x-x} \\ &= \frac{150 \times 10^3}{12} + 150 \times 10 \times (41.2 - 5)^2 \\ &= 12,500 + 19,65,660 \\ &= 19,78,160\end{aligned}$$

M.I. of the 2nd portion about the axis x-x

$$\begin{aligned}&\frac{10 \times 140^3}{12} + 10 \times 40 \times (108.8 - 70)^2 \\ &= 22,86,666.7 + 21,07,616 \\ &= 43,94,282.7 \text{ mm}^4\end{aligned}$$

$\therefore$ M.I. of the T section about xx - axis

$$\begin{aligned}I_{xx} &= 1978160 + 4394282.7 \\ &= 63,72,442.7 \text{ mm}^4\end{aligned}$$

SIMPLE MACHINES

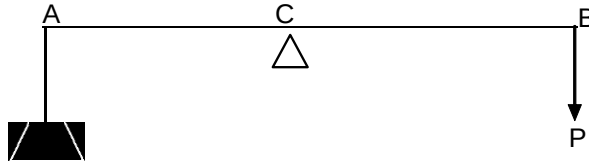
Before going to details of Simple Machines, we should know the following terms and their definitions.

Important Terms :

1. Simple Machine :

It is a contrivance in which an external force is applied at some point in order to overcome a bigger resistance at some other point.

Example



Taking moment about C, we get $W \times AC = P \times BC$

$BC > AC$

$$\frac{W}{P} = \frac{BC}{AC}$$

$W > P$ or $P < W$

W - resistance or load

P - external force or effort

2. Lifting Machine :

It is a Simple machine in which the load lifted acts as a resistance.

3. Mechanical Advantage :

Briefly known as M.A. is the ratio of the weight lifted to the effort applied.

$$M.A = \frac{W}{P}$$

No unit, it is a pure number.

4. Input of a Machine

The input of a machine is the work done on the machine.

In a lifting machine, it is measured by the product of the effort and the distance through which it has moved.

-

Input = P x Y

N.M.

Where - P = Effort in N and Y = distance moved by effort in M.

5. Output of a Machine

In a lifting machine, it is measured by the product of the weight lifted and the distance through which it has been lifted.

-

Input = W x X

N.M.

Where W is load in N and X is distance in M which the load is lifted.

6. Efficiency of a Machine

It is the ratio of output to the input of a machine.

Mathematically, $\eta = \frac{\text{Output}}{\text{Input}}$

$$\therefore \frac{W \times X}{P \times Y} = \frac{\left(\frac{W}{P}\right)}{\left(\frac{Y}{X}\right)} = \frac{M.A}{V.R}$$

7. Ideal Machine

If the efficiency is 100% i.e., output = Input, the machine is called as a perfect or an ideal machine.

8. Velocity Ratio (V.R)

It is the ratio of the distance moved by the effort (y) to the distance moved by the load (x).

$$\text{Mathematically, } V.R = \frac{Y}{X}$$

9. Effort lost in Friction

Let P_1 = effort required to lift the same load W under ideal condition the Effort lost in Friction.

$$P - P_1$$

In ideal case, input = output

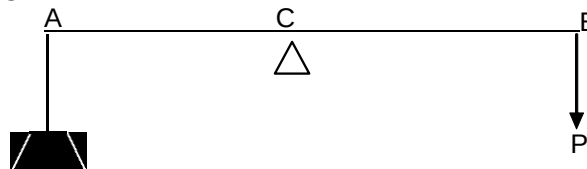
$$P_1 \times Y = W \times X$$

$$P_1 \frac{WX}{Y} = \frac{W}{\left(\frac{Y}{X}\right)} \frac{W}{V.R}$$

$$\text{Effort lost in Friction} = P - P_1$$

$$= P - \frac{W}{V.R}$$

10. Reversibility of a lifting Machine



When the effort P is withdrawn the end A will go down wards raising the end B upwards. We say that the lever is reversible.

So a reversible machine is that machine in which the load moved in the reverse direction after withdrawal of the effort which was applied to raise the load.

$$\text{Work lost in Friction} = P \times Y - Wx$$

In order that, the load may fall back after withdrawal of the effort.

$$\text{So } W.X > P.Y - Wx$$

$$\text{or } 2Wx > P.Y$$

$$\text{or } \frac{Wx}{Py} > \frac{1}{2}$$

$$\text{Efficiency} > \frac{1}{2} \text{ or Efficiency} > 50\%$$

Condition of Reversibility

The machine will be reversible when its efficiency is $> 50\%$

11. Self locking Machine

If a Machine is not capable of doing any work in the reverse direction after the effort is withdrawn and such a machine is called a non-reversible or self locking machine.

For this $\eta < 50\%$

12. Law of a lifting Machine

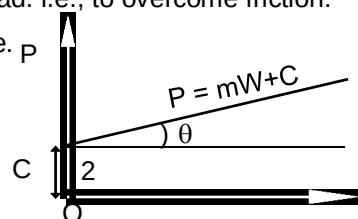
Where P = effort applied, W = load lifted, M and C are constants.

C - the effort required to move the machine under no load. i.e., to overcome friction.

13. Maximum Mechanical Advantage of a lifting Machine. P

$$\text{we know, } M.A = \frac{W}{P}$$

$$\text{Also we know, } P = MW + C$$



Then putting it

$$M.A = \frac{W}{MW + C} = \frac{1}{M + \frac{C}{W}}$$

neglecting $\frac{C}{W}$

$$(M.A)_{\max} = \frac{1}{M}$$

14. Maximum Efficiency of a lifting Machine

we know that

$$\eta = \frac{W/P}{VR} = \frac{W}{P \times VR}$$

Putting for max efficiency

$$\eta_{\max} = \frac{W}{(MW + C)VR} = \frac{1}{\left(M + \frac{C}{W}\right)VR}$$

$$\text{neglecting } \frac{C}{W}, \eta_{\max} = \frac{1}{M \times VR}$$

Example

In a lifting machine, an effort of 31 N raised a load of 1 KN. If efficiency of the machine is 0.75, what is its VR ? If on this machine, an effort of 61 N raised a load of 2 KN, what is now the efficiency ? what will be the effort required to raise a load of 5 KN ?

Solution

Data given $P_1 = 31 \text{ N}$, $W_1 = 1 \text{ KN} = 1000 \text{ N}$.

$\eta = 0.75$, $P_2 = 61 \text{ N}$, $W_2 = 2 \text{ KN} = 2000 \text{ N}$.

$\eta = ?$, $VR = ?$, $P_3 = ?$, $W_3 = 5 \text{ KN} = 5000 \text{ N}$.

VR

$$\eta_1 = \frac{MA}{VR} = \frac{1000}{\frac{31}{VR}}$$

$$\text{or } 0.75 = \frac{1000}{31 \times VR}$$

$$\text{or } VR = \frac{1000}{31 \times 0.75} = 43 \text{ (Ans)}$$

η_2

$$\eta_2 = \frac{MA}{VR} = \frac{2000}{61 \times 43} = 0.763 \text{ or } 76.3\% \text{ (Ans)}$$

$P_3 = ?$

$P = mw + c$

$$31 = m \times 1000 + c \dots\dots\dots (i)$$

$$61 = m \times 2000 + c \dots\dots\dots (ii)$$

subtracting (i) from (ii)

$$61 - 31 = 2000m - 1000m$$

$$\Rightarrow 30 = 1000m$$

$$\Rightarrow m = \frac{30}{1000} = 0.03$$

We have $31 = 1000M + C$

$$31 = 1000 \times 0.03 + C$$

$$\Rightarrow 31 = 30 + C$$

$$\Rightarrow C = 1$$

Then $P = 0.03W + 1$

Now for 5000N, $P = 0.03 \times 5000 + 1$

$$\Rightarrow \boxed{P = 151 \text{ N}} \text{ (Ans)}$$

Study of Simple Machine

Types

1. Simple wheel and axle.
2. Differential wheel and axle.
3. Weston's differential pulley block.
4. General pulley block.
5. Worm and wormwheel.
6. Worm geared pulleyblock.
7. Single purchase crabwinch.
8. Double purchase crab winch.
9. Pulleys :
 - (a) First system of Pulleys
 - (b) Second system of Pulleys
 - (c) Third system of Pulleys
10. Simple ScrewJack
11. Differential ScrewJack
12. Worm geared ScrewJack.

1. Simple wheel and Axle

In Fig 2.1 is shown a simple when and axle, in which the wheel A and axle B are keyed to the same shaft. A string is wound round the axle B, which carries the load W to be lifted. A Second string is wound round the wheel A in the opposite direction to that of the string on B.

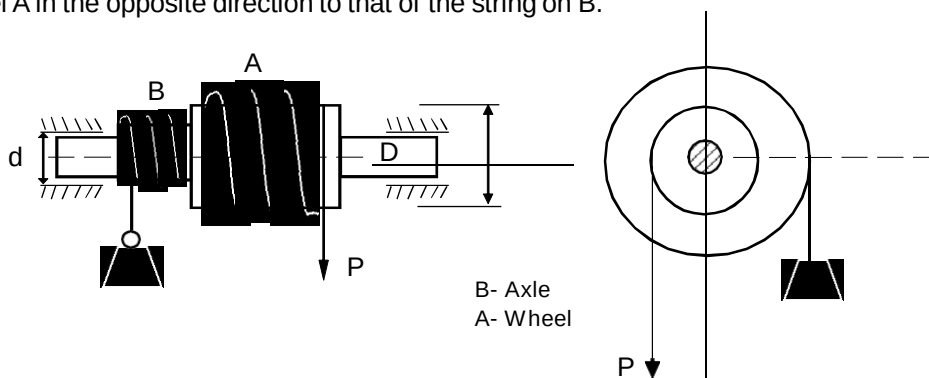


Fig 2.1 Simple wheel and axle

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since th two strings are wound in opposite directions, therefore, a downward motion of P will raise the load W.

Let D = Diameter of effort wheel

d = Diameter of the load axle

w = Load lifted, and

p = Effort applied to the load.

Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel makes one revolution the axle will also make one revolution.

In one revolution of effort wheel A, displacement of effort similarly the load displacement = πD

$$VR = \frac{D}{\pi d} \cdot \frac{D}{d}$$

$$MA = \frac{W}{P}$$

$$\eta = \frac{M.A}{V.R}$$

If t_1 = thickness of the rope used over the wheel.

and t_2 = thickness of the rope used over the axle.

$$\text{then } VR = \frac{D + T_1}{d + T_2}$$

Example

In a wheel and axle machine, the diameter of the wheel is 100cm and that of the axle is 10 cm. The thickness of the load on the wheel is 3 mm and that on the drum is 6 mm. In this machine a load of 500 N is lifted as an effort of 100 N. Determine the efficiency and state whether the machine is reversible at this load.

Data given : $D = 100 \text{ cm}$, $W = 500\text{N}$

$d = 10 \text{ cm}$, $P=100\text{N}$

$T_1 = 3\text{mm}$

$T_2 = 6\text{mm}$

To find, $n = ?$

$$MA = \frac{W}{P} = \frac{500}{100} = 5$$

Solution

$$M.A = VR = \frac{D + t_1}{d + t_2}$$

$$\Rightarrow VR = \frac{(100 + 0.3)\text{cm}}{(10 + 0.6)\text{cm}} = 9.46$$

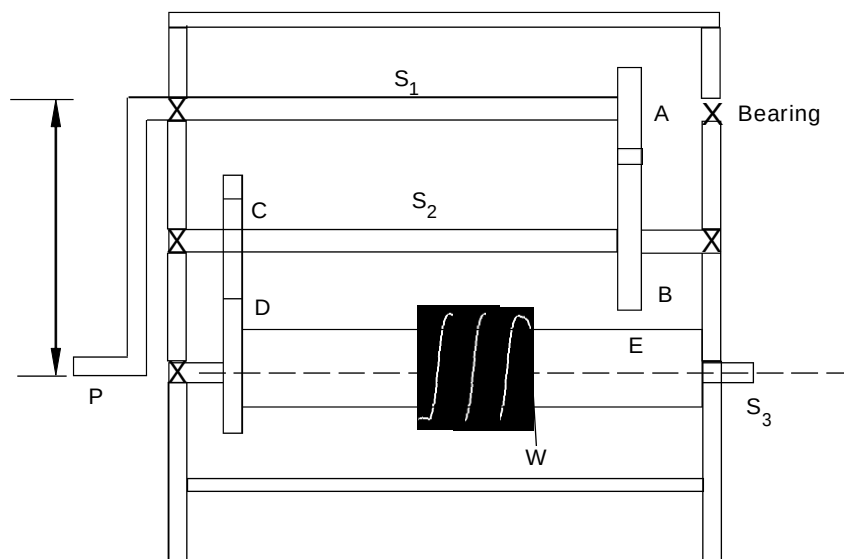
$$\text{and } \eta = \frac{MA}{VR} = \frac{5}{9.46} \times 100 = 52.85\% \text{ (Ans)}$$

Since efficiency is greater than 50%, the machine is reversible.

Double Purchase crab

Double or Purchase winch

Diagram



Single Purchase crab

Single purchase crab consists of a Pinion (A), which is engaged with a Spur gear B. Gear A and the effort handle are fixed to the same shaft (G_1). Also, a drum C and the gear b are fixed to the same shaft (S_2). Load is lifted by a string wound round the drum.

Calculation

Let T_A = number of teeth of Pinion(A)

T_B = number of teeth of the gear (B)

L = Length of the effort handle

D = diameter of the load drum.

When the effort handle is rotated once, the distance moved by effort = $2\pi L$ when the handle is rotated once, the Pinion (A) also rotates once, i.e., $N_A = 1$ rev, where N_A = revolution of Pinion A.

Let N_B = revolution of gear B

Then we know for gear Drive

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\therefore N_B \times \frac{T_A}{T_B} = 1 \times \frac{T_A}{T_B} = \frac{T_A}{T_B}$$

Revolution of Pinion C is the same as that of gear (B) as both are connected to same shaft.

$$\text{So } N_C = \frac{T_A}{T_B}$$

Again we know,

$$\frac{N_D}{N_C} = \frac{T_C}{T_D}$$

$$\therefore N_D = N_C \times \frac{T_C}{T_D}$$

Putting N_C value

$$\Rightarrow N_D = \frac{T_A}{T_B} \times \frac{T_C}{T_D}$$

Spur gear (D) and a drum (E) are mounted on a third shaft S_3 .

So when the effort handle makes one revolution, the gear D and drum E make $\frac{T_A T_C}{T_B T_D}$ revolution.

$$\text{So, the load is lifted through a distance} = \pi D \times \frac{T_A T_C}{T_B T_D}$$

Where D = diameter of the load drum E

$$\text{Hence Velocity Ratio} = \frac{2L}{\pi D \times \frac{T_A T_C}{T_B T_D}}$$

$$\boxed{VR = \frac{2L}{D} \times \frac{T_B T_D}{T_A T_C}}$$

Example

In a double purchase crab, the pinions have 15 and 20 teeth while the spur wheels have 45 and 42 teeth. The effort handle is 40 cm. While the effective diameter of the drum is 15 cm, if the efficiency of the winch is 40%, what load will be lifted by an effort of 250N applied at the end of the handle ?

Data Given

$$T_A = 15, T_B = 45$$

$$T_C = 20, T_D = 40$$

$$L = 40 \text{ cm } D = 15 \text{ cm}$$

$$\eta = 40\%, P = 250 \text{ N}$$

To find $W = ?$

Solution

we know,

$$VR = \frac{2\ell}{0} \times \frac{T_B T_D}{T_A T_C}$$

$$\text{Or } VR = \frac{2 \times 40}{15} \times \frac{45 \times 40}{15 \times 20} = 32$$

$$\eta = \frac{M.A}{VR} = \frac{W / 250}{15}$$

$$\text{Or } 0.4 = \frac{W}{250 \times 32}$$

$$\Rightarrow W = 3200 \text{ N} = 32 \text{ kN (Ans)}$$



SINGLE PURCHASE CRAB WINCH

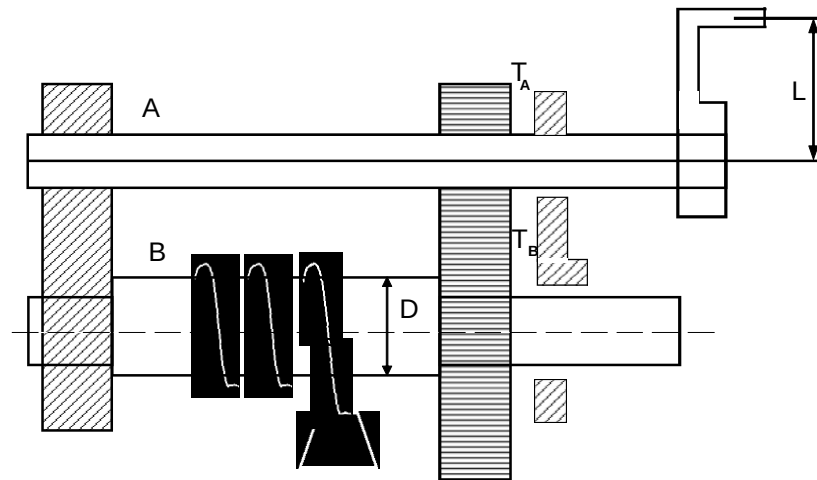


Fig 2:3 Single Purchase Crab Winch. In a Single Purchase Crab Winch, a rope is fixed to the drum and wound. A spur gear (B) is mounted on the load drum. Another pinion A is geared with (B) and connect to effort wheel.

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

If, $N_A = 1$

$$N_B = \frac{T_A}{T_B}$$

Distance moved by load in N_B revolution

$$= \pi D \times \frac{T_A}{T_B}$$

$$VR = \frac{Y}{X} = \frac{2\ell}{\pi D \times \frac{T_A}{T_B}} = \frac{2\ell}{D} \times \frac{T_B}{T_A}$$

Example

In a Single Purchase Crab, the number of Teeth on Pinion is 25 and on the Spur wheel 250, Radio of the drum and handle are 1500 mm and 300 mm respectively. Find the efficiency of the machine and the effect of friction, if an effort of 20 N can lift a load of 300 N.

Data given

$$T_A = 25$$

$$T_B = 250$$

$$D = 2 \times 150 = 300 \text{ mm} = 30 \text{ cm}$$

$$\ell = 300 \text{ mm} = 30 \text{ cm}$$

To Find, $\eta = ?$ and Friction = ?

Solution

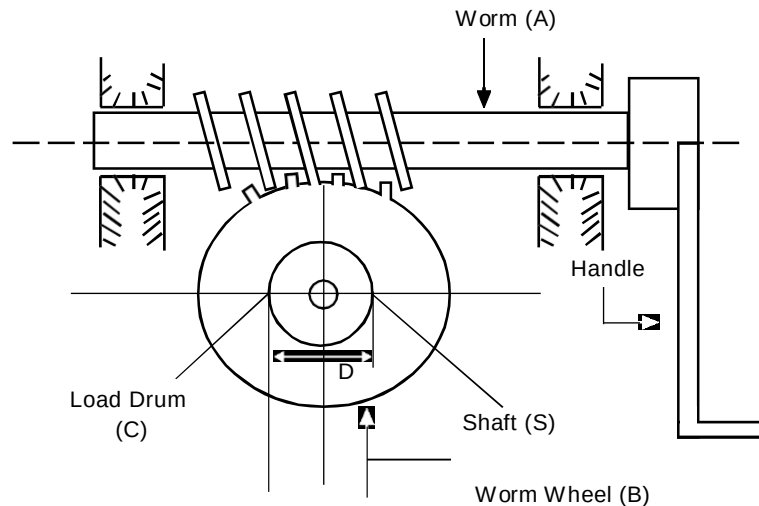
$$VR = \frac{2\ell}{D} \times \frac{T_B}{T_A}$$

$$= \frac{2 \times 30}{30} \times \frac{250}{25} = 20$$

$$\eta = \frac{MA}{VR} = \frac{300}{20} = 75 = 75\% \text{ (Ans)}$$

$$\begin{aligned} \text{Effort lost in Friction} &= P - \frac{W}{VR} \\ &= 20 - \frac{300}{20} = 5 \text{ N (Ans)} \end{aligned}$$

Worm and Worm Wheel



Description

Worm is threaded spindle (A)

Worm wheel is a spur gear (B)

The threads of the worm are engaged within the teeth of the worm wheel. A load drum (C) is mounted on the same shaft (S) as that of the worm wheel. A string hangs vertically from the load drum string hangs vertically from the load drum (C) and the load (W) to be lifted is attached to the string at its free end and as shown above. The effort (P) is applied at the end of a handle fitted at one end of the worm.

When the worm is rotated by application of effort at the end of the handle, the worm wheel and the drum (C) rotate winding the string round the surface of the drum. In this way the load is lifted the Worm (A) may have single start threads as 'multi start threads'.

If the worm has single start threads, in one rotation of the worm, one teeth.

If the worm has double start threads is one rotation of the worm, two teeth of the worm wheel will move, and so on.

Culcation of VR

Let ℓ = Length of the effort handle

n = no. of starts of the worm

$n = 1$, single start thread

$n = 2$, double start thread

T = number of teeth of the worm wheel.

D = Diameter of the load drum (C)

Considering on rotation of the effort handle, the distance through which effort (P) moves $= 2\pi\ell$

We Know

$$\frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$

If $N_A = 1$ rpm

$$\Rightarrow N_B = 1 \times \frac{T_A}{T_B} = \frac{T_A}{T_B}$$

Here for single start $T_A = 1$

$$\therefore N_B = \frac{1}{T_B} = \frac{1}{T}$$

In one rotation of the load drum the load is lifted to a height D

In N_B rotation the load lifted will be $N_B \times \pi D = \pi D \times \frac{1}{T}$

In Multi start thread the load lifted will be = $\frac{\pi D n}{T}$

$$VR = \frac{2\pi L}{\pi D \times \frac{n}{T}} = \frac{2L}{D} \times \frac{T}{n}$$

$$\therefore VR = \frac{2LT}{nD}$$

Example

In a worm and worm wheel, the worm is triple threaded and the drum which is rigidly fixed to the wheel having common axis of rotation, has a diameter of 50 cm. Determine the number of teeth on wheel of 40 turns of the worm move the load up by 60 cm. If the handle attached to the worm has an effective length of 30 cm and the load lifted weighs 25 kN. Calculate the M.A. and take efficiency as 40 %.

Solution

Given $n = 3$, $D = 50$ cm, $L = 30$ cm, $W = 25$ kN, $\eta = 40\%$

In 40 turns of the worm, distance described by effort = $40 \times 2\pi \times 30 = 7539.83$ cm

$$\text{Hence } VR = \frac{7539.82}{60} = 125.66$$

$$\text{But } VR = \frac{2LT}{Dn} = \frac{2 \times 30}{50} \times \frac{T}{3}$$

$$\Rightarrow T = 314.15$$

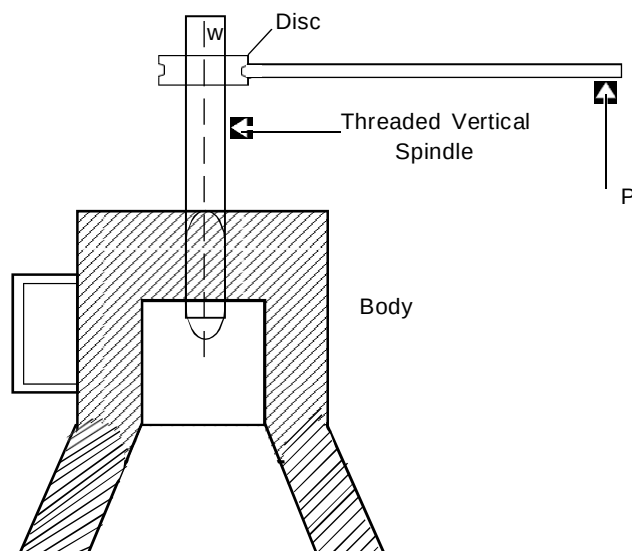
As teeth can not be fraction, $T = 314$.

$$\frac{MA}{VR} = \eta$$

$$\frac{MA}{125.66} = 0.4$$

$$\Rightarrow MA = 50.264 \text{ (Ans)}$$

SIMPLE SCREW JACK



A Simple Screw Jack consists a vertical threaded spindle. The threaded portion of the spindle passes through a nut cut in the body of the Screw Jack.

The Load W is placed on a disc fitted at the top of the vertical spindle. The disc is fitted with a handle. Effort P is applied at the end of this handle. When the screw is turned by P , the screw either moves up or comes down through a nut, depending upon the direction of rotation of the handle.

A Screw Jack is used to raise and hold up heavy machinery like motor car, truck, etc.

Calculation

Let ℓ = Length of the lever

P = Pitch of the Screw

$$\text{Velocity Ratio} = \frac{2\ell}{P}$$

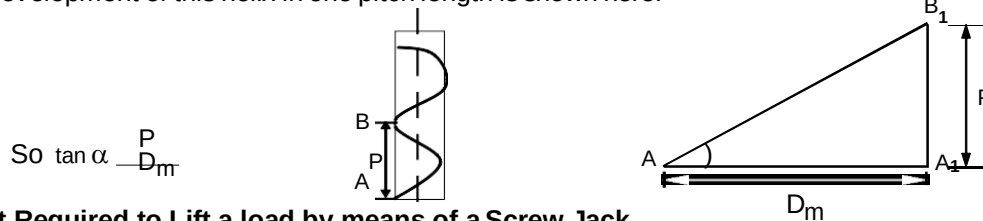
Relation between Mean diameter Pitch and Heli x angle of a Screw thread

$$D_m = \frac{D + d}{2}$$

Where D- outside diameter

d - Inside diameter

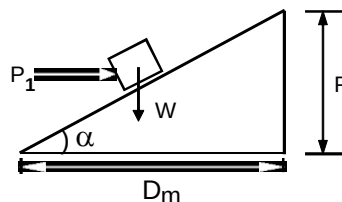
Threads of a Screw may be considered to be a metal strip wrapped round a cylinder in the form of helix. The development of this helix in one pitch length is shown here.



$$\text{So } \tan \alpha = \frac{P}{D_m}$$

Effort Required to Lift a load by means of a Screw Jack

When a load is lifted by means of a Screw Jack the case becomes equivalent to lifting a load (W) up an inclined plane of inclination α by applying a horizontal force. The equivalent inclined plane.



Let P_1 = horizontal force required to move a body of weight W up the inclined plane.

Then $P_1 = W \tan(\alpha + \theta)$

Where θ = Angle of Friction.

In case of Screw Jack, P_1 , acts at a distance of mean radius from the axis of the screw.

$$\begin{aligned} P \times \ell &= P_1 \times \frac{D_m}{2} \\ &= W \tan(\alpha + \theta) \times \frac{D_m}{2} \\ &= W \times \frac{D_m}{2} \left(\frac{\tan \alpha + \tan \theta}{1 - \tan \alpha \times \tan \theta} \right) \\ &= W \times \frac{D_m}{2} \left(\frac{\frac{P}{\pi D_m} + \mu}{1 - \frac{P}{\pi D_m} \times \mu} \right) \\ &= W \times \frac{D_m}{2} \left(\frac{P + \mu \pi D_m}{\pi D_m - \mu P} \right) \\ \text{or } P &= W \times \frac{D_m}{2\ell} \times \frac{P + \mu \pi D_m}{\pi D_m - \mu P} \end{aligned}$$

Effort Required to Lower a load by Screw Jack

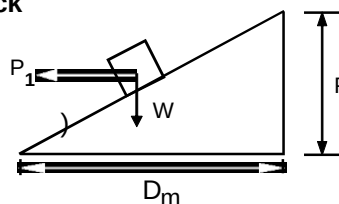
The Equivalent plane is shown below :

$$\begin{aligned} \text{Here } P \times \ell &= P_1 \times \frac{D_m}{2} = W \tan(\theta - \alpha) \times \frac{D_m}{2} \\ \text{or } P &= W \times \frac{D_m}{2\ell} \times \frac{\mu \pi D_m - P}{\pi D_m - \mu P} \end{aligned}$$

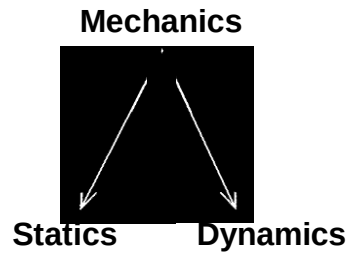
Efficiency when load is lifted

$$\eta = \frac{\text{Output work}}{\text{Input work}} = \frac{W \times P}{P \times 2\ell}$$

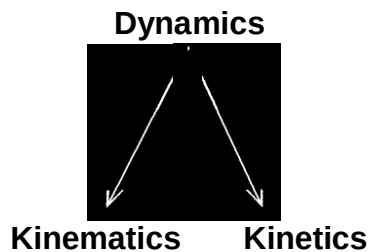
$$\text{A gain } P \times \ell = P_1 \times \frac{D_m}{2}$$



DYNAMICS



In contrast to statics, that deals with rigid bodies at rest, in dynamics we consider the bodies that are in motion. For convenience dynamics is divided into two branches called Kinematics and kinetics.



Kinematics :

kinematics is concerned with the space time relationship of given motion of a body without reference to the force that cause the motion.

Example : When a wheel rolls along a straight level track with uniform speed, the determination of the shape of the path described by a point on its rim and of the position along with path that the chosen point occupy at any given instant are problems of kinematics.

Kinetics : Kinetics is concerned with the motion of a body or system of bodies under the action of forces causing the motion.

Example : 1) When a constant horizontal force is applied to a body that rests on a smooth horizontal plane, the prediction of the way in which the body moves is a problem of kinetics.

2) Determination of the constant torque that must be applied to the shaft of a given rotor in order to bring it up to a desired speed of rotation in a given interval of time is a problem of kinetics

Particle :

The whole science of dynamics is based on the natural laws governing motion of a particle or particles. A particle is defined as a material point without dimensions but containing a definite quantity of matter.

But in true sense of term there can be no such thing as a particle, since a definite amount of matter must occupy some space. However, when the size of a body is extremely small compared with its range of motion, it may in certain cases, be considered as a particle.

Example : 1) Star & planets, although many thousands of kilometers in diameter are as small compared with their range of motion that they may be considered as particles in space.

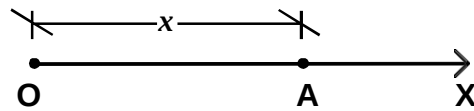
2) The dimensions of a rifle bullet are so small compared with those of its trajectory that ordinarily it may be considered as a particle.

Whenever a particle moves through space, it describes a trace that is called the path. Which may be either a space curve / tortuous path or a plane curve / plane path. In the simplest case when the plane path is a straight line the particle is said to have rectilinear motion or else, it is a curvilinear motion.

Displacement, velocity and acceleration function

These are the three quantities that are necessary to completely describe the motion of a particle in dynamics.

Displacement :

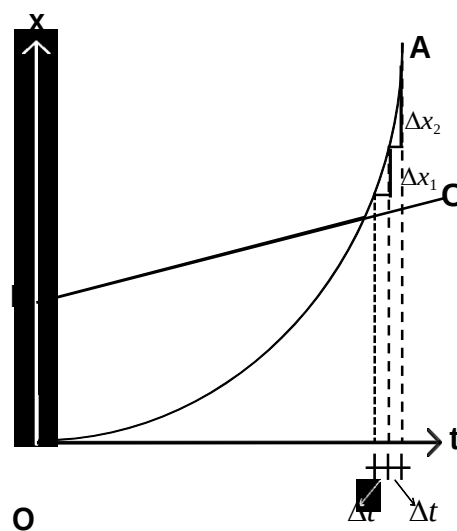


Assuming the motion of the particle along X-Axis, we can define the displacement of a particle by its x-coordinate, measured from the fixed reference point O. This displacement is considered as positive when the particle is to the right of the origin O and as negative when to the left.

As the particle moves, the displacement varies with time, and the motion of the particle is completely defined if we know the displacement x at any instant of time t . Analytically, this relation can be expressed by the general displacement – time equation, $x = f(t)$, where $f(t)$ stands for any function of time. This equation will take different forms depending upon how the particle moves along the x-axis. The displacement is measured by the unit of length.

Velocity :

In rectilinear motion, considering the case of uniform rectilinear motion, as represented graphically by the straight line BC, we see that for equal intervals of time Δt , the particle receives equal increments of displacement Δx . This velocity v of uniform motion is given by the equation $v = \Delta x / \Delta t$. this velocity is considered as positive if the displacement x is increasing with time and negative if it is decreasing with time.



In the more general case of non-uniform rectilinear motion of a particle as represented graphically by the curve OA, in equal intervals of time Δt , the particle receives unequal increments of displacement. Δx_1 & Δx_2 . Thus, if Δx denotes the increment of displacement received during the interval of time Δt , the average velocity during this time is given by the equation $v_{av} = \Delta x / \Delta t$. As the time interval Δt is taken

smaller and smaller, the motion during the interval becomes more & more nearly uniform so that this ratio $\Delta x / \Delta t$ approaches more and more closely to the velocity at any particular instant of time interval. Taking the limit approach we obtain the instantaneous

velocity of the particle $v = \Delta t \rightarrow 0 \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x}$.

Thus we see that the velocity time relationship of a moving particle can always be obtained by differentiating the displacement time equation. If the derivative is (+)ve i.e. the displacement increases with time, the velocity is positive and has a positive direction of the x-axis other wise it is negative. The velocity is measured by the unit of length divided by the unit of time.

Acceleration :

If the rectilinear motion of particle is non uniform its velocity is changing with time and we have acceleration. But in case of uniform motion, the velocity remains constant and there is zero acceleration. When the particle receives equal increments of velocity "v in equal intervals of time Δt , we have motion with constant acceleration as given by the equation $a = \frac{\Delta v}{\Delta t}$. Acceleration is consider positive when the velocity obtains positive increments in successive intervals of time and negative if the velocity is decreasing. However the acceleration of a particle may be positive when its velocity is negative of vice versa.

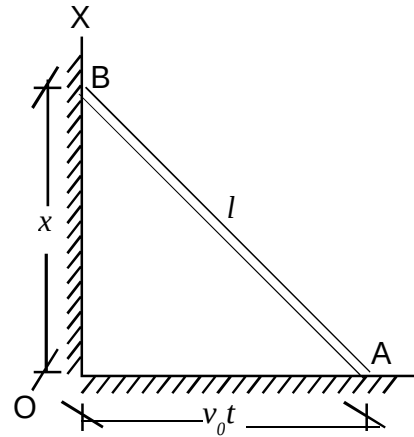
It the more general case when in equal intervals of time Δt , the increments of velocity Δv_1 and Δv_2 are unequal, we have a motion of the particle with variable acceleration. To obtain the acceleration of the particle at any instant t in such a case, we take the limit approach using the equation

$$a = \Delta t \rightarrow 0 \frac{\Delta v}{\Delta t} = \frac{d^2x}{dt^2} = \ddot{x}.$$

Thus, in any case of rectilinear motion of a particle, the acceleration time relationship can be expressed analytically by taking second derivative with respective time of the displacement time equation. The acceleration is measured by the unit of length divided by square of unit of time.

Problem 1:

A slender bar AB of length l which remains always in the same vertical plane has its ends A & B constrained to remain in contact with a horizontal floor and a vertical wall respectively as shown in the figure. The bar starts from the vertical position and the end A is moving along the floor with constant velocity v_0 . So that its displacement $OA = v_0 t$. Write the displacement – time, velocity-time and acceleration-time equations for the vertical motion of the end B of the bar.

**Solution :**

Let us choose x-axis along the vertical line of motion of end B, i.e. along OB with origin at O.

From the geometrical relationship of the figure, the displacement x of the end b is given by the equation.

$$x = \sqrt{l^2 - (v_0 t)^2}$$

which is the desired displacement time equation.

Differentiating w.r.t. time once, we get

$$\dot{x} = (-) \frac{v_0^2 t}{\sqrt{l^2 - v_0^2 t^2}} \quad \text{----- (2)}$$

the velocity time equation.

Differentiating w.r.t. time twice, we get

$$\ddot{x} = (-) \frac{v_0^2 l^2}{(l^2 - v_0^2 t^2)^{3/2}} \quad \text{----- (3)}$$

the acceleration time equation.

These expressions are valid in the interval $(0 < t < l/v_0)$

Problem 2 :

A particle starting from rest moves rectilinearly with constant acceleration a and acquires a velocity v in time t , traveling a total distance s . Develop formulae showing the relationship that must exist among any three of these quantities.

Solution :

Let us draw the velocity time diagram. Here

- i) The acceleration a is represented by the slope of the straight line OA.
- ii) The velocity v by the ordinate BA.
- iii) The time t by the abscissa OB.
- iv) The distance traveled s by the area OAB.

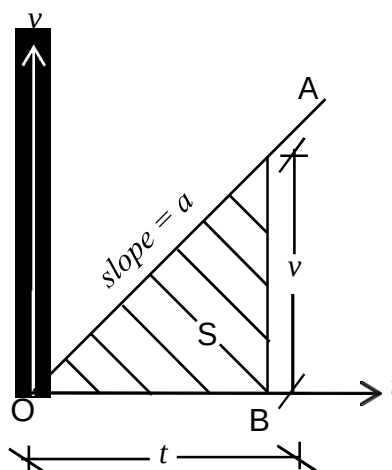
From geometry of the figure we may

write $v = at$ -----(1), $s = \frac{1}{2} vt$ -----(2)

Putting (1) in (2), $s = \frac{1}{2} at^2$ ----- (3)

Eliminating t from equations (1) & (2), $t = \frac{v}{a}$ & $s = \frac{1}{2} v \cdot \frac{v}{a}$ or $v = \sqrt{2as}$ (4)

But these formulae do not account for any initial displacement or initial velocity.



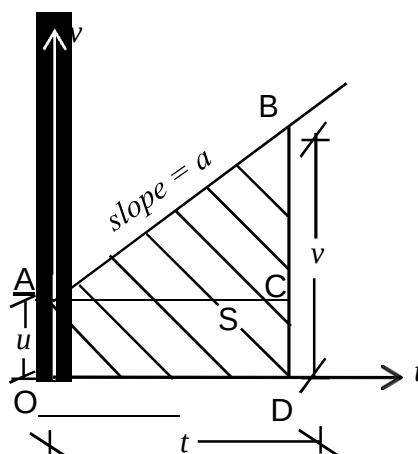
Problem 3 :

A particle moving with initial velocity u moves with constant acceleration a and acquires a velocity v after time t during which it travels a total distance s . Develop formulae showing relationship between these four quantities. Also give the expression for displacement, if it has an initial displacement s_0 before it starts moving.

Solution :

Acceleration a = slope of AB

Initial velocity u = OA along velocity axis.



Velocity v = ordinate BD

Time t = ordinate OD

Distance s = area of OABCD

From the geometry of the figure, we may write,

$$v = CD + CB = OA + CB = u + at \text{ ----- (1)}$$

$$s = \triangle ABC + \square OACD = ut + \frac{1}{2} at^2 \text{ ----- (2)}$$

$$\text{Eliminating } t, \quad t = \frac{v - u}{a}, \quad s = u \left(\frac{v - u}{a} \right) + \frac{1}{2} a \times \left(\frac{v - u}{a} \right)^2 = \frac{uv - u^2}{a} + \frac{(v - u)^2}{2a}$$

$$= \frac{2uv - 2u^2 + v^2 + u^2 - 2uv}{2a} = \frac{v^2 - u^2}{2a}$$

$$\text{or } v^2 - u^2 = 2as \text{ ----- (3)}$$

When there is initial displacement, $s = s_0 + ut + \frac{1}{2} at^2$.

Principles of dynamics

There are several axioms called the principle of dynamics which are concerned with the relationship between the kind of motion of a particle and the force producing it. These axioms are in fact, broad generalization of Kepler's observation on the motion of heavenly bodies and of carefully conducted experiment with the motion of earthly bodies. The first reliable experiments were made by Galileo in this connection who discovered the first two laws of motion of a particle. However, the complete set of principles and their final formulation were made by Newton after the name of which they are commonly called the Newton's laws of motion.

Newton's Laws of Motion :

First Law : This law also some times called the law of inertia is stated as "Every body continues in a state of rest or of uniform motion in a straight line unless and until it is acted upon by a force to change that state. For practical problems, we consider the surface of the earth as immovable and refer the motion of the particle with respect to the earth i.e. surface of the earth is taken as inertial frame of reference. But for the motion of heavenly bodies (planets & satellites), a system of coordinates defined by stars is taken as the immovable system of reference and their motion with respective these stars considered. From the first law it follows that any change in the motion of a particle is the effect of a force, that form the concept of force. However, the relation

between this change in velocity and the force that produces it given by the second law of dynamics.

Second Law :

The observations made by Galileo on the basis of his experiments on falling bodies and bodies moving along inclined planes, was verified by experiment on pendulums of various materials by Newton and generalized as second law of dynamics which states that the acceleration of a given particle / body is proportional to the force applied to it and acts in the direction of force.

In this law, as formulated above, there is no mention of the motion of the particle before it was acted upon by the force and hence the acceleration of a particle produced by a given force is independent of the motion of the particle. Thus a given force acting on particle produces the same acceleration regardless of whether the particle is at rest or in motion and also regardless of the direction of motion.

Again, there is no reference as to how many forces are acting on the particle. So if a system of concurrent forces is acting on the particle, then each force is expected to produce exactly the same acceleration as it would have if acting alone.

Thus, the resultant acceleration of a particle may be obtained as the vector sum of the accelerations produced separately by each of the forces acting upon it. Moreover, since the acceleration produced by each force is in the direction of the force and proportional to it, the resultant acceleration act in the direction of, and proportional to, the resultant force.

By using Newton's Second law, the general equation of motion of particle can be established as follows :

Let the gravity force i.e. weight of the particle W , acting alone produces an acceleration equal to g .

If instead of weight W , a force F acts on the same particle, then from the 2nd Law it follows that the acceleration a produced by this force will act the direction of the force and will be in the same ratio to the gravitational acceleration g as the force F to gravity force W .

$$\Rightarrow \frac{a}{g} = \frac{F}{W} \text{ or } \frac{W}{g} = \frac{F}{a} \text{ or } F = \frac{W}{g}a$$

This is the general equation of motion of a particle for which its acceleration at any instant can be obtained if the magnitude of force F is known. Also it is evident that for a given magnitude of force f , the acceleration produced is inversely proportional to the factor W/g , which is called mass of the particle denoted by m . This factor measures the degree of sluggishness (inherent resistance against motion) with which the particle yields (responds) to the action of applied force and is a measure of the inertia of the particle. Thus the second law of dynamics forms the basis of concept of mass. Thus using the notation $m = W/g$, the general equation of motion of a particle becomes $F=ma$.

Third Law :

By using the first two laws formulated above the motion of a single particle subjected to the action of given forces can be investigated. However in more complicated cases where it was required to deal with a system of particles or rigid bodies the mutual actions & reactions among them must be taken into account, which is given by Newton's third law. It states that "To every action there is always an equal and opposite reaction or in other words, the mutual action of any two bodies are always equal and oppositely directed".

This implies that if one body presses another, it is in turn is pressed by the other with an equal force in the opposite direction and if one body attracts another from a distance, this other attracts it with an equal and opposite force.

Example :

Sun attracts the earth with a certain force and therefore the earth attracts the sun with exactly the same force.

This holds good not only for the forces of gravitation but also for the other kind of forces such as magnetic or electrical forces. Thus a magnet attracts a piece of iron no more than the iron attracts the magnet.

The equation of motion can be used so solve two kinds of problems.

- 1) Motion of particle is given, to find the force necessary to produce such a motion.
- 2) The force is given and it is required to find the motion of the particle.

Let us discuss the first kind of problem :

Problem 4 :

A balloon of gross weight W falls vertically downward with constant acceleration a . What amount of ballast Q must be thrown out in order to give the balloon an equal upward acceleration a . Neglect air resistance.

Solution :

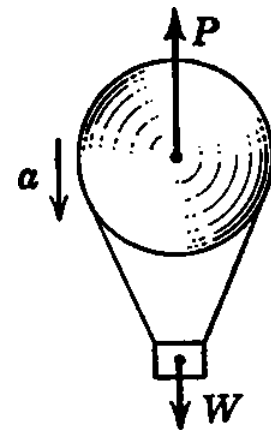
Case 1 : When the balloon is falling

The active forces on the balloon are :

- i) The total weight ' W ' including the ballast, that acts vertically down ward and.
- ii) The buoyant force P , acting upward representing the weight of the volume of air displaced.

The equation of motion in this case may be written as

$$\frac{W}{g}a = W - P \text{ ----- (1)}$$



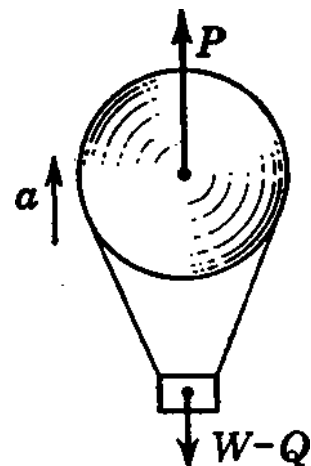
Case II : When the balloon is rising :

The active forces on the balloon are :

- i) The balance weight $(W-Q)$ excluding the ballast acting vertically downward.
- ii) The same buoyant force P acting upward as in the previous case.

The equation of motion in this case may similarly be written as :

$$\frac{W}{g}a = P - (W - Q) \text{ ----- (2)}$$



Solving equations (1) & (2), we obtain

$$Q = \frac{2Wa}{g + a} \text{ ----- (3)}$$

Particular cases :

i) When $a=0$, if the balloon was in equilibrium, $Q=0$ & $W=P$

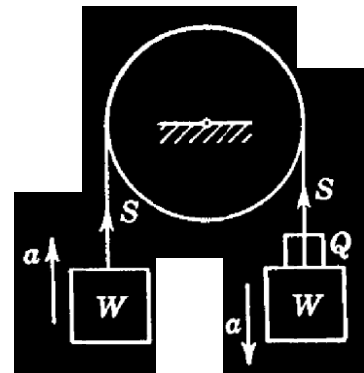
No ballast would have to be thrown out to rise with the same zero acceleration.

ii) When $a=g$, if the balloon was falling freely, $Q=W$ & $P=0$

It is impossible to throw out sufficient ballast to cause the balloon to rise.

Problem 5 :

Two equal weights W and a single weight Q are attached to the ends of the flexible but in extensible cord over hanging a pulley as shown in the figure. If the system moves with constant acceleration a as indicated by arrows, find the magnitude of the weight Q . Neglect air resistance and the inertia of the pulley.

**Solution :**

Here we have a system of two particles for which there are two equations of motion.

For the weight on the left :

The active forces are the tension in the cord 'S' acting upward and the weight of the body 'W' down ward.

The equation of the motion in this case may be written as

$$\frac{W}{g}a = S - W \text{ ----- (1)}$$

For the weight on the right :

The active forces are the same tension in the cord 'S' acting upwards but a down ward force of $(W+Q)$.

The equation of motion in this case may similarly be written as :

$$\left(\frac{W+Q}{g} \right) a = (W+Q) - S \text{ ----- (2)}$$

Solving the equations (1) & (2), we have

$$Q = \frac{2Wa}{g - a} \text{ ----- (3)}$$

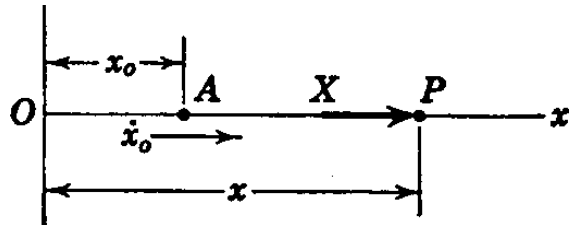
Particular Case : (Interpretation of the solution of the problem)

To produce an acceleration $a=g$ of the system would require a weight $Q =$, which implies that it is impossible for the weight on the RHS to attain practically a free fall condition.

Motion of a particle acted upon by a constant force :

Now, let us discuss the second kind of dynamics problem, in which the acting force is given and the resulting motion of the particle is required. To start with, let us consider the simplest case of a particle acted upon by a constant force, the direction of which remains unchanged. In this case, the particle moves rectilinearly in the direction of force with a constant acceleration.

Let the line of motion be along x-axis and the magnitude of force be denoted by X . Also the particle starts from A with initial displacement x_0 , when $t = 0$.



The equation of motion can be written as :

$$X = \frac{W}{g} a = \frac{W}{g} \dot{x} \text{ or } \dot{x} = \frac{X}{W/g} = a = \frac{d^2x}{dt^2} \text{ -----(1)}$$

To find the velocity $\dot{x}$ and displacement x as function of time, integrating this differential equation,

we get, $d\left(\frac{dx}{dt}\right) = d(\dot{x}) = a dt$ or $\frac{dx}{dt} = \dot{x} = at + C_1$, where C_1 is the constant of integration.

Boundary condition :

At $t = 0$, $\dot{x} = \dot{x}_0 \Rightarrow C_1 = \dot{x}_0$

$$\therefore \dot{x} = \dot{x}_0 + at \text{ -----(2)}$$

This is the general velocity time equation for the rectilinear motion of a particle under the action of a constant force X.

Writing equation (2) in the form - $\frac{dx}{dt} = \dot{x} + at$ or $dx = \dot{x}_0 dt + at dt$ and integrating again,

we obtain, $x = x_0 + \dot{x}_0 t + \frac{1}{2} at^2 + C_2$ -----(3)

Where C_2 again a constant of integration.

Boundary Condition :

At $t = 0$, $x = x_0 \Rightarrow C_2 = x_0$

$$\therefore x = x_0 + \dot{x}_0 t + \frac{1}{2} at^2$$

This is the general displacement time equation for the rectilinear motion of a particle under the action of a constant force X.

Thus the initial conditions influence the motion of a particle quite as much as does the acting force. Infact the initial condition represents the heredity of motion, while the acting force represents its environment.

Particular case :

Case – I – Freely falling body :

Acting force $X = W$ and $a = g$

Then from equations (2) & (3), $\dot{x} = \dot{x}_0 + gt$ ----- (2)'

$$x = x_0 + \dot{x}_0 t + \frac{1}{2} gt^2$$
 -----(3)'

Case II – Body starts to fall from rest and from the origin :

In this case $x_0 = 0$ and $\dot{x}_0 = 0$ and the equations (2) & (3) reduces to

$$\dot{x} = gt \text{ -----(2)''}$$

$$x = \frac{1}{2} gt^2 \text{ -----(3)''}$$

Problem 6 :

A particle projected vertically upward is at a height h after t_1 seconds and again after t_2 seconds. Find the height h and also the initial velocity v_0 with which the particle was projected.

Solution :

Neglecting air resistance, the active force in the particle is its own gravity force W which is always directed vertically downward.

Taking x-axis along the vertical line of motion, the origin at the starting point and considering the upward displacement as positive, from displacement time equation we have :

$$\text{When } t = t_1, h = v_0 t_1 - \frac{1}{2} g t_1^2 \text{ ----- (1)}$$

$$\text{\& When } t = t_2, h = v_0 t_2 - \frac{1}{2} g t_2^2 \text{ ----- (2)}$$

Multiply equation (1) with t_2 & equation (2) with t_1 & subtracting

$$h(t_2 - t_1) = \frac{1}{2} g t_1 t_2 (t_2 - t_1) \text{ or } h = \frac{1}{2} g t_1 t_2 \text{ ----- (3)}$$

$$\text{Equating equations (1) \& (2), } v_0 t_1 - \frac{1}{2} g t_1^2 = v_0 t_2 - \frac{1}{2} g t_2^2$$

$$\text{or } v_0(t_2 - t_1) = \frac{1}{2} g (t_2 - t_1)(t_2 + t_1) \text{ or } v_0 = \frac{1}{2} g (t_2 + t_1) \text{ ----- (4)}$$

Problem 7 :

From the top of a tower of height $h = 40\text{m}$ a ball is dropped at the same instant that another is projected vertically upward from the ground with initial velocity $v_0 = 20 \text{ m/s}$. However from the top do they pass and with what relative velocity ?

Solution :**Case 1 : For the ball dropped from the top :**

The top of the tower is taken as the origin and downward displacement is considered as positive. The initial displacement and initial velocity are zero. Acceleration due to its own gravitational force is downward. Thus the displacement time equation will be

$$x_1 = \frac{1}{2}gt^2 \text{ (1)}$$

Case 2 : For the ball projected from the ground :

The ground is taken as the origin and the upward displacement is considered as positive. Here initial displacement = 0 and initial velocity = v_0 upward while the acceleration due to gravity is down ward. Thus the displacement time equation will be

$$x_2 = v_0 t - \frac{1}{2}gt^2 \text{ (2)}$$

When the balls pass each other, we must have

$$x_1 + x_2 = h \text{ (3)}$$

Substituting the values of x_1 and x_2 from equation (1) & (2) in (3), we obtain,

$$\frac{1}{2}gt^2 + v_0 t - \frac{1}{2}gt^2 = h \text{ or } h = v_0 t \text{ (4)}$$

Putting numerical values, we have $t = \frac{v_0}{h} = \frac{40}{20} = 2 \text{ sec}$

Putting this value of t in equation (1) we find

$$(x_1)_{t=2} = \frac{1}{2} \times 9.81 \times 4 = 19.62m.$$

Relative Velocity :

The velocity time equation for case - I,

$$x_1' = \frac{dx_1}{dt} = \frac{1}{2}g \cdot 2t = gt \quad \downarrow$$

At time $t = 2 \text{ Sec}$, Velocity $x_1' = 9.81 \times 2 = 19.62m/s \downarrow$

The velocity time equation for case - II,

$$x_2' = \frac{dx_2}{dt} = v_0 - gt \uparrow$$

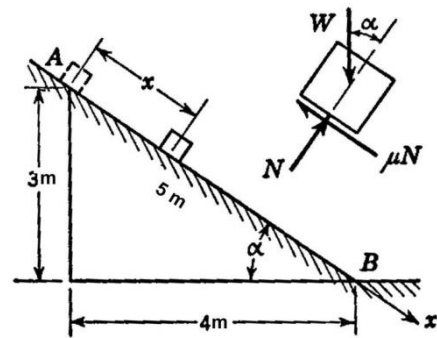
At time $t = 2 \text{ Sec}$, Velocity $x_2' = 20 - 19.62 = 0.38m/s \uparrow$

$\therefore$ Their relative velocity $v_R = x_1' - x_2' = 19.62 - (-0.38) = 20m/s$

Hence the balls pass 19.62 m. below the top of the tower with a relative velocity of 20 m/s. i.e. same as the initial velocity.

Problem 8 :

A small block of weight W is placed on an inclined plane as shown in the figure. What time interval t will be required for the block to traverse the distance AB , if it is released from rest at A and the coefficient of kinetic friction on the plane is $\mu=0.3$. What is the velocity at B .

**Solution :**

From the free body diagram of the block, the resultant force along the plane $X = W(\sin \alpha - \mu \cos \alpha)$ acting down ward.

Resulting acceleration $a = \frac{X}{W/g} = g(\sin \alpha - \mu \cos \alpha)$

For sliding of the block, a must be positive $(\sin \alpha - \mu \cos \alpha) > 0$ or $\tan \alpha \geq \mu$

Here $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$ & putting other numerical values

$$a = 9.81 \left(\frac{3}{5} - 0.3 \times \frac{4}{5} \right) = 3.53 \text{ m/s}^2$$

Again the initial displacement = 0 and initial velocity = 0

The displacement time equation becomes $x = 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2$

Putting numerical values $50 = \frac{1}{2} \times 3.53 \times t^2$ or $t^2 = 28.32$ or $t = 5.32 \text{ s}$

Substituting the value of t in the velocity time equation,

Velocity at B, $v_b = 0 + at = at = 3.53 \times 5.32 = 18.78 \text{ m/s}$

D' Alembert's Principle :

The differential equation of rectilinear motion of a particle $X = m\ddot{x}$ can be written in the form $X - m\ddot{x} = 0$, where x denotes the resultant in the direction of X-axis of all applied forces and m , the mass of the particle.

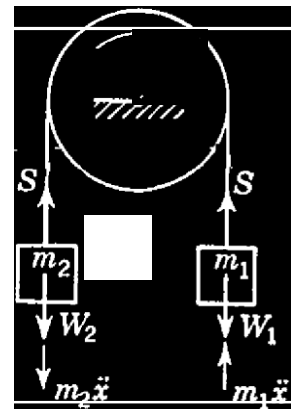
This equation of motion of particle is of the same form as an equation of static equilibrium and may be considered as an equation of dynamic equilibrium. For writing this equation, we need only to consider in addition to the real forces acting on the

particle, an imaginary force $-m\ddot{x}$. This force equal to the product of the mass of the particle and its acceleration and directed oppositely to the acceleration, is called inertia force of the particle.

D' Alembert was first to point out this fact that equation of motion could be written as equilibrium equations simply by introducing inertia forces in addition to the real forces acting on a system. This idea is known as the D' Alembert's principles and has definite advantage in the solution of engineering problems of dynamics.

Due to application of this principle, when dealing with a system having one degree of freedom, we need to write only one equation of dynamic equilibrium in stead of writing as many equation of motion as there are particles. It particularly becomes very useful when used in conjunction with the method of virtual work.

Problem 9 : Two unequal weights W_1 and W_2 are attached to the ends of a flexible but inextensible cord overhanging a pulley as shown in the figure ($W_1 > W_2$). Neglecting air resistance and inertia of the pulley, find the magnitude of acceleration of the weights.



Solution :

Here we have a system of particle connected between themselves and so constrained that each particle can have only a rectilinear motion.

Assuming motion of the system in the direction shown by the arrow on the pulley, there will be upward acceleration $\ddot{x}$ of the weight W_2 and downward acceleration $\ddot{x}$ of the weight W_1 . Denoting the masses by m_1 and m_2 respectively, the corresponding inertia forces act as shown in the figure. Assuming the tension in the string (String Reaction) to be S on both sides of the pulley, neglecting friction on the pulley, we can have a system of forces in equilibrium for each particle by adding the inertia forces to the real forces.

$$S - W_2 - m_2 \ddot{x} = 0 \text{-----(1)}$$

$$W_1 - S - m_1 \ddot{x} = 0 \text{----- (2)}$$

Eliminating S from these equations, $W_2 + m_2\ddot{x} = W_1 + m_1\ddot{x}$ ----- (3)

Again this problem can be conceived in another way i.e. since each particle is in equilibrium we can say that the entire system of forces is in equilibrium. So instead of writing separate equations of equilibrium for each particle, we can write one equation equilibrium for the entire system by equating to zero the algebraic sum of moment of all the forces (including the inertia forces) with respect of the axis of the pulley.

In this case, we need not consider the internal forces or reaction S of the system and can directly write.

$$(W_2 + m_2\ddot{x})r = (W_1 - m_1\ddot{x})r \text{ or } W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

from which
$$\ddot{x} = \left(\frac{W_1 - W_2}{W_1 + W_2} \right) g = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$$

Problem 10 :

Let us solve the previous problem No. 5 by D' Alembert's Principle.

Solution :

Here also we have a system of two particles so connected that it has single degree of freedom.

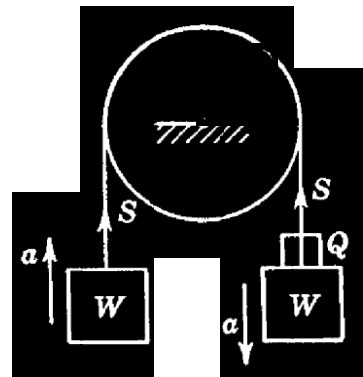
Taking moment about the centre of the pulley O,

$$\text{we have } \left(W + \frac{W}{g}a \right) r = \left\{ (W + Q) - \frac{(W + Q)}{g}a \right\} r$$

$$\text{or } W + \frac{W}{g}a = (W + Q) - \frac{(W + Q)}{g}a$$

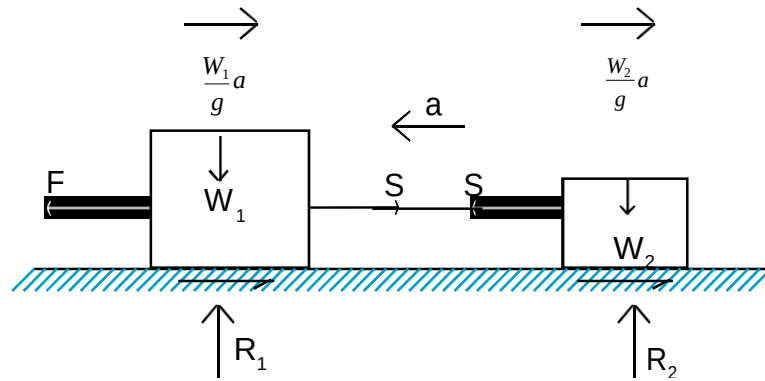
$$\text{or } \frac{W}{g}a = Q - \frac{Wa}{g} - \frac{Qa}{g} \text{ or } \frac{2Wa}{g} = Q \left(1 - \frac{a}{g} \right)$$

$$\text{or } \frac{2Wa}{g} = Q \left(\frac{g - a}{g} \right) \text{ or } Q = 2 \frac{Wa}{(g - a)}$$



Problem 11 :

Two bodies of weight W_1 & W_2 are connected by a thread and move along a rough horizontal plane under the action of a force F applied to the first body. If the coefficient of friction between the sliding surface of bodies and the plane is 0.3, determine the acceleration of the bodies and tension in the thread.



Given $m_1 = 80 \text{ Kg.}$, $m_2 = 20 \text{ Kg.}$, $\mu = 0.3$, $F = 400 \text{ N}$.

Solution :

Here also we have a system of two particles so connected that, it has a single degree of freedom. So we can apply D' Alembert's principle to the system and write only one equation of equilibrium for the entire system taking only the active forces but without considering the internal forces (reactions).

$$\text{Along horizontal direction, } F = \frac{W_1}{g}a + \frac{W_2}{g}a + \mu W_1 + \mu W_2$$

$$= \left(\frac{W_1 + W_2}{g} \right) a + \mu (W_1 + W_2)$$

$$\text{or, } \left(\frac{W_1 + W_2}{g} \right) a = F - \mu (W_1 + W_2)$$

$$\text{or, } a = \frac{F - \mu (W_1 + W_2)}{\left(\frac{W_1 + W_2}{g} \right)} \quad \text{or, } a = \frac{F - \mu g (m_1 + m_2)}{(m_1 + m_2)} = \frac{400 - 0.3 \times 9.81 (80 + 20)}{(80 + 20)} = 1.06 \text{ m/s}^2$$

Considering the equilibrium of second body

$$S = \frac{W_2}{g}a + \mu W_2 = W_2 \left(\frac{a}{g} + \mu \right) = m_2 (a + \mu g) = 20 (1.06 + 0.3 \times 9.81) = 80 \text{ N}$$

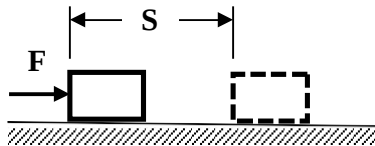
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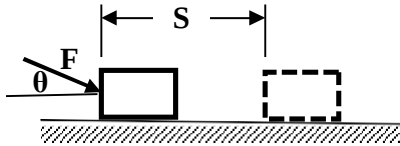
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Work, Energy and Power

Work: When a force is acting on a body and the body undergoes a displacement then some work is said to be done. Thus the work done by a force on a moving body is defined as *the product of the force and the distance moved in the direction of the force*.



Body moving with direction of force



Body not moving with direction of force

When force F acts on the body and body displaces in the direction of force as shown in fig. The work done by the force $F = \text{Force} \times \text{Distance} = F \times S$

When the force acts on the body at an angle θ to the horizontal and the body moves in horizontal direction by distance s , then work done by the force $F = \text{Component of the force in the direction of displacement} \times \text{distance} = F \cos \theta \times S$.

From definition of work it is obvious that unit of work is obtained by multiplying unit of force by unit of length. Hence, if unit of force is newton and unit of distance is meter then unit of work is N.m. So one N-m of work is denoted by one Joule(J). Hence one joule may be defined as *the amount of work done by one newton force when the particle moves one meter in the direction of the force*.

Energy: Energy is defined as the capacity to do work. There are many forms of energy like heat energy, mechanical energy, electrical energy, and chemical energy. In mechanics mostly discussed about mechanical energy. The mechanical energy may be classified into two forms i.e Potential energy and Kinetic energy.

Potential Energy is the capacity to do work due to position of the body. A body of weight 'W' held at a height 'h' possesses an energy Wh .

Kinetic Energy is the capacity to do work due to motion of the body. Consider a car moving with a velocity u m/s. If the engine is stopped, it still moves forward doing work against frictional resistance and stops at a certain distance s .

From the kinematic of the motion, $0 - u^2 = 2as$

$$a = -\frac{u^2}{2s}$$

From D'Alembert's principle,

$$F + \frac{W}{g}a = 0$$

$$F - \frac{W}{g} \times \frac{u^2}{2s} = 0$$

$$F = \frac{Wu^2}{2gs}$$

$$\text{Then, Work done} = F \times s = \frac{Wu^2}{2g}$$

This work is done by the energy stored initially in the body.

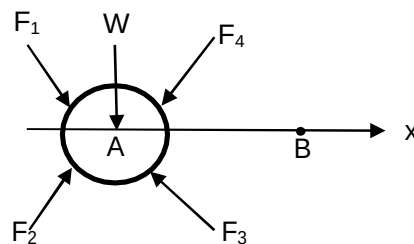
$$\text{Kinetic Energy} = \frac{1}{2} \times \frac{W}{g} v^2 = \frac{1}{2} m v^2$$

Where, m is the mass of the body and v is the velocity of the body.

Unit of energy is same as that of work, since it is nothing but capacity to do work. It is measured in joule (N-m) or kilo Joule (kN-m).

Power: It is defined as the time rate of doing work. Unit of power is watt (w) and is defined as one joule of work done in one second. In practice kilowatt is the commonly used unit which is equal to 1000 watts.

Work Energy equation: Consider the body shown in figure below subjected to a system of forces $F_1, F_2, \dots$ and moving with acceleration a in x-direction. Let its initial velocity at A be u and final velocity when it moves distance $AB=s$ be v . Then the resultant of system of the forces must be in x- direction. Let $F = \sum X$



$$\text{From Newton's second law, } F = \frac{W}{g}a$$

Multiplying both side of the equation by elementary distance ds , then

$$Fds = \frac{W}{g} a ds = \frac{W}{g} \frac{dv}{dt} ds = \frac{W}{g} dv \frac{ds}{dt}$$

$$Fds = \frac{W}{g} v dv$$

Integrating both sides for the motion from A to B,

$$\int_0^s F ds = \int_u^v \frac{W}{g} v dv$$

$$Fs = \frac{W}{g} \left[\frac{v^2}{2} \right]_u^v = \frac{W}{2g} (v^2 - u^2)$$

Work done = Final kinetic energy- Initial kinetic energy (It is called work energy equation)

This work energy principle may be stated as *the work done by system of forces acting on a body during a displacement is equal to the change in kinetic energy of the body during the same displacement.*

Conservation of Energy:

Conservative force: A force is said to be conservative if the work done by the force on a system that moves between two configurations is independent of the path of the system takes.

Non-Conservative forces: A force is said to be non-conservative if the work done by a force on a system depends on the path of the system follows.

Principle of conservation of energy, “ when a rigid body or a system of rigid bodies, moves under the action of the conservative forces, the sum of the kinetic energy (T) and the potential energy (V) of the system remains constant.

$$T + V = \text{constant or } \Delta T + \Delta V = 0 \text{ or } T_1 + V_1 = T_2 + V_2$$

Consider a conservative system; say a particle moving from position 1 to position 2.

As the particle moves from position 1 to position 2 it undergoes a change in potential energy,

$$\Delta V = V_2 - V_1$$

Where: V_1 and V_2 are the potential energies in positions 1 and 2 respectively.

As the particle displace from position 1 to 2, let U_{1-2} be the work of the forces that act on the particle. The work U_{1-2} does not depend on the path or the speed with which the particle moves on the path.

Consequently, $U_{1-2} = V_1 - V_2 = -\Delta V$

Then, the principle of work energy ($U_{1-2} = \Delta T$), yields $-\Delta V = \Delta T$

Where; $\Delta T = T_2 - T_1$ is change in kinetic energy from position 1 to position 2.

Thus relationship may be written as $\Delta T + \Delta V = 0$

$T + V = \text{constant}$

Where $T + V$ is called the mechanical energy of the particle.

Define Momentum & Impulse, Explain Conservation of energy and Linear momentum, Explain collision of elastic bodies and different co-efficient of restitution.

Solved Examples:

Example-1: A pump lifts 40m^3 of water to a height of 50m and delivers it with a velocity of 5m/sec. What is the amount of energy spent during the process? If the job is done in half an hour, what is the input power of the pump which has an overall efficiency of 70%?

Solution: Output energy of the pump is spent in lifting 40m^3 of water to a height of 50m and deliver it with the given kinetic energy of delivery.

Work done in lifting 40m^3 water to a height of 50m = Wh

Where W=weight of 40m^3 of water= 40×9810 N

(Note: 1m^3 of water weighs 9810 Newton)

Work done= Wh = $40 \times 9810 \times 50 = 19620000\text{Nm}$

Kinetic energy at delivery = $\frac{1}{2} \times \frac{W}{g} \times v^2 = \frac{1}{2} \times \frac{40 \times 9810}{9.81} \times 5^2 = 500000\text{Nm}$

Total Energy spent= $19620000 + 500000 = 20120000\text{Nm} = 20120\text{kJ} = 20.12 \times 10^6 \text{ Nm}$.

This energy is spent by the pump in half an hour i.e. in $30 \times 60 = 1800\text{sec}$.

Output power of pump = Output energy spent per second = $\frac{20120000}{1800} = 11177.8\text{watts} = 11.1778\text{kW}$.

Input power = $\frac{\text{Output power}}{\text{efficiency}} = \frac{11.1778}{0.7} = 15.9583\text{kW}$.

Example 2: For the same kinetic energy of a body, what should be the change in its velocity if its mass is increased four times?

Answer: Let ' m_1 ' be the mass of a body moving with a velocity ' v_1 '.

$$\text{Kinetic energy } T_1 = \frac{1}{2} m_1 v_1^2$$

When mass is increased four times let the velocity be v_2 .

$$m_2 = 4m_1$$

$$v_2 = ?$$

$$\therefore T_2 = \frac{1}{2} m_2 v_2^2$$

Substitute the value of m_2 in the above equation

$$T_2 = \frac{1}{2} 4m_1 v_2^2$$

Given

$$T_1 = T_2$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} 4m_1 v_2^2$$

$$v_1^2 = 4v_2^2$$

$$v_2^2 = \frac{v_1^2}{4}$$

$$v_2 = \sqrt{\frac{v_1^2}{4}}$$

$$v_2 = \frac{v_1}{2}$$

The velocity of the body is halved when its mass is increased four times.

Example 3: Calculate the time taken by a water pump of power 500 W to lift 2000 kg of water to a tank, which is at a height of 15 m from the ground?

Solution: Given $g = 10 \text{ m/s}^2$

Power of the water pump = 500 W

$$P = \frac{W}{t}$$

Since the water is lifted through a height of 15 m, work done is equal to the potential energy.

$$\therefore P = \frac{mgh}{t}$$

Mass of water (m) = 2000 kg

$$g = 10 \text{ m/s}^2$$

Height (h) = 15 m

$$t = ?$$

$$t = \frac{mgh}{P}$$

$$t = \frac{2000 \times 10 \times 15}{500}$$

$$t = \frac{20 \times 10 \times 15}{5}$$

$$t = 40 \times 15 = 600 \text{ s}$$

Time required to lift water = 600 s.

Example 4: A car weighing 1000 kg and travelling at 30 m/s stops at a distance of 50 m decelerating uniformly. What is the force exerted on it by the brakes? What is the work done by the brakes?

solution: Mass of the car (m) = 1000 kg

Initial velocity (u) = 30 m/s

Distance traveled (S) = 50 m

Since the car stops, final velocity (v) = 0

Work done by the brakes = kinetic energy of the car

$$W = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1000 \times (30)^2$$

$$= 500 \times 900$$

$$= 450000 \text{ J}$$

$$W = F \times S$$

$$F = \frac{W}{S}$$

$$= \frac{450000}{50}$$

$$= 9000 \text{ N}$$

The force exerted by the brakes is 9000N

Momentum and Impulse

It is clear from the discussion of the previous chapters that for solving kinetic problems, involving force and acceleration, D'Alembert's principle is useful and that for problems involving force, velocity and displacement the work energy method is useful. The Impulse-Momentum method is useful for solving the problems involving force, time and velocity.

Momentum: Momentum is the motion contained in a moving body. It is also defined as the product of an object's mass and velocity. It is a vector.

The SI units of momentum are $\text{kg} \cdot \text{m/s}$.

Impulse: Impulse is defined as a force multiplied by the amount of time it acts over. In calculus terms, the impulse can be calculated as the integral of force with respect to time. Alternately, impulse can be calculated as the difference in momentum between two given instances.

The SI units of impulse are $\text{N} \cdot \text{s}$ or $\text{kg} \cdot \text{m/s}$.

Impulse-Momentum Equation: As stated earlier impulse can be calculated as the difference in momentum between two given instances.

If F is the resultant force acting on a body of mass m in the direction of motion of the body, then according to Newton's 2nd law of motion,

$$F = ma$$

But, acceleration, $a = \frac{dv}{dt}$, v is velocity of body.

$$\text{Then, } F = m \frac{dv}{dt}$$

$$\text{i.e. } F dt = m dv$$

$$\int F dt = \int m dv$$

If initial velocity is u and after time interval t the velocity becomes v then

$$\int_0^t F dt = \int_u^v m dv$$

$$\int_0^t F dt = m[v]_u^v = mv - mu = \text{Impulse Momentum Equation}$$

The term $\int_0^t F dt$ is called the **impulse**. If the resultant force is in newton and time in second, the unit of impulse is N*sec.

If the resultant force F is constant during time interval t then, impulse is equal to $F \times t$.

The term $[mv - mu]$ is called change in momentum.

So, Impulse= Change in momentum= Final momentum- Initial momentum.

Impulse momentum equation can be stated as: *The component of the impulse along any direction is equal to change in the component of momentum in that direction.*

Note: Since the velocity is a vector, impulse is also a vector. The impulse momentum equation holds good when the direction of F , u and v are the same.

The impulse momentum equation can be applied in any convenient direction and the kinetic problem involving force, velocity and time can be solved.

Conservation of Linear Momentum:

The impulse momentum equation is: $\int_0^t F dt = mv - mu$

If the resultant force F is zero the equation reduces to $mv = mu$ i.e. Final Momentum = Initial Momentum, Such situation arises in many cases because the force system consists of only action and reaction on the elements of the system. The resultant force is zero, only when entire system is considered.

The principle of conservation of momentum may be defined as: *the momentum is conserved in a system in which resultant force is zero. In other words, in a system if resultant force is equal to zero, the initial momentum is equal to final momentum i.e. momentum is conserved.*

Example: When a person jumps off a boat, the action of the person is equal and opposite to reaction of boat. Hence the resultant force of system is zero. If w_1 is weight of the person and w_2 is weight of the boat, v is velocity of the person and the boat before the person jumps out of the boat and v_1, v_2 are the velocities of the person and the boat after jumping. According to conservation of momentum:

$$\frac{w_1 + w_2}{g} v = \frac{w_1}{g} v_1 + \frac{w_2}{g} v_2$$

Similar, equation also holds good when the system of a shell and gun is considered.

Note: It must be noted that conservation of momentum applies to entire system and not to individual elements of the system.

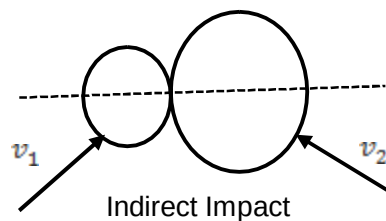
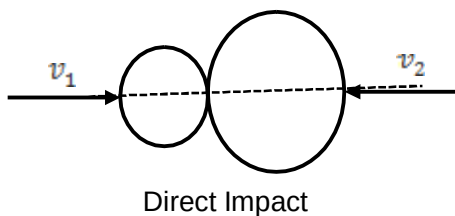
Collision: Collision is a dynamic event consisting of the close approach of two or more bodies resulting in an abrupt change of momentum and exchange of energy.

- A collision between two bodies is said to be impact, if the bodies are in contact for a short interval of time and exerts very large force on each other during this short period.
- On impact, the bodies deform first and then recover due to elastic properties and start moving with different velocities.
- The velocities with which they separate depend not only on their velocities of approach but also on the shape, size, elastic properties and line of impact.

Types of Impact:

According to Motion of Bodies:

- (a) **Direct impact:** If the motion of two colliding bodies is directed along the line of impact (common normal to the colliding surfaces), the impact is said to be direct impact.
- (b) **Indirect or Oblique impact:** If the motion of the one or both of the colliding bodies is not directed along the line of impact, the impact is known as oblique impact.



According to properties of bodies:

- (a) **Plastic impact:** The material properties of the colliding bodies are plastic or inelastic like putty, so the plastic deformation takes place at point of contact and both bodies move as a single body after impact. In this case only the principle of conservation of momentum hold good.

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

- (b) **Elastic collision:** The material properties of the colliding bodies are perfectly elastic, so the bodies regain its original shape after impact at the point of contact. In this case both the principle of conservation of momentum and energy hold good.

For conservation of momentum:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

For conservation of energy:

$$m_1 \frac{u_1^2}{2} + m_2 \frac{u_2^2}{2} = m_1 \frac{v_1^2}{2} + m_2 \frac{v_2^2}{2}$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

From above two equation, we get $u_1 + v_1 = u_2 + v_2$

$$u_1 - u_2 = v_2 - v_1$$

$$\text{Velocity of approach} = \text{Velocity of separation}$$

- (c) **Semi elastic collision:** The material properties of the colliding bodies are not perfectly elastic but partly elastic, so the bodies regain part of its original shape after impact at the point of contact. In this case the momentum is conserved but energy partly conserved.

$$v_2 - v_1 = e(u_1 - u_2)$$

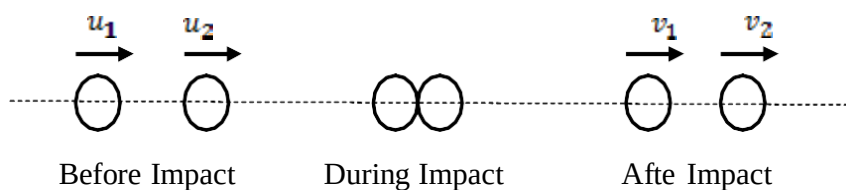
Where: e- coefficient of restitution which value is in between 0 to 1.

Coefficient of Restitution: During the collision the colliding bodies initially under goes a deformation for a small time interval and then recover the deformation in a further small time interval due to elastic property of the body. So the period of collision or time of impact consists of two time intervals i.e. *Period of Deformation* and *Period of Restitution*.

Period of Deformation: It is the time elapse between the instant of initial contact and the instant of maximum deformation of the bodies.

Period of Restitution: It is the time elapse between the instant of maximum deformation condition and the instant of separation of the bodies

Consider two bodies collide and move after collision as shown in Figure below.



Let m_1 –mass of the first body

m_2 –mass of the second body

u_1 –velocity of the first body before impact

u_2 –velocity of the second body before impact

v_1 –velocity of the first body after impact

v_2 –velocity of the second body after impact

Therefore, Impulse during deformation = $F_D dt$

Where, F_D refers to the force that acts during the period of deformation

The magnitude of F_D is varies from zero at the instant of initial contact to the maximum value at the instant of maximum deformation.

Similarly, Impulse during restitution= $F_R dt$

Where, F_R refer to the force that acts during period of restitution.

The magnitude of F_R is varies from maximum value at the instant of maximum deformation to zero at the instant of just separation of bodies.

At the instant of maximum deformation the colliding bodies will have same velocity. Let the velocity of bodies at the instant of maximum deformation is $U_{D max}$

Applying Impulse-Momentum principle for first body

$$F_D dt = m_1 U_{D max} - m_1 u_1 \quad (1)$$

And

$$F_R dt = m_1 v_1 - m_1 U_{D max} \quad (2)$$

Dividing Equation (2) by (1)

$$\frac{F_R dt}{F_D dt} = \frac{m_1 v_1 - m_1 U_{D max}}{m_1 U_{D max} - m_1 u_1}$$

i.e.

$$\frac{F_R dt}{F_D dt} = \frac{v_1 - U_{D max}}{U_{D max} - u_1}$$

(3)

Similarly for second body,

$$\frac{F_R dt}{F_D dt} = \frac{U_{D \max} - v_2}{u_2 - U_{D \max}} \quad (4)$$

From Equation (3) and (4)

$$\begin{aligned} \frac{F_R dt}{F_D dt} &= \frac{v_1 - U_{D \max}}{U_{D \max} - u_1} = \frac{U_{D \max} - v_2}{u_2 - U_{D \max}} \\ &= \frac{v_1 - U_{D \max} + U_{D \max} - v_2}{U_{D \max} - u_1 + u_2 - U_{D \max}} \\ &= \frac{v_1 - v_2}{u_2 - u_1} = \frac{v_2 - v_1}{u_1 - u_2} \\ &= \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} \end{aligned}$$

Newton conducted the experiments and observed that when collision of two bodies takes place relative velocity of separation bears a constant ratio to the relative velocity of approach, the relative velocities being measured along the line of impact. This constant ratio is called as the coefficient of restitution and is denoted by “e”. Hence

$$\text{Coefficient of restitution} = e = \frac{v_2 - v_1}{u_1 - u_2}$$

For perfectly elastic bodies the magnitude of relative velocity after impact will be same as that before impact and hence coefficient of restitution will be 1.

For perfectly plastic bodies the velocity of separation is zero as both bodies moving together after impact, so coefficient of restitution is zero.

For semi-elastic collision where some part of energy is conserved the value of coefficient of restitution is between 0 to 1.

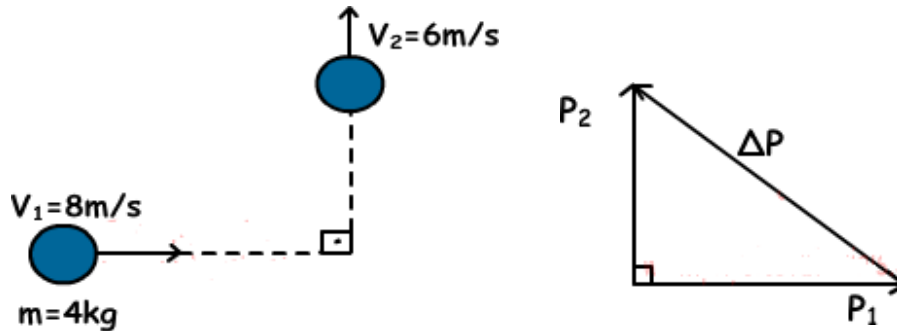
So the coefficient of restitution always lies between 0 to 1. The value of coefficient of restitution depends not only on the material property, but it also depends on shape and size of the body. Hence the coefficient of restitution is the property of two colliding bodies but not merely of material of colliding bodies.

Solved Examples:

Example 1: Ball having mass 4kg and velocity 8m/s travels to the east. Impulse given at point O, makes it change direction to north with velocity 6m/s. Find the given impulse and change in the momentum.

Solution:

Initial and final momentum vectors of ball are shown in the figure below.



$$P_1 = m \cdot V_1 = 4 \text{ kg} \cdot 8 \text{ m/s} = 32 \text{ kg} \cdot \text{m/s}$$

$$P_2 = m \cdot V_2 = 4 \text{ kg} \cdot 6 \text{ m/s} = 24 \text{ kg} \cdot \text{m/s}$$

$$\Delta P = P_2 + P_1 \text{ (vector addition)}$$

$$\Delta P^2 = P_2^2 + P_1^2 = m^2 (V_2^2 + V_1^2)$$

$$\Delta P^2 = 16 \cdot 100$$

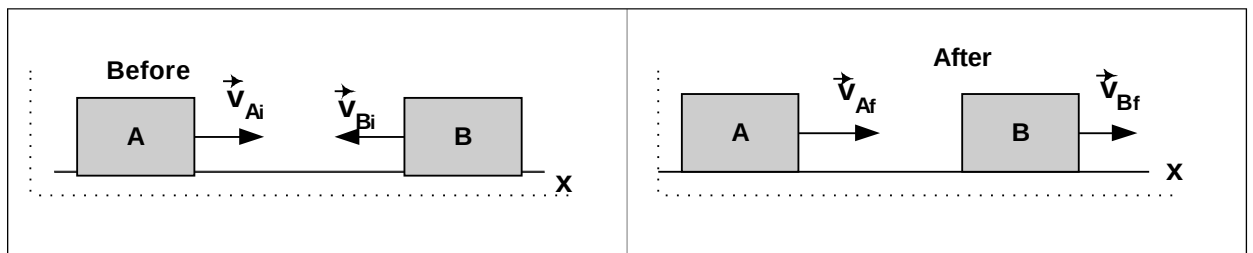
$$\Delta P = 40 \text{ kg} \cdot \text{m/s}$$

Impulse = change in momentum

$$I = \Delta P = 40 \text{ kg} \cdot \text{m/s}$$

Example 2: Two blocks are travelling toward each other. The first has a speed of 10 cm/sec and the second a speed of 60 cm/sec. After the collision the second is observed to be travelling with a speed of 20 cm/sec in a direction opposite to its initial velocity. If the weight of the first block is twice that of the second, determine: (a) the velocity of the first block after collision; (b) whether the collision was elastic or inelastic.

Solution: We have a collision problem in 1-dimension. We draw both 'before' and 'after' pictures and select a coordinate system as shown.



Since the surface is frictionless, and since no work is performed by either mg or the normal, then the net force acting on the system is 0, and we have conservation of linear momentum:

Thus adding the x-components we have: $m_1 v_{1i} - m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Since $m_1 = 2 m_2$ we find: $2 v_{1i} - v_{2i} = 2 v_{1f} + v_{2f} \rightarrow (2)(10) - (60) = 2 v_{1f} + 20$

Thus $2 v_{1f} = -60$ and $v_{1f} = -30$ cm/sec ('-' means to left).

The initial KE is given by: $KE_i = (1/2) m_1 (v_{1i})^2 + (1/2) m_2 (v_{2i})^2$. This gives:

$$= (1/2)(2 m_2)(10)^2 + (1/2) m_2 (60)^2 = (1/2)(200 + 3600) m_2 = 1900 m_2$$

The final KE is: $KE_f = (1/2) m_1 (v_{1f})^2 + (1/2) m_2 (v_{2f})^2$. This gives:

$$= (1/2)(2 m_2)(30)^2 + (1/2) m_2 (20)^2 = (1/2)(1800 + 400) m_2 = 1100 m_2$$

Since KE_f is not equal to KE_i , the collision is inelastic.

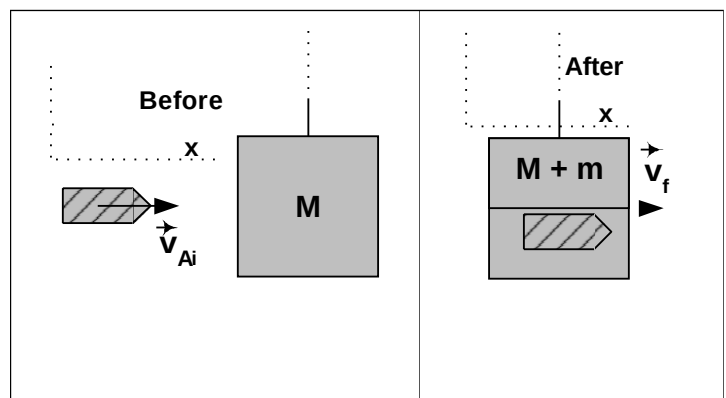
Example 3: A 0.005 kg bullet going 300 m/sec strikes and is imbedded in a 1.995 kg block which is the bob of a ballistic pendulum. Find the speed at which the block and bullet leave the equilibrium position, and the height which the center of gravity of the bullet-block system reaches above the initial position of the center of gravity.

Solution: A 'ballistic pendulum' is a device which can be used to measure the muzzle velocity of a gun. The bullet is fired horizontally into the block of wood and becomes imbedded in the block. The block is attached to a light rod and can swing like a pendulum. After the collision the 'bob' swings upward and the maximum height it reaches is determined. From this information plus the masses of the bullet and block, one can determine the velocity of the bullet. The critical point to note in this problem is that we have two distinct problems:

Problem 1: A collision problem. Apply Conservation of Linear Momentum and energy relation. Problem 2: A work-energy type problem. Apply conservation of total mechanical energy.

Part 1 We draw before&afte pictures, label the velocities, and choose a CS. Conservation of total linear momentum is:

$$m v_{Ai} + M v_{Bi} = (m + M) v_f$$



In component form (for x-components) this gives:

$$m v_{Aix} + M v_{Bix} = (m + M) v_{fx}$$

Since $v_{Bi} = 0$, we have:

$$v_f = (m v_{Ai}) / (m + M) = (.005)(300) / (2.00) = 0.75 \text{ m/sec.}$$

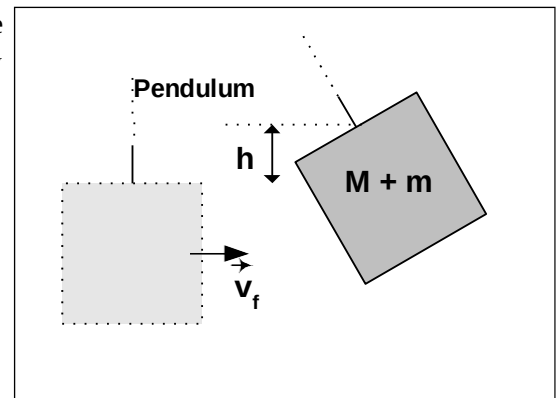
(Note: We were able to 'solve' the collision problem without an 'energy relation' since the collision was a perfectly inelastic collision. That is the two objects had the same final velocity. This condition is equivalent to an 'energy relation', since for such a collision the loss of KE is a maximum possible amount).

Part 2: In the work-energy part of the problem we note that the only force which performs work is gravity. Hence, we have only conservative forces present, and we have conservation of total mechanical energy.

We draw the figure indicating 'initial' and 'final' situations. We may choose the 0 level for gravitational potential energy anywhere we like. Hence, select $U_i = 0$.

Then: $KE_i + U_i = KE_f + U_f$.

$$(1/2)(m + M) v_f^2 = 0 + (m + M)g h$$



Here v_f is the 'initial' velocity in this part of the problem (0.75 m/sec) and $(m+M)$ is the combined mass of bullet & block.

Thus: $h = v_f^2 / 2g = (.75)^2 / (2)(9.8) = .0287$ m or 2.87 cm.

Note the reversal of this problem. If we know the masses and measure 'h', then from part 2 we can calculate ' v_f ' (the initial velocity of bullet & block in the 2nd part of the problem). This is the same as v_f , the final velocity in the collision problem. Thus using this we can calculate v_{Ai} the 'muzzle' velocity of the bullet.

Example 4: A pile hammer weighing 20 kN drops from a height of 750mm on a pile of 10kN. The pile penetrates 100mm per blow. Assuming that the motion of the pile is resisted by a constant force, find the resistance to penetrate of the ground.

Solution: Initial velocity of hammer= $u=0$

Distance moved= $h= 750\text{mm}=0.75\text{m}$

Acceleration, $g= 9.81\text{m/sec}^2$

Velocity at the time of strike = $\sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.75} = 3.836\text{m/sec}$

Applying the principle of conservation of momentum to pile and hammer, we get velocity V of the pile and hammer immediately after the impact,

$$\frac{20}{9.81} \times 3.836 = \frac{20 + 10}{9.81} V$$

$$V = \frac{20}{30} \times 3.836 = 2.557 \text{ m/sec}$$

Applying work energy equation to the motion of the hammer and pile, resistance R of the ground can be obtained,

$$(20 + 10 - R)s = \frac{20 + 10}{2g} (0 - V^2)$$

$$(30 - R)0.1 = \frac{20 + 10}{2 \times 9.81} (-2.557^2)$$

$$R = 130 \text{ kN}$$

Mechanics

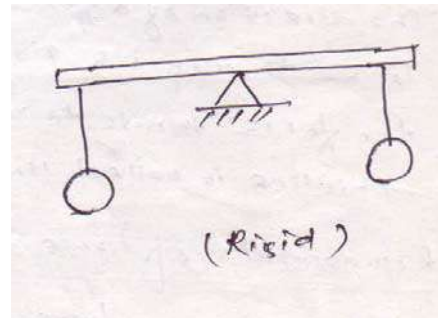
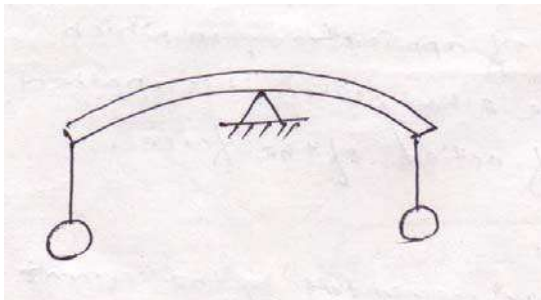
It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

Statics

Statics deal with the condition of equilibrium of bodies acted upon by forces.

Rigid body

A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid.

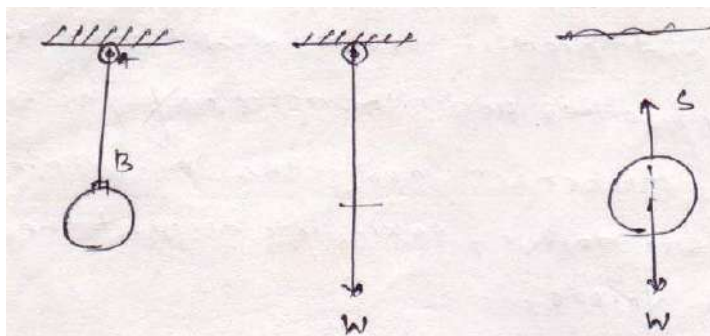


Force

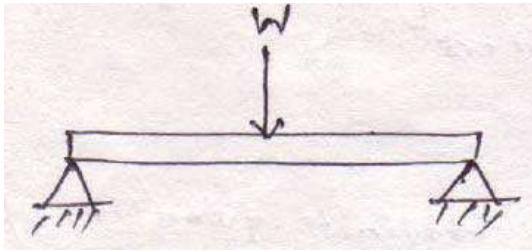
Force may be defined as any action that tends to change the state of rest or motion of a body to which it is applied.

The three quantities required to completely define force are called its specification or characteristics. So the characteristics of a force are:

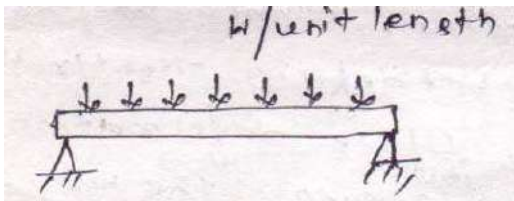
1. Magnitude
2. Point of application
3. Direction of application



Concentrated force/point load



Distributed force

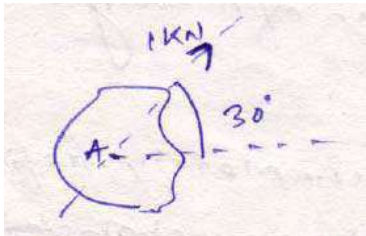


Line of action of force

The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

Representation of force

Graphically a force may be represented by the segment of a straight line.

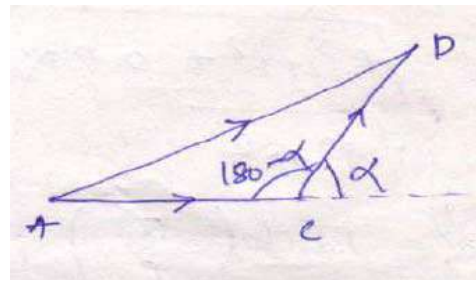
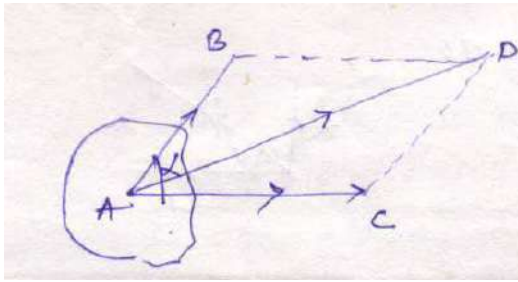


Composition of two forces

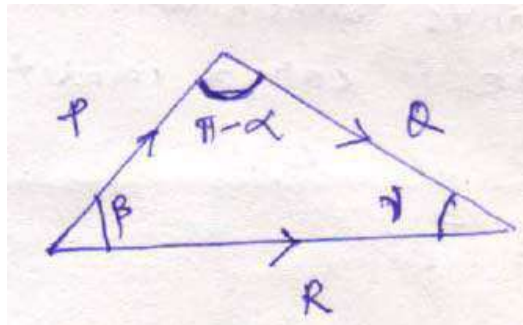
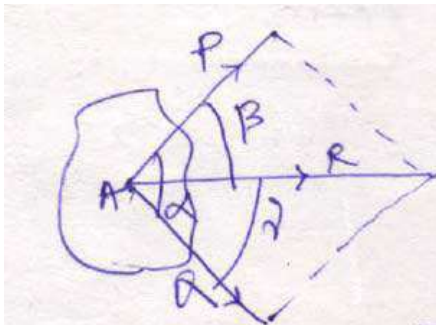
The reduction of a given system of forces to the simplest system that will be its equivalent is called the problem of composition of forces.

Parallelogram law

If two forces represented by vectors AB and AC acting under an angle α are applied to a body at point A. Their action is equivalent to the action of one force, represented by vector AD, obtained as the diagonal of the parallelogram constructed on the vectors AB and AC directed as shown in the figure.



Force AD is called the resultant of AB and AC and the forces are called its components.



$$R = \sqrt{(P^2 + Q^2 + 2PQ \times \cos \alpha)}$$

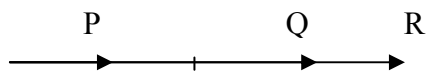
Now applying triangle law

$$\frac{P}{\sin \gamma} = \frac{Q}{\sin \beta} = \frac{R}{\sin(\pi - \alpha)}$$

Special cases

Case-I: If $\alpha = 0^\circ$

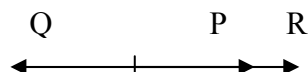
$$R = \sqrt{(P^2 + Q^2 + 2PQ \times \cos 0^\circ)} = \sqrt{(P+Q)^2} = P+Q$$



$$R = P+Q$$

Case- II: If $\alpha = 180^\circ$

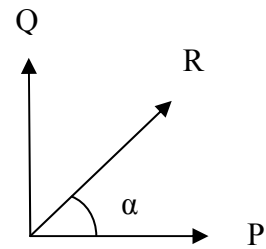
$$R = \sqrt{(P^2 + Q^2 + 2PQ \times \cos 180^\circ)} = \sqrt{(P^2 + Q^2 - 2PQ)} = \sqrt{(P-Q)^2} = P-Q$$



Case-III: If $\alpha = 90^\circ$

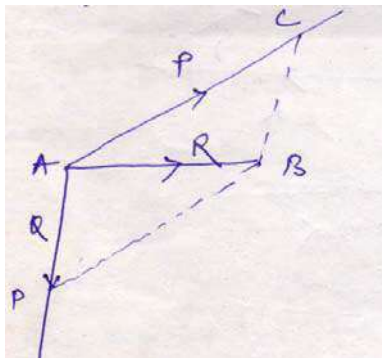
$$R = \sqrt{(P^2 + Q^2 + 2PQ \times \cos 90^\circ)} = \sqrt{P^2 + Q^2}$$

$$\alpha = \tan^{-1} (Q/P)$$



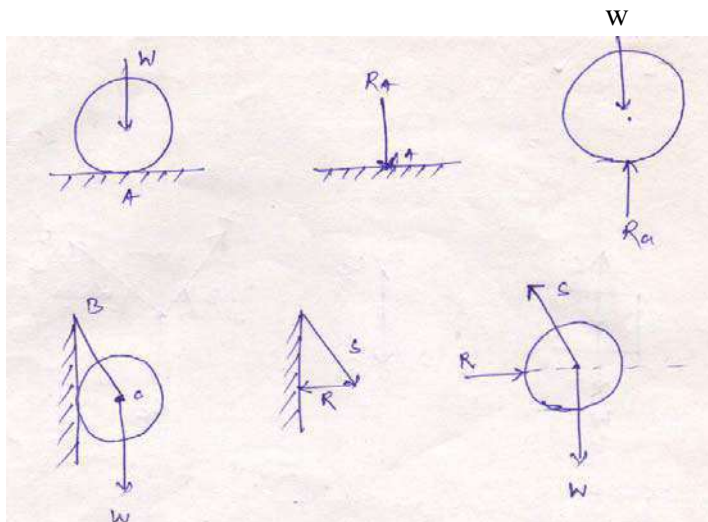
Resolution of a force

The replacement of a single force by a several components which will be equivalent in action to the given force is called resolution of a force.



Action and reaction

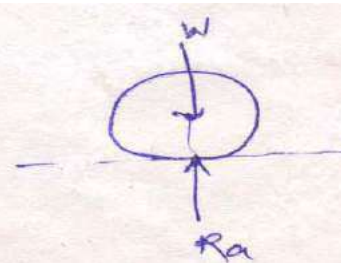
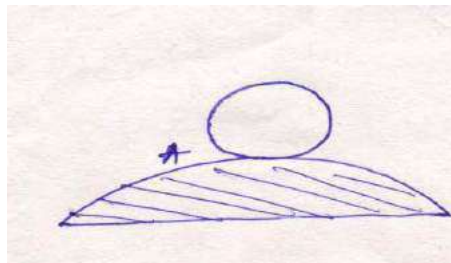
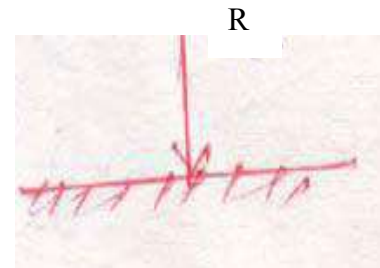
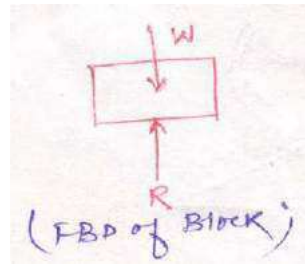
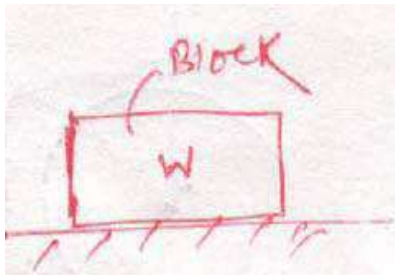
Often bodies in equilibrium are constrained to investigate the conditions.



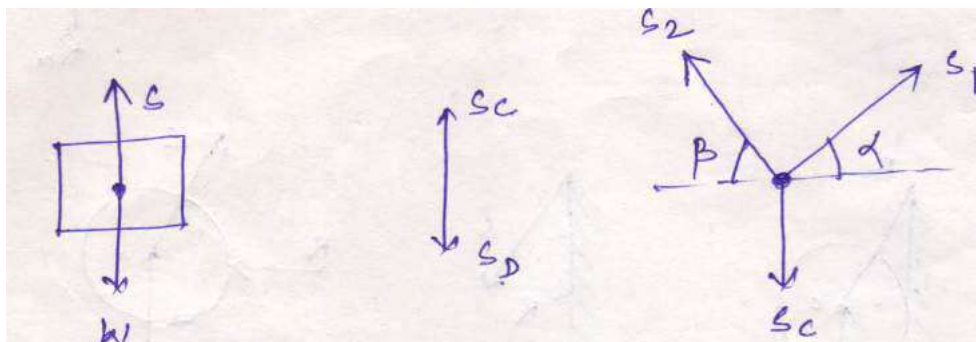
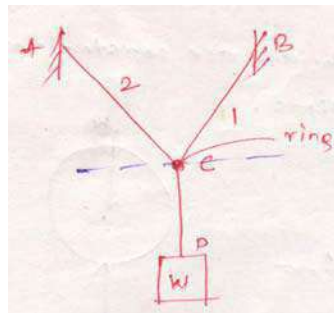
Free body diagram

Free body diagram is necessary to investigate the condition of equilibrium of a body or system. While drawing the free body diagram all the supports of the body are removed and replaced with the reaction forces acting on it.

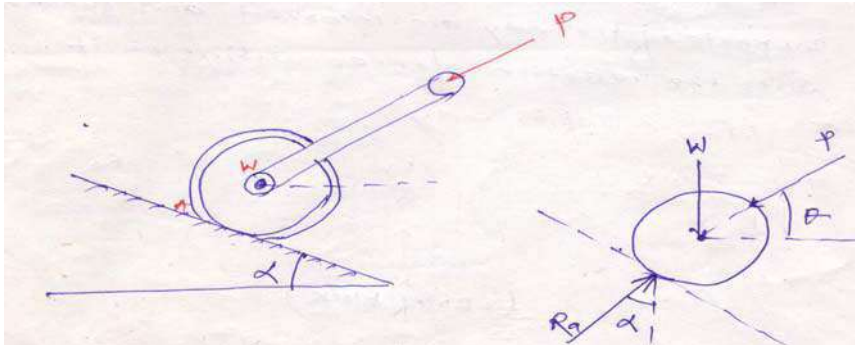
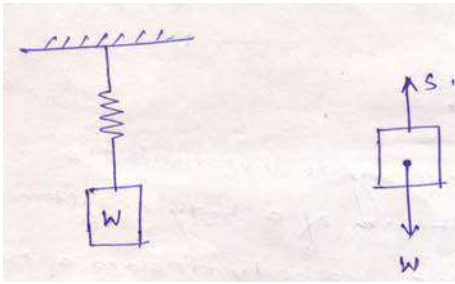
1. Draw the free body diagrams of the following figures.



2. Draw the free body diagram of the body, the string CD and the ring.

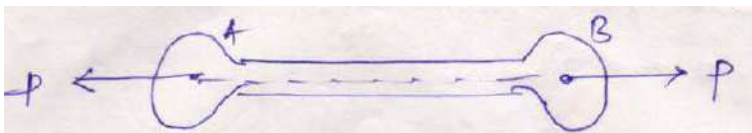


3. Draw the free body diagram of the following figures.

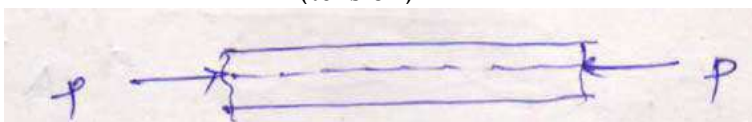


Equilibrium of colinear forces:

Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



(tension)



(compression)

Superposition and transmissibility

Problem 1: A man of weight $W = 712 \text{ N}$ holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight $Q = 534 \text{ N}$. Find the force with which the man's feet press against the floor.

Tension in the string S is equal to the load attached to it

$Q = 534 \text{ N}$

So $S = 534 \text{ N}$

Now applying parallelogram law resultant force

$$R = \sqrt{W^2 + S^2 + 2WS \cos 180^\circ}$$

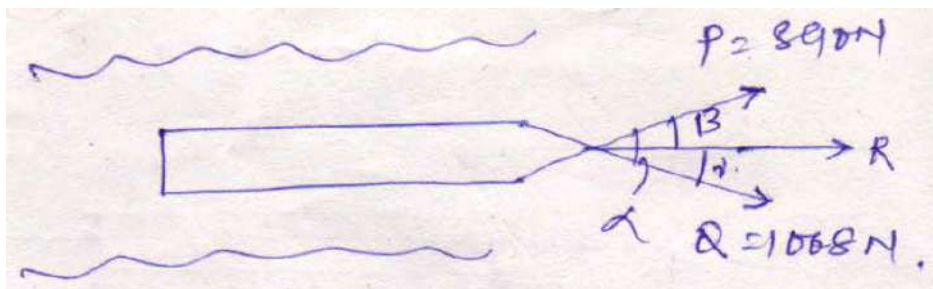
$$= \sqrt{W^2 + S^2 - 2WS}$$

$$= \sqrt{(W - S)^2} = W - S$$

$\Rightarrow R = 712 - 534 = 178 \text{ N} (\downarrow)$

Reaction on the man's feet $= 178 \text{ N} (\uparrow)$

Problem 2: A boat is moved uniformly along a canal by two horses pulling with forces $P = 890 \text{ N}$ and $Q = 1068 \text{ N}$ acting under an angle $\alpha = 60^\circ$. Determine the magnitude of the resultant pull on the boat and the angles β and γ .



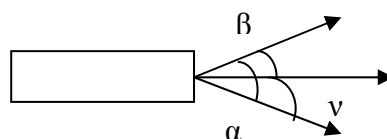
$$P = 890 \text{ N}, \alpha = 60^\circ$$

$$Q = 1068 \text{ N}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$= \sqrt{(890^2 + 1068^2 + 2 \times 890 \times 1068 \times 0.5)}$$

$$= 1698.01 \text{ N}$$

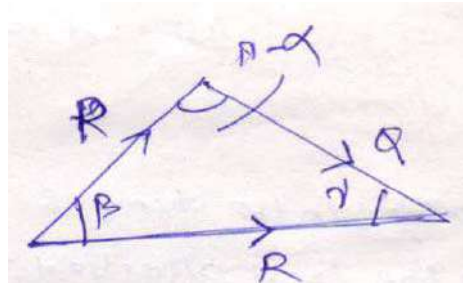


$$\frac{Q}{\sin \beta} = \frac{P}{\sin \nu} = \frac{R}{\sin(\pi - \alpha)}$$

$$\sin \beta = \frac{Q \sin \alpha}{R}$$

$$= \frac{1068 \times \sin 60^\circ}{1698.01}$$

$$= 33^\circ$$



$$\sin \nu = \frac{P \sin \alpha}{R}$$

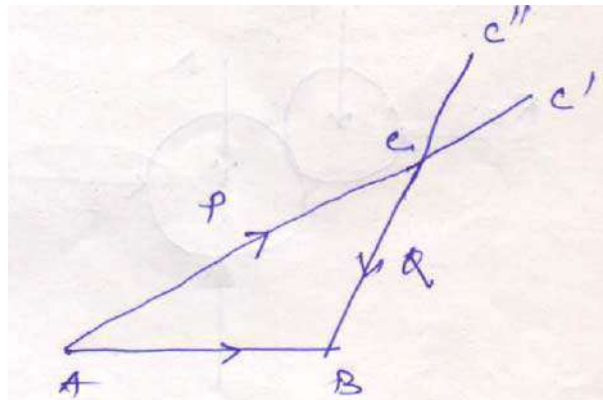
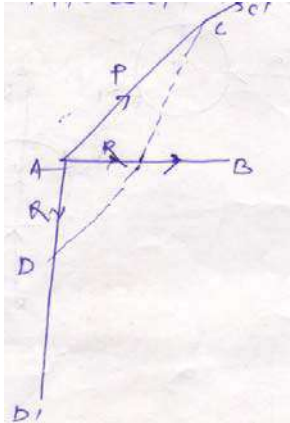
$$= \frac{890 \times \sin 60^\circ}{1698.01}$$

$$= 27^\circ$$

Resolution of a force

Replacement of a single force by several components which will be equivalent in action to the given force is called the problem of resolution of a force.

By using parallelogram law, a single force R can be resolved into two components P and Q intersecting at a point on its line of action.



Equilibrium of collinear forces:

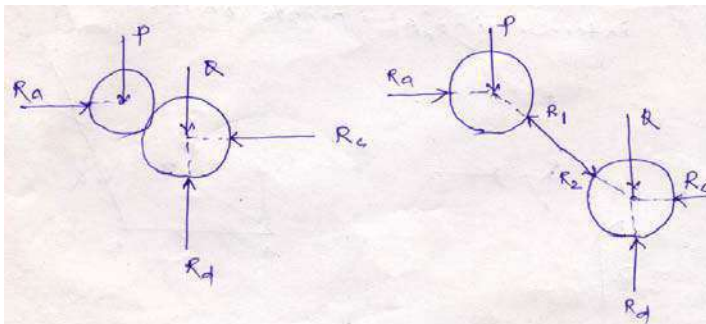
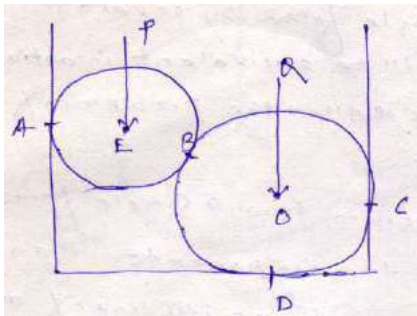
Equilibrium law: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.



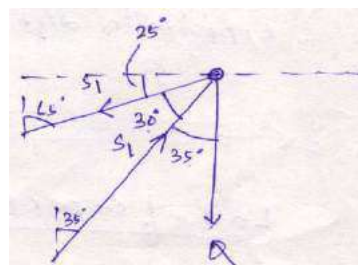
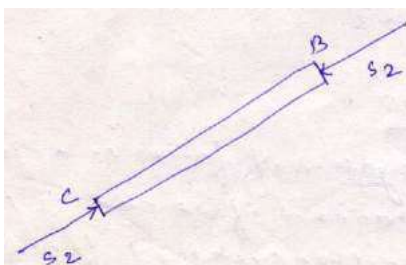
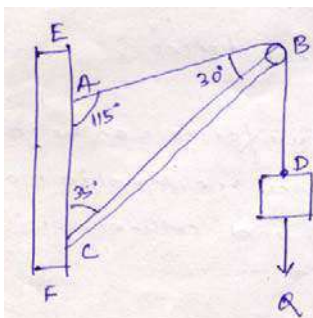
Law of superposition

The action of a given system of forces on a rigid body will no way be changed if we add to or subtract from them another system of forces in equilibrium.

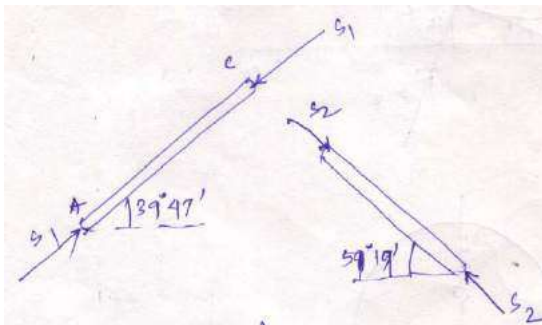
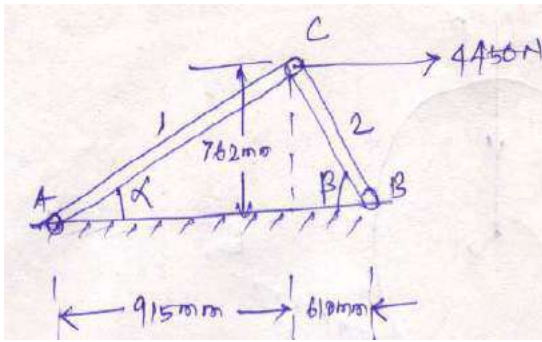
Problem 3: Two spheres of weight P and Q rest inside a hollow cylinder which is resting on a horizontal force. Draw the free body diagram of both the spheres, together and separately.



Problem 4: Draw the free body diagram of the figure shown below.



Problem 5: Determine the angles α and β shown in the figure.

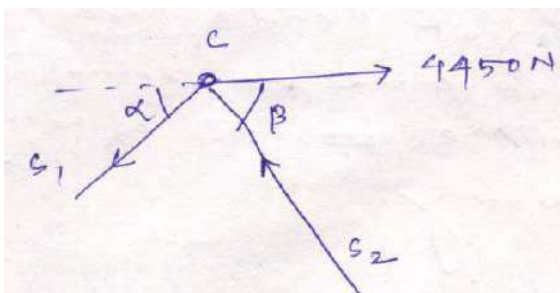


$$\alpha = \tan^{-1}\left(\frac{762}{915}\right)$$

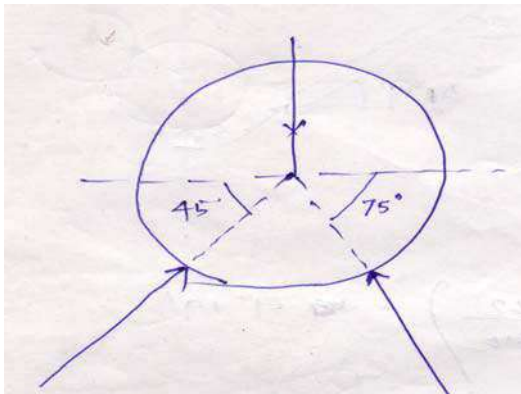
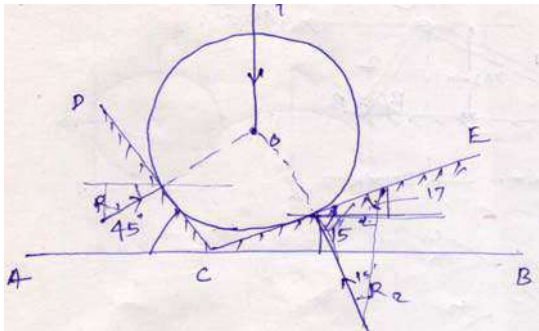
$$= 39^\circ 47'$$

$$\beta = \tan^{-1}\left(\frac{762}{610}\right)$$

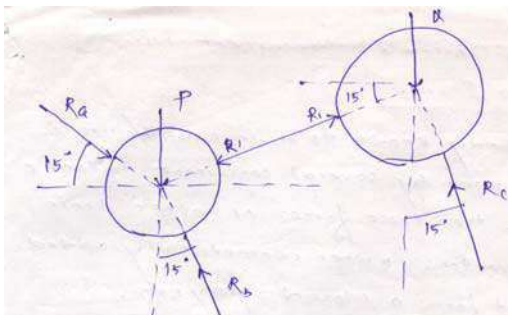
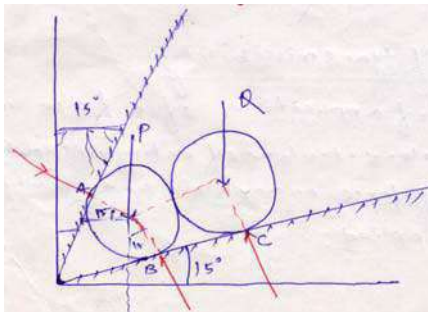
$$= 51^\circ 19'$$



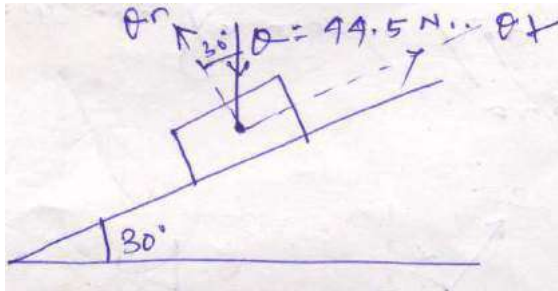
Problem 6: Find the reactions R_1 and R_2 .



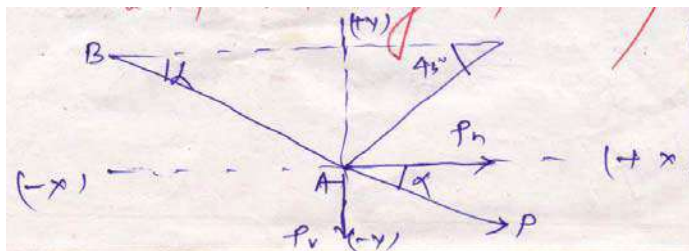
Problem 7: Two rollers of weight P and Q are supported by an inclined plane and vertical walls as shown in the figure. Draw the free body diagram of both the rollers separately.



Problem 8: Find θ_n and θ_t in the following figure.



Problem 9: For the particular position shown in the figure, the connecting rod BA of an engine exert a force of $P = 2225 \text{ N}$ on the crank pin at A. Resolve this force into two rectangular components P_h and P_v horizontally and vertically respectively at A.

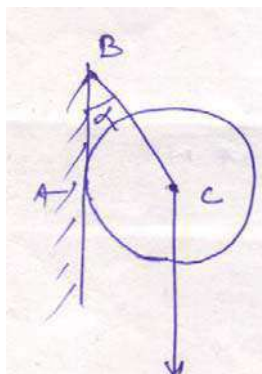


$$P_h = 2081.4 \text{ N}$$

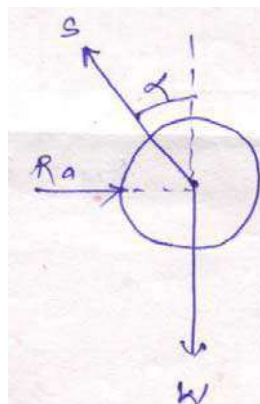
$$P_v = 786.5 \text{ N}$$

Equilibrium of concurrent forces in a plane

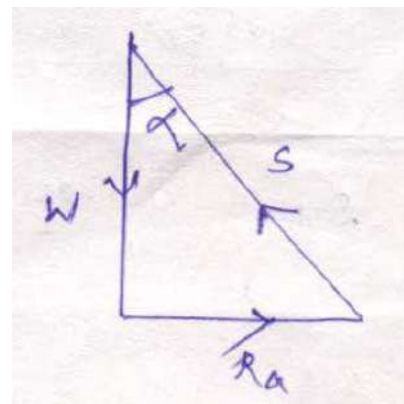
- If a body known to be in equilibrium is acted upon by several concurrent, coplanar forces, then these forces or rather their free vectors, when geometrically added must form a closed polygon.
- This system represents the condition of equilibrium for any system of concurrent forces in a plane.



W

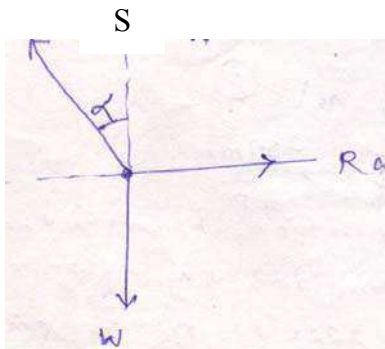


W



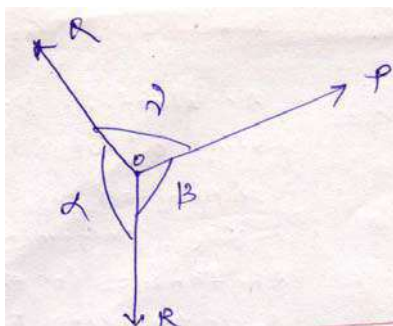
$$R_a = w \tan \alpha$$

$$S = w \sec \alpha$$

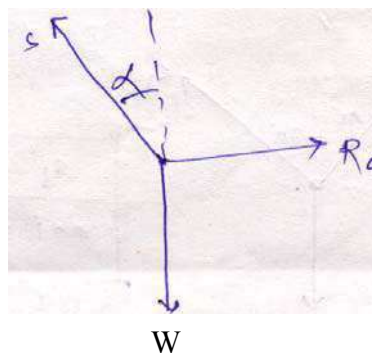
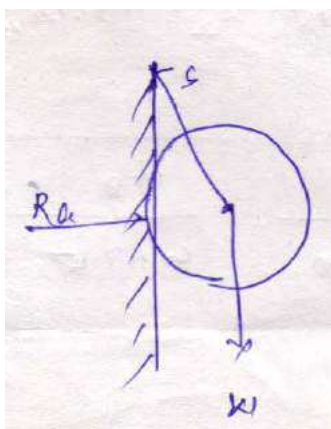


Lami's theorem

If three concurrent forces are acting on a body kept in an equilibrium, then each force is proportional to the sine of angle between the other two forces and the constant of proportionality is same.

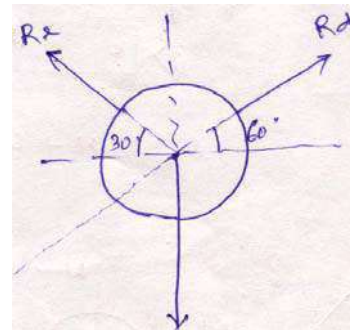
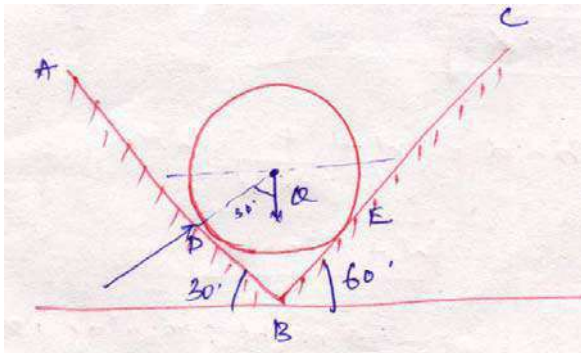


$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$



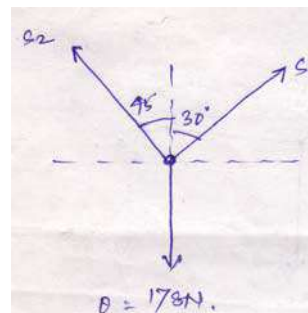
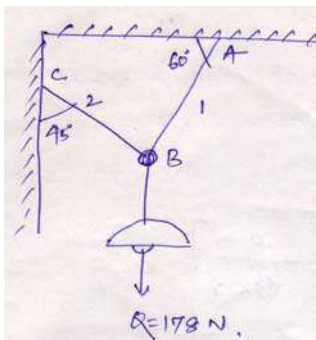
$$\frac{S}{\sin 90} = \frac{R_a}{\sin(180 - \alpha)} = \frac{W}{\sin(90 + \alpha)}$$

Problem: A ball of weight $Q = 53.4\text{N}$ rest in a right angled trough as shown in figure. Determine the forces exerted on the sides of the trough at D and E if all the surfaces are perfectly smooth.

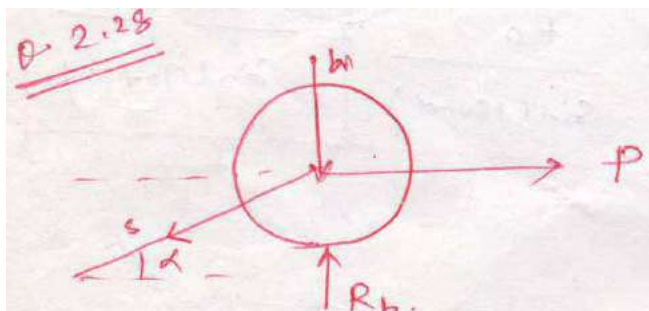


W

Problem: An electric light fixture of weight $Q = 178\text{ N}$ is supported as shown in figure. Determine the tensile forces S_1 and S_2 in the wires BA and BC, if their angles of inclination are given.



$$\frac{S_1}{\sin 135} = \frac{S_2}{\sin 150} = \frac{178}{\sin 75}$$

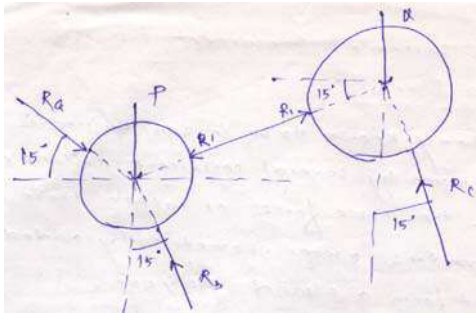


$$S_1 \cos \alpha = P$$

$$S = P \sec \alpha$$

$$\begin{aligned}
 R_b &= W + S \sin \alpha \\
 &= W + \frac{P}{\cos \alpha} \times \sin \alpha \\
 &= W + P \tan \alpha
 \end{aligned}$$

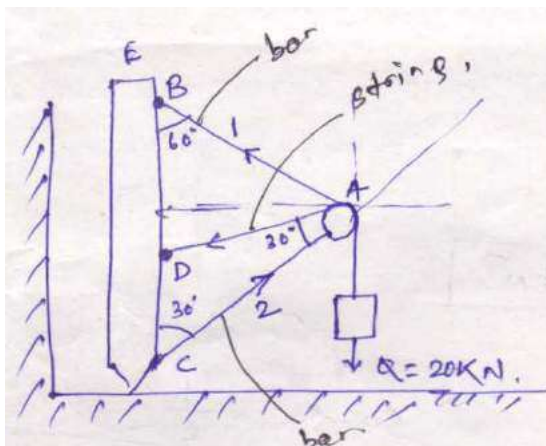
Problem: A right circular roller of weight W rests on a smooth horizontal plane and is held in position by an inclined bar AC . Find the tensions in the bar AC and vertical reaction R_b if there is also a horizontal force P is active.

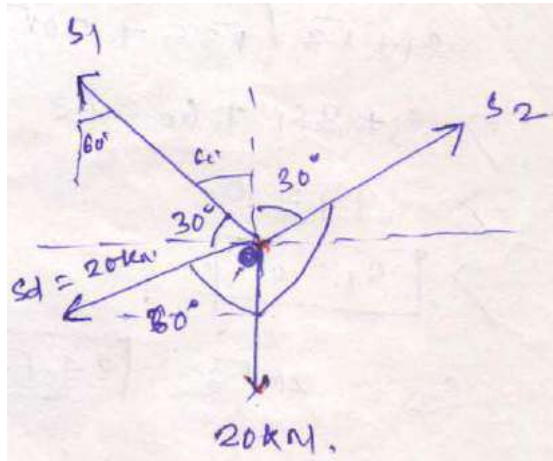


Theory of transmissibility of a force:

The point of application of a force may be transmitted along its line of action without changing the effect of force on any rigid body to which it may be applied.

Problem:





$$\sum X = 0$$

$$S_1 \cos 30 + 20 \sin 60 = S_2 \sin 30$$

$$\frac{\sqrt{3}}{2} S_1 + 20 \frac{\sqrt{3}}{2} = \frac{S_2}{2}$$

$$\frac{S_2}{2} = \frac{\sqrt{3}}{2} S_1 + 10\sqrt{3}$$

$$S_2 = \sqrt{3} S_1 + 20\sqrt{3}$$

(1)

$$\sum Y = 0$$

$$S_1 \sin 30 + S_2 \cos 30 = S_d \cos 60 + 20$$

$$\frac{S_1}{2} + S_2 \frac{\sqrt{3}}{2} = \frac{20}{2} + 20$$

$$\frac{S_1}{2} + \frac{\sqrt{3}}{2} S_2 = 30$$

$$S_1 + \sqrt{3} S_2 = 60$$

(2)

Substituting the value of S_2 in Eq.2, we get

$$S_1 + \sqrt{3} (\sqrt{3} S_1 + 20\sqrt{3}) = 60$$

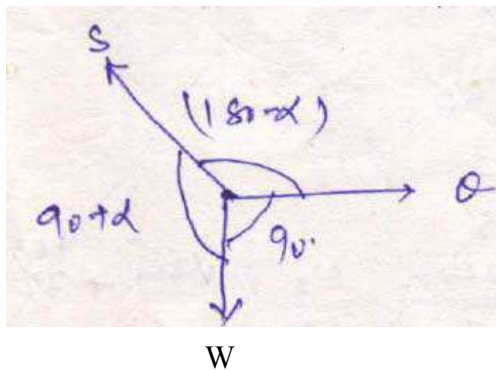
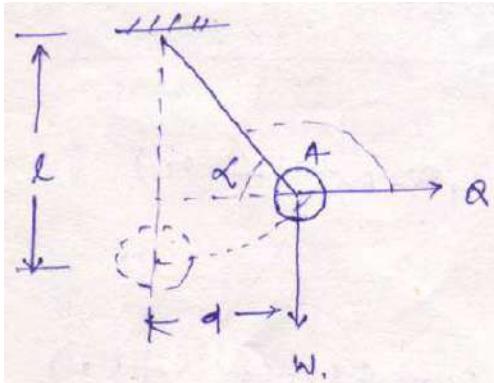
$$S_1 + 3S_1 + 60 = 60$$

$$4S_1 = 0$$

$$S_1 = 0 \text{ kN}$$

$$S_2 = 20\sqrt{3} = 34.64 \text{ kN}$$

Problem: A ball of weight W is suspended from a string of length l and is pulled by a horizontal force Q . The weight is displaced by a distance d from the vertical position as shown in Figure. Determine the angle α , forces Q and tension in the string S in the displaced position.



$$\cos \alpha = \frac{d}{l}$$

$$\alpha = \cos^{-1} \left(\frac{d}{l} \right)$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow \sin \alpha = \sqrt{1 - \cos^2 \alpha}$$

$$= \sqrt{1 - \frac{d^2}{l^2}}$$

$$= \frac{1}{l} \sqrt{l^2 - d^2}$$

Applying Lami's theorem,

$$\frac{S}{\sin 90} = \frac{Q}{\sin(90 + \alpha)} = \frac{W}{\sin(180 - \alpha)}$$

$$\frac{Q}{\sin(90 + \alpha)} = \frac{W}{\sin(180 - \alpha)}$$

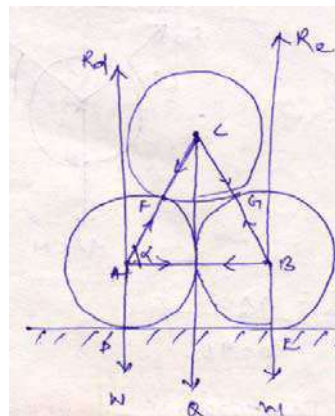
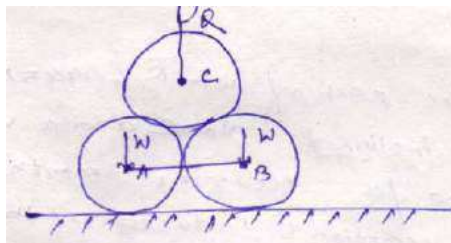
$$\Rightarrow Q = \frac{W \cos \alpha}{\sin \alpha} = \frac{W \left(\frac{d}{l} \right)}{\frac{1}{l} \sqrt{l^2 - d^2}}$$

$$\Rightarrow Q = \frac{Wd}{\sqrt{l^2 - d^2}}$$

$$S = \frac{W}{\sin \alpha} = \frac{W}{\frac{1}{l} \sqrt{l^2 - d^2}}$$

$$= \frac{Wl}{\sqrt{l^2 - d^2}}$$

Problem: Two smooth circular cylinders each of weight $W = 445 \text{ N}$ and radius $r = 152 \text{ mm}$ are connected at their centres by a string AB of length $l = 406 \text{ mm}$ and rest upon a horizontal plane, supporting above them a third cylinder of weight $Q = 890 \text{ N}$ and radius $r = 152 \text{ mm}$. Find the forces in the string and the pressures produced on the floor at the point of contact.

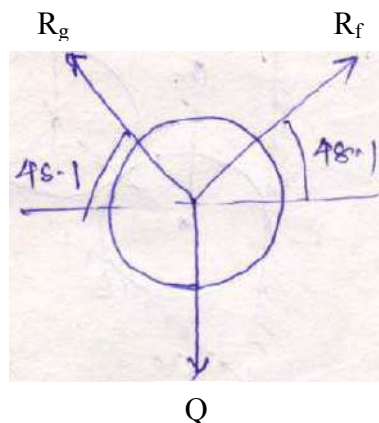


$$\cos \alpha = \frac{203}{304}$$

$$\Rightarrow \alpha = 48.1^\circ$$

$$\frac{R_g}{\sin 138.1} = \frac{R_e}{\sin 138.1} = \frac{Q}{83.8}$$

$$\Rightarrow R_g = R_e = 597.86 \text{ N}$$



Resolving horizontally

$$\sum X = 0$$

$$S = R_f \cos 48.1$$

$$= 597.86 \cos 48.1$$

$$= 399.27 N$$

Resolving vertically

$$\sum Y = 0$$

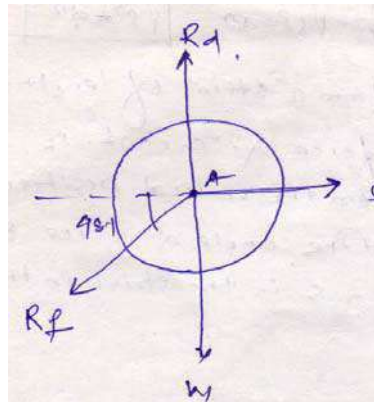
$$R_d = W + R_f \sin 48.1$$

$$= 445 + 597.86 \sin 48.1$$

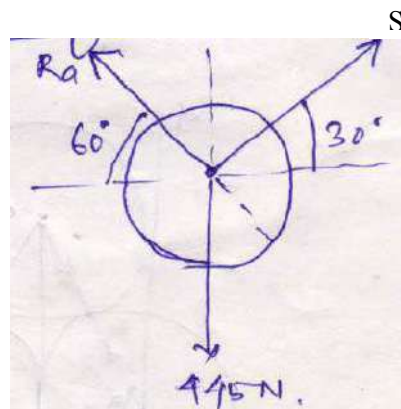
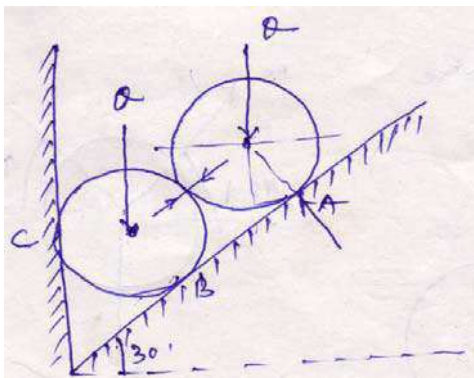
$$= 890 N$$

$$R_e = 890 N$$

$$S = 399.27 N$$



Problem: Two identical rollers each of weight $Q = 445 \text{ N}$ are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.



$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$

$$\Rightarrow R_a = 385.38 N$$

$$\Rightarrow S = 222.5 N$$

Resolving vertically

$$\sum Y = 0$$

$$R_b \cos 60 = 445 + S \sin 30$$

$$\Rightarrow R_b \frac{\sqrt{3}}{2} = 445 + \frac{222.5}{2}$$

$$\Rightarrow R_b = 642.302 N$$

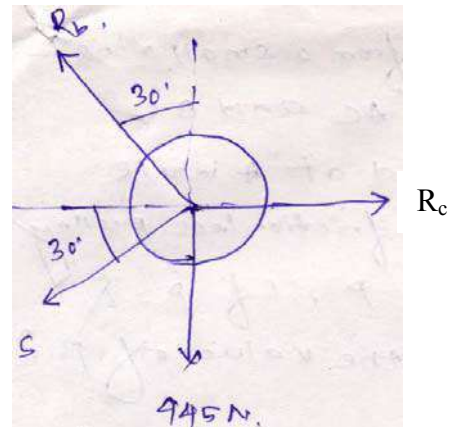
Resolving horizontally

$$\sum X = 0$$

$$R_c = R_b \sin 30 + S \cos 30$$

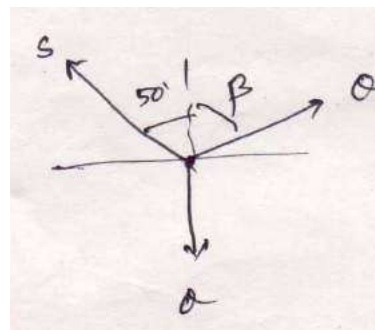
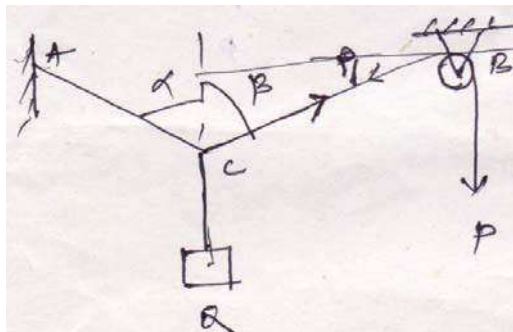
$$\Rightarrow 642.302 \sin 30 + 222.5 \cos 30$$

$$\Rightarrow R_c = 513.84 N$$



Problem:

A weight Q is suspended from a small ring C supported by two cords AC and BC . The cord AC is fastened at A while cord BC passes over a frictionless pulley at B and carries a weight P . If $P = Q$ and $\alpha = 50^\circ$, find the value of β .



Resolving horizontally

$$\sum X = 0$$

$$S \sin 50 = Q \sin \beta$$

(1)

Resolving vertically

$$\sum Y = 0$$

$$S \cos 50 + Q \sin \beta = Q$$

$$\Rightarrow S \cos 50 = Q(1 - \cos \beta)$$

Putting the value of S from Eq. 1, we get

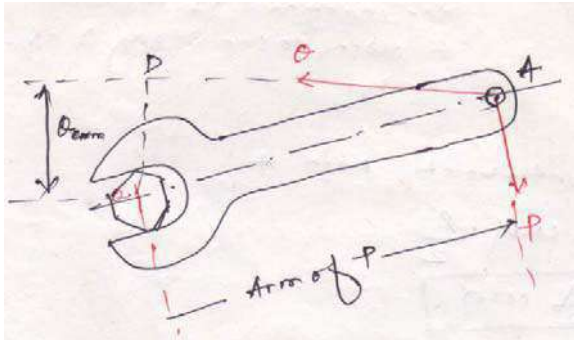
$$\begin{aligned}
S \cos 50 + Q \sin \beta &= Q \\
\Rightarrow S \cos 50 &= Q(1 - \cos \beta) \\
\Rightarrow Q \frac{\sin \beta}{\sin 50} \cos 50 &= Q(1 - \cos \beta) \\
\Rightarrow \cot 50 &= \frac{1 - \cos \beta}{\sin \beta} \\
\Rightarrow 0.839 \sin \beta &= 1 - \cos \beta
\end{aligned}$$

Squaring both sides,

$$\begin{aligned}
0.703 \sin^2 \beta &= 1 + \cos^2 \beta - 2 \cos \beta \\
0.703(1 - \cos^2 \beta) &= 1 + \cos^2 \beta - 2 \cos \beta \\
0.703 - 0.703 \cos^2 \beta &= 1 + \cos^2 \beta - 2 \cos \beta \\
\Rightarrow 1.703 \cos^2 \beta - 2 \cos \beta + 0.297 &= 0 \\
\Rightarrow \cos^2 \beta - 1.174 \cos \beta + 0.297 &= 0 \\
\Rightarrow \beta &= 63.13^\circ
\end{aligned}$$

Method of moments

Moment of a force with respect to a point:



- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards its tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- $\text{Moment} = \text{Magnitude of the force} \times \text{Perpendicular distance of the line of action of force.}$
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m

Theorem of Varignon:

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to some centre.

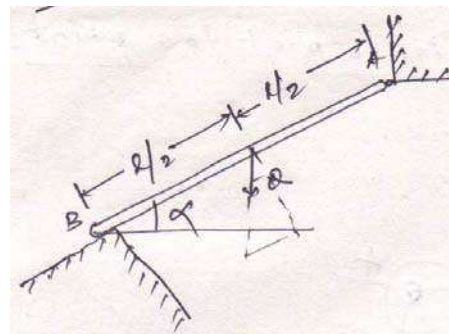
Problem 1:

A prismatic bar of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

Taking moment about point A,

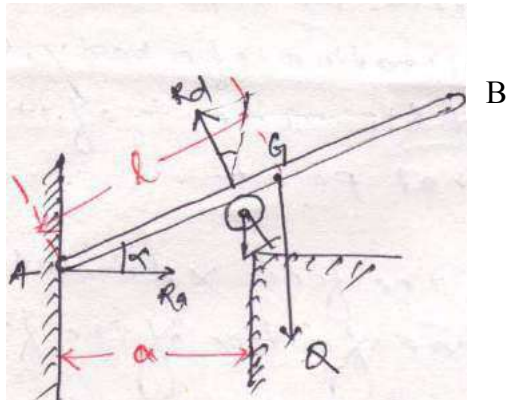
$$R_b \times l = Q \cos \alpha \cdot \frac{l}{2}$$

$$\Rightarrow R_b = \frac{Q}{2} \cos \alpha$$



Problem 2:

A bar AB of weight Q and length $2l$ rests on a very small frictionless roller at D and against a smooth vertical wall at A . Find the angle α that the bar must make with the horizontal in equilibrium.



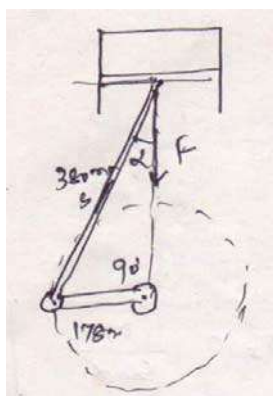
Resolving vertically,
 $R_d \cos \alpha = Q$

Now taking moment about A,

$$\begin{aligned} \frac{R_d \cdot a}{\cos \alpha} - Q \cdot l \cos \alpha &= 0 \\ \Rightarrow \frac{Q \cdot a}{\cos^2 \alpha} - Q \cdot l \cos \alpha &= 0 \\ \Rightarrow Q \cdot a - Q \cdot l \cos^3 \alpha &= 0 \\ \Rightarrow \cos^3 \alpha &= \frac{Q \cdot a}{Q \cdot l} \\ \Rightarrow \alpha &= \cos^{-1} \sqrt[3]{\frac{a}{l}} \end{aligned}$$

Problem 3:

If the piston of the engine has a diameter of 101.6 mm and the gas pressure in the cylinder is 0.69 MPa. Calculate the turning moment M exerted on the crankshaft for the particular configuration.



Area of cylinder

$$A = \frac{\pi}{4} (0.1016)^2 = 8.107 \times 10^{-3} \text{ m}^2$$

Force exerted on connecting rod,

$$\begin{aligned} F &= \text{Pressure} \times \text{Area} \\ &= 0.69 \times 10^6 \times 8.107 \times 10^{-3} \\ &= 5593.83 \text{ N} \end{aligned}$$

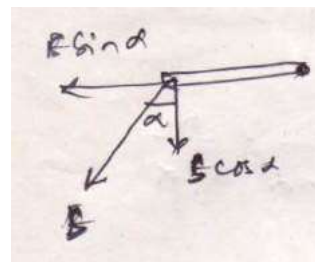
$$\text{Now } \alpha = \sin^{-1} \left(\frac{178}{380} \right) = 27.93^\circ$$

$$S \cos \alpha = F$$

$$\Rightarrow S = \frac{F}{\cos \alpha} = 6331.29 \text{ N}$$

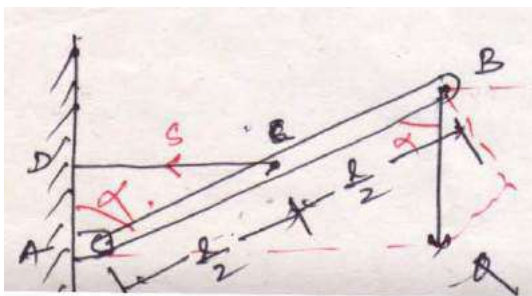
Now moment entered on crankshaft,

$$S \cos \alpha \times 0.178 = 995.7 \text{ N} = 1 \text{ kN}$$



Problem 4:

A rigid bar AB is supported in a vertical plane and carrying a load Q at its free end. Neglecting the weight of bar, find the magnitude of tensile force S in the horizontal string CD.



Taking moment about A,

$$\sum M_A = 0$$

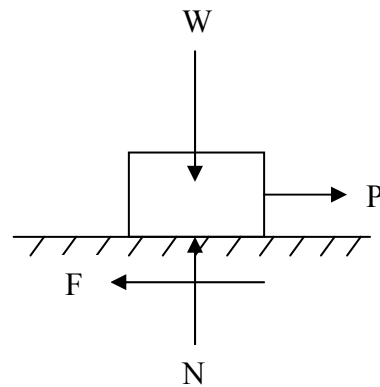
$$S \cdot \frac{l}{2} \cos \alpha = Q \cdot l \sin \alpha$$

$$\Rightarrow S = \frac{Q \cdot l \sin \alpha}{\frac{l}{2} \cos \alpha}$$

$$\Rightarrow S = 2Q \cdot \tan \alpha$$

Friction

- The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannot increase.
- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **Limiting Friction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limiting friction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limiting friction.
- Dynamic friction is classified into following two types:
 - a) Sliding friction
 - b) Rolling friction
- Sliding friction is the friction experienced by a body when it slides over the other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient of Friction**.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ .

$$\text{Thus, } \mu = \frac{F}{N}$$

Laws of friction

1. The force of friction always acts in a direction opposite to that in which body tends to move.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.
4. The force of friction depends upon the roughness/smoothness of the surfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called **coefficient of dynamic friction**.

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is given by

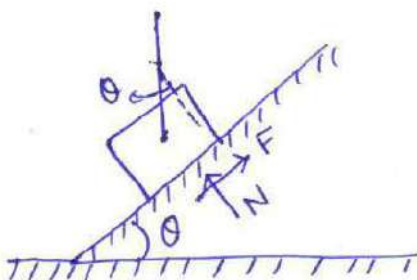
$$\tan \theta = \frac{F}{N}$$

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage,

$$\tan \alpha = \frac{F}{N} = \mu$$

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose



Consider the block of weight W resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is gradually increased, a stage is reached at which the block start sliding down the plane. The angle θ for which the motion is impending, is called the angle of repose. Thus, the maximum inclination of the plane on which a body, free from external forces, can repose is called **Angle of Repose**.

Resolving vertically,
 $N = W \cdot \cos \theta$

Resolving horizontally,
 $F = W \cdot \sin \theta$

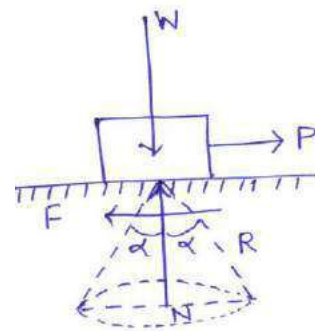
$$\text{Thus, } \tan \theta = \frac{F}{N}$$

If ϕ is the value of θ when the motion is impending, the frictional force will be limiting friction and hence,

$$\begin{aligned} \tan \phi &= \frac{F}{N} \\ &= \mu = \tan \alpha \\ \Rightarrow \phi &= \alpha \end{aligned}$$

Thus, the value of angle of repose is same as the value of limiting angle of repose.

Cone of friction

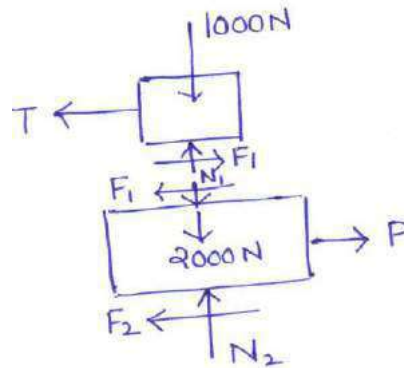
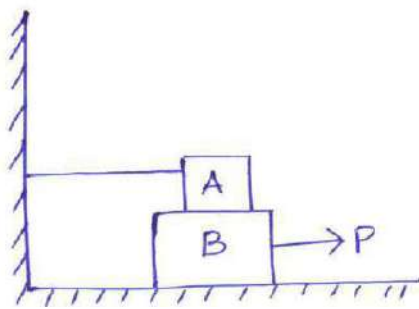


- When a body is having impending motion in the direction of force P , the frictional force will be limiting friction and the resultant reaction R will make limiting angle α with the normal.
- If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle α with the normal to that direction. Thus, when the direction of force P is gradually changed through 360° , the resultant R generates a right circular cone with semi-central angle equal to α .

Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is $\frac{1}{3}$, what should be the value of P to move the block (B), if

- (a) P is horizontal.
- (b) P acts at 30° upwards to horizontal.

Solution: (a)



Considering block A,

$$\sum V = 0$$

$$N_1 = 1000N$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu = 0.25$$

$$F_1 = 0.25N_1 = 0.25 \times 1000 = 250N$$

$$\sum H = 0$$

$$F_1 - T = 0$$

$$T = F_1 = 250N$$

Considering equilibrium of block B,

$$\sum V = 0$$

$$N_2 - 2000 - N_1 = 0$$

$$N_2 = 2000 + N_1 = 2000 + 1000 = 3000N$$

$$\frac{F_2}{N_2} = \mu = \frac{1}{3}$$

$$F_2 = 0.3N_2 = 0.3 \times 3000 = 900N$$

$$\sum H = 0$$

$$P = F_1 + F_2 = 250 + 1000 = 1250 \text{ N}$$

(b) When P is inclined:

$$\sum V = 0$$

$$N_2 - 2000 - N_1 + P \sin 30 = 0$$

$$\Rightarrow N_2 + 0.5P = 2000 + 1000$$

$$\Rightarrow N_2 = 3000 - 0.5P$$

From law of friction,

$$F_2 = \frac{1}{3} N_2 = \frac{1}{3} (3000 - 0.5P) = 1000 - \frac{0.5}{3} P$$

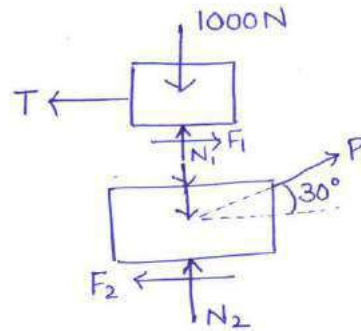
$$\sum H = 0$$

$$P \cos 30 = F_1 + F_2$$

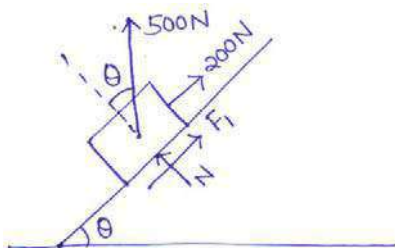
$$\Rightarrow P \cos 30 = 250 + \left(1000 - \frac{0.5}{3} P \right)$$

$$\Rightarrow P \left(\cos 30 + \frac{0.5}{3} \right) = 1250$$

$$\Rightarrow P = 1210.43 \text{ N}$$



Problem 2: A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between the inclined plane and the block.



$$\sum V = 0$$

$$N = 500 \cos \theta$$

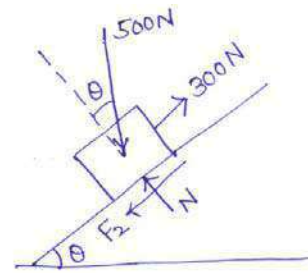
$$F_1 = \mu N = \mu \cdot 500 \cos \theta$$

$$\begin{aligned}\sum H &= 0 \\ 200 + F_1 &= 500 \cdot \sin \theta \\ \Rightarrow 200 + \mu \cdot 500 \cos \theta &= 500 \cdot \sin \theta\end{aligned}\tag{1}$$

$$\begin{aligned}\sum V &= 0 \\ N &= 500 \cdot \cos \theta \\ F_2 &= \mu N = \mu \cdot 500 \cdot \cos \theta\end{aligned}$$

$$\begin{aligned}\sum H &= 0 \\ 500 \sin \theta + F_2 &= 300 \\ \Rightarrow 500 \sin \theta + \mu \cdot 500 \cos \theta &= 300\end{aligned}\tag{2}$$

Adding Eqs. (1) and (2), we get



$$\begin{aligned}500 &= 1000 \cdot \sin \theta \\ \sin \theta &= 0.5 \\ \theta &= 30^\circ\end{aligned}$$

Substituting the value of θ in Eq. 2,

$$500 \sin 30 + \mu \cdot 500 \cos 30 = 300$$

$$\mu = \frac{50}{500 \cos 30} = 0.11547$$

Parallel forces on a plane

Like parallel forces: Coplanar parallel forces when act in the same direction.



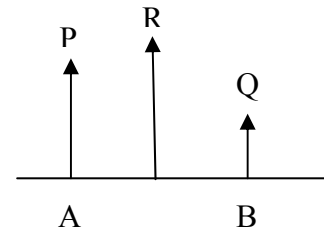
Unlike parallel forces: Coplanar parallel forces when act in different direction.



Resultant of like parallel forces:

Let P and Q are two like parallel forces act at points A and B.

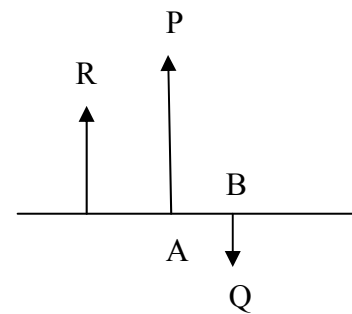
$$R = P + Q$$



Resultant of unlike parallel forces:

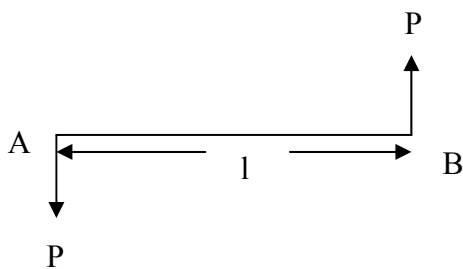
$$R = P - Q$$

R is in the direction of the force having greater magnitude.



Couple:

Two unlike equal parallel forces form a couple.

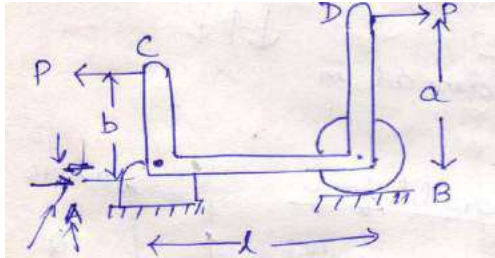


The rotational effect of a couple is measured by its moment.

$$\text{Moment} = P \times l$$

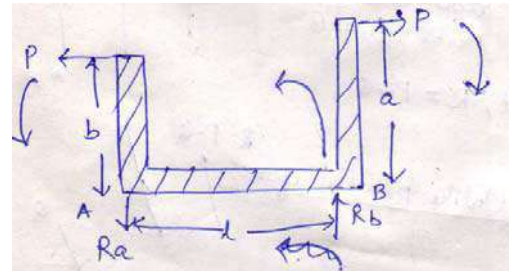
Sign convention: Anticlockwise couple (Positive)
Clockwise couple (Negative)

Problem 1 : A rigid bar CABD supported as shown in figure is acted upon by two equal horizontal forces P applied at C and D . Calculate the reactions that will be induced at the points of support. Assume $l = 1.2$ m, $a = 0.9$ m, $b = 0.6$ m.



$$\sum V = 0$$

$$R_a = R_b$$



Taking moment about A,

$$R_a = R_b$$

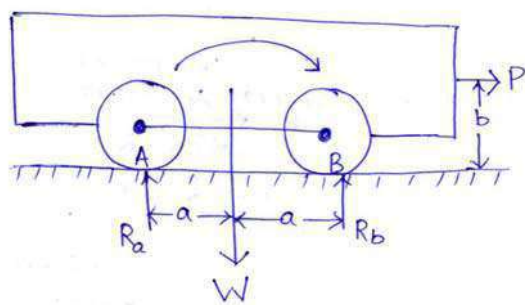
$$R_b \times l + P \times b = P \times a$$

$$\Rightarrow R_b = \frac{P(0.9 - 0.6)}{1.2}$$

$$\Rightarrow R_b = 0.25P(\uparrow)$$

$$\Rightarrow R_a = 0.25P(\downarrow)$$

Problem 2: Owing to weight W of the locomotive shown in figure, the reactions at the two points of support A and B will each be equal to $W/2$. When the locomotive is pulling the train and the drawbar pull P is just equal to the total friction at the points of contact A and B , determine the magnitudes of the vertical reactions R_a and R_b .



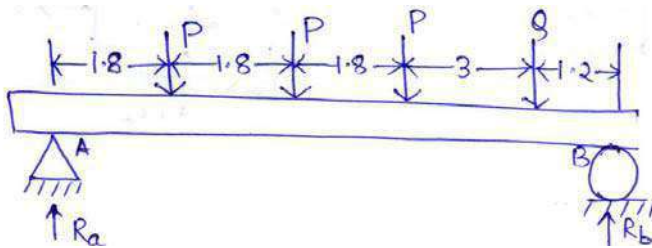
$$\sum V = 0$$

$$R_a + R_b = W$$

Taking moment about B,

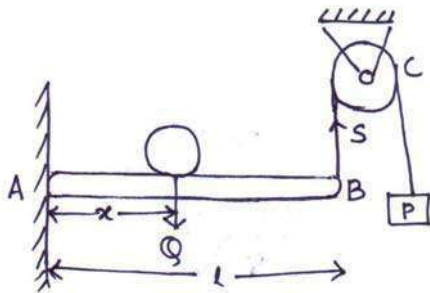
$$\begin{aligned}
\sum M_B &= 0 \\
R_a \times 2a + P \times b &= W \times a \\
\Rightarrow R_a &= \frac{W \cdot a - P \cdot b}{2a} \\
\therefore R_b &= W - R_a \\
\Rightarrow R_b &= W - \left(\frac{W \cdot a - P \cdot b}{2a} \right) \\
\Rightarrow R_b &= \frac{W \cdot a + P \cdot b}{2a}
\end{aligned}$$

Problem 3: The four wheels of a locomotive produce vertical forces on the horizontal girder AB. Determine the reactions R_a and R_b at the supports if the loads $P = 90$ KN each and $Q = 72$ KN (All dimensions are in m).

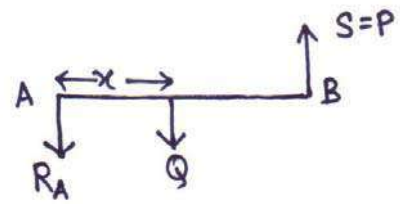
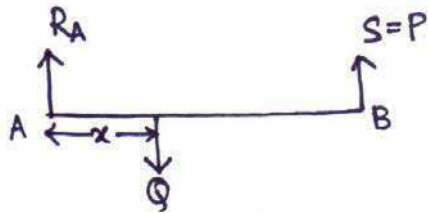


$$\begin{aligned}
\sum V &= 0 \\
R_a + R_b &= 3P + Q \\
\Rightarrow R_a + R_b &= 3 \times 90 + 72 \\
\Rightarrow R_a + R_b &= 342 \text{ KN} \\
\sum M_A &= 0 \\
R_b \times 9.6 &= 90 \times 1.8 + 90 \times 3.6 + 90 \times 5.4 + 72 \times 8.4 \\
\Rightarrow R_b &= 164.25 \text{ KN} \\
\therefore R_a &= 177.75 \text{ KN}
\end{aligned}$$

Problem 4: The beam AB in figure is hinged at A and supported at B by a vertical cord which passes over a frictionless pulley at C and carries at its end a load P. Determine the distance x from A at which a load Q must be placed on the beam if it is to remain in equilibrium in a horizontal position. Neglect the weight of the beam.



FBD

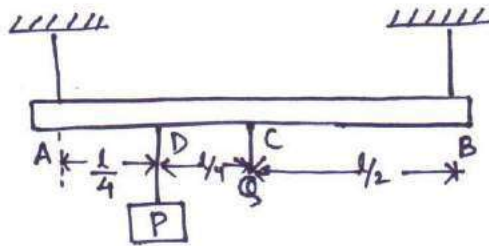


$$\sum M_A = 0$$

$$S \times l = Q \times x$$

$$\Rightarrow x = \frac{P \cdot l}{Q}$$

Problem 5: A prismatic bar AB of weight $Q = 44.5 \text{ N}$ is supported by two vertical wires at its ends and carries at D a load $P = 89 \text{ N}$ as shown in figure. Determine the forces S_a and S_b in the two wires.



$$Q = 44.5 \text{ N}$$

$$P = 89 \text{ N}$$

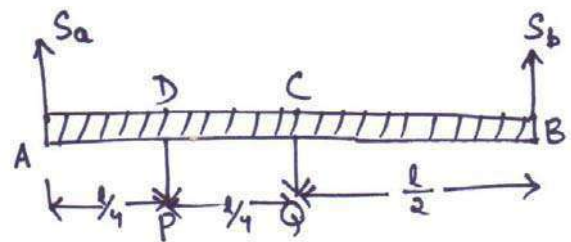
Resolving vertically,

$$\sum V = 0$$

$$S_a + S_b = P + Q$$

$$\Rightarrow S_a + S_b = 89 + 44.5$$

$$\Rightarrow S_a + S_b = 133.5 \text{ N}$$



$$\sum M_A = 0$$

$$S_b \times l = P \times \frac{l}{4} + Q \times \frac{l}{2}$$

$$\Rightarrow S_b = \frac{P}{4} + \frac{Q}{2}$$

$$\Rightarrow S_b = \frac{89}{4} + \frac{44.5}{2}$$

$$\Rightarrow S_b = 44.5$$

$$\therefore S_a = 133.5 - 44.5$$

$$\Rightarrow S_a = 89N$$

Centre of gravity

Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

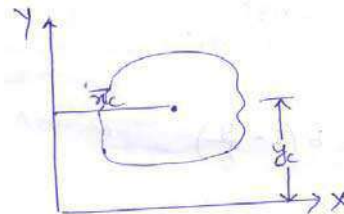
- As the point through which resultant of force of gravity (weight) of the body acts.

Centroid: Centroid of an area lies on the axis of symmetry if it exists.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

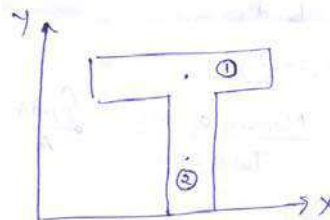
$$x_c = \sum A_i x_i$$

$$y_c = \sum A_i y_i$$



$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

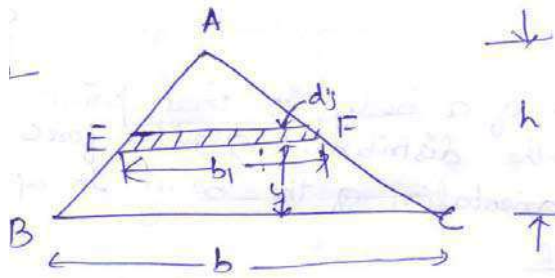


$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

$$x_c = \frac{\int x.dA}{A}$$

$$y_c = \frac{\int y.dA}{A}$$

Problem 1: Consider the triangle ABC of base 'b' and height 'h'. Determine the distance of centroid from the base.



Let us consider an elemental strip of width 'b₁' and thickness 'dy'.

$$\triangle AEF \sim \triangle ABC$$

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$\Rightarrow b_1 = b \left(\frac{h-y}{h} \right)$$

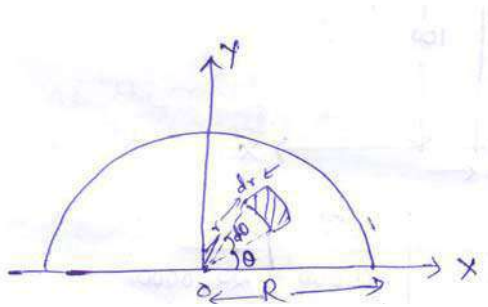
$$\Rightarrow b_1 = b \left(1 - \frac{y}{h} \right)$$

$$\begin{aligned} \text{Area of element EF (dA)} &= b_1 \times dy \\ &= b \left(1 - \frac{y}{h} \right) dy \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\int y \cdot dA}{A} \\ &= \frac{\int_0^h y b \left(1 - \frac{y}{h} \right) dy}{\frac{1}{2} b \cdot h} \\ &= \frac{b \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h}{\frac{1}{2} b \cdot h} \\ &= \frac{2}{h} \left[\frac{h^2}{2} - \frac{h^3}{3} \right] \\ &= \frac{2}{h} \times \frac{h^2}{6} \\ &= \frac{h}{3} \end{aligned}$$

Therefore, y_c is at a distance of h/3 from base.

Problem 2: Consider a semi-circle of radius R. Determine its distance from diametral axis.



Due to symmetry, centroid ' y_c ' must lie on Y-axis.

Consider an element at a distance ' r ' from centre ' o ' of the semicircle with radial width dr .

Area of element = $(r.d\theta) \times dr$

Moment of area about x = $\int y.dA$

$$= \int_0^\pi \int_0^R (r.d\theta).dr \times (r.\sin \theta)$$

$$= \int_0^\pi \int_0^R r^2 \sin \theta .dr.d\theta$$

$$= \int_0^\pi \int_0^R (r^2.dr) .\sin \theta .d\theta$$

$$= \int_0^\pi \left[\frac{r^3}{3} \right]_0^R .\sin \theta .d\theta$$

$$= \int_0^\pi \frac{R^3}{3} .\sin \theta .d\theta$$

$$= \frac{R^3}{3} [-\cos \theta]_0^\pi$$

$$= \frac{R^3}{3} [1+1]$$

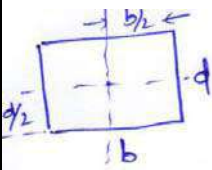
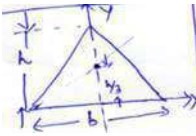
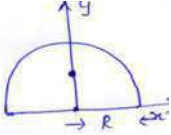
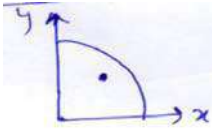
$$= \frac{2}{3} R^3$$

$$y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

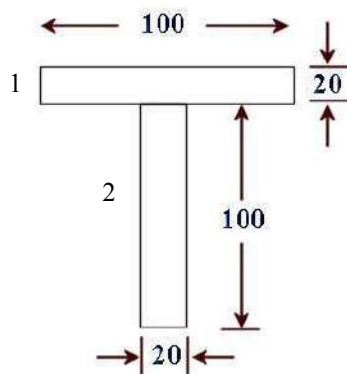
$$\begin{aligned}
 &= \frac{\frac{2}{3}R^3}{\pi R^2 / 2} \\
 &= \frac{4R}{3\pi}
 \end{aligned}$$

Therefore, the centroid of the semicircle is at a distance of $\frac{4R}{3\pi}$ from the diametric axis.

Centroids of different figures

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

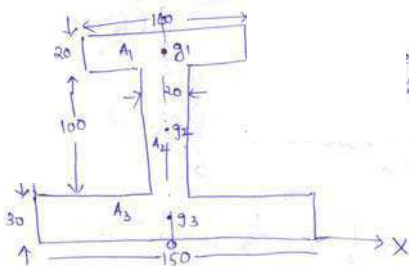


Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



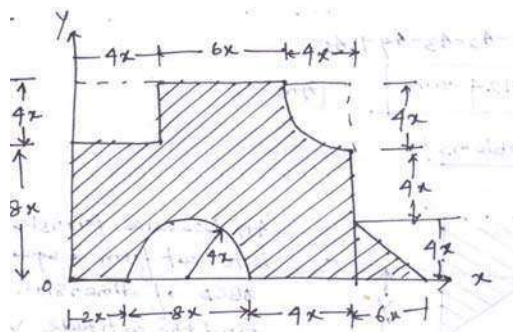
As the figure is symmetric, centroid lies on y-axis. Therefore, $\bar{x} = 0$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71 \text{ mm}$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take $x = 40$ mm.



$$A_1 = \text{Area of rectangle} = 12x \cdot 14x = 168x^2$$

$$A_2 = \text{Area of rectangle to be subtracted} = 4x \cdot 4x = 16x^2$$

$$A_3 = \text{Area of semicircle to be subtracted} = \frac{\pi R^2}{2} = \frac{\pi(4x)^2}{2} = 25.13x^2$$

$$A_4 = \text{Area of quartercircle to be subtracted} = \frac{\pi R^2}{4} = \frac{\pi(4x)^2}{4} = 12.56x^2$$

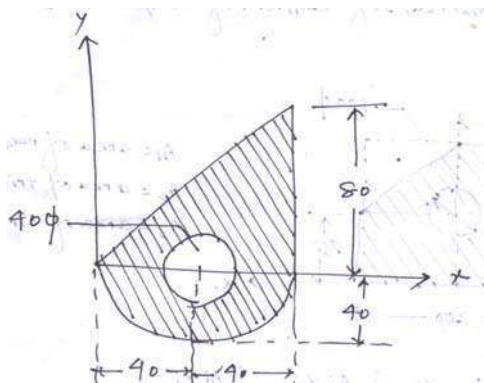
$$A_5 = \text{Area of triangle} = \frac{1}{2} \times 6x \times 4x = 12x^2$$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
$A_1 = 268800$	$7x = 280$	$6x = 240$	75264000	64512000
$A_2 = 25600$	$2x = 80$	$10x = 400$	2048000	10240000
$A_3 = 40208$	$6x = 240$	$\frac{4 \times 4x}{3\pi} = 67.906$	9649920	2730364.448
$A_4 = 20096$	$10x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 492.09$	$8x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 412.093$	9889040.64	8281420.926
$A_5 = 19200$	$14x + \frac{6x}{3} = 16x = 640$	$\frac{4x}{3} = 53.33$	12288000	1023936

$$x_c = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3 - A_4 x_4 + A_5 x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404 \text{ mm}$$

$$y_c = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3 - A_4 y_4 + A_5 y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124 \text{ mm}$$

Problem 6: Determine the centroid of the following figure.



$$A_1 = \text{Area of triangle} = \frac{1}{2} \times 80 \times 80 = 3200 \text{ m}^2$$

$$A_2 = \text{Area of semicircle} = \frac{\pi d^2}{8} - \frac{\pi R^2}{2} = 2513.274 \text{ m}^2$$

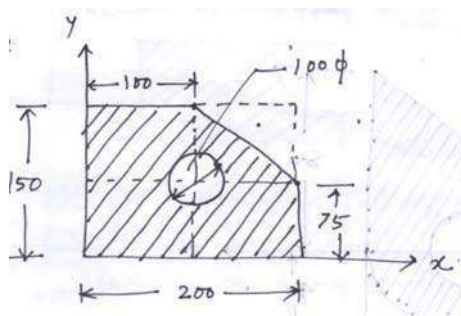
$$A_3 = \text{Area of semicircle} = \frac{\pi D^2}{2} = 1256.64 \text{ m}^2$$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
3200	$2 \times (80/3) = 53.33$	$80/3 = 26.67$	170656	85344
2513.274	40	$\frac{-4 \times 40}{3\pi} = -16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 + A_3} = 49.57 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3} = 9.58 \text{ mm}$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle

A_2 = Area of triangle

A_3 = Area of circle

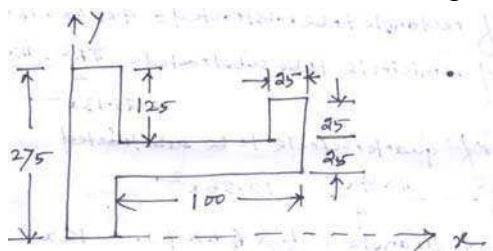
Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
30,000	100	75	3000000	2250000
3750	$100 + 200/3 = 166.67$	$75 + 150/3 = 125$	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3} = 86.4 \text{ mm}$$

$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A_1 - A_2 - A_3} = 64.8 \text{ mm}$$

Module –I

2. Find the centroid of the following figure.



-

-
- The diagram shows a quarter circle sector in the first quadrant of a Cartesian coordinate system. The center is at the origin O . The radius is 150 . The angle between the two radii is 60° . A horizontal line segment of length 75 is drawn from the center to the arc, and a vertical line segment of length 125 is drawn from the center to the arc. The area between the arc and these two segments is shaded with diagonal lines.

Truss/ Frame: A pin jointed frame is a structure made of slender (cross-sectional dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints.

Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of space frames.

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03 members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j , and the number of members m in a perfect frame.

$$m = 2j - 3$$

- (a) When $LHS = RHS$, Perfect frame.
- (b) When $LHS < RHS$, Deficient frame.
- (c) When $LHS > RHS$, Redundant frame.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

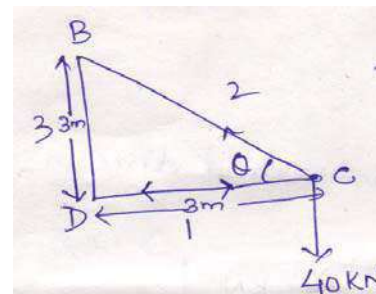
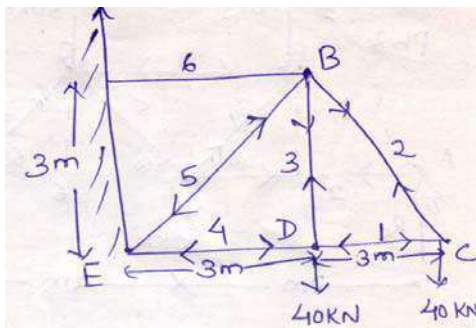
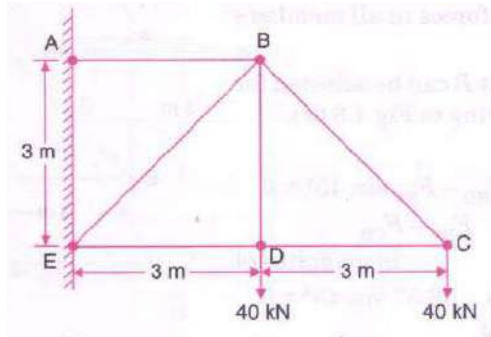
- 1. The ends of the members are pin jointed (hinged).
- 2. The loads act only at the joints.
- 3. Self weight of the members is negligible.

Methods of analysis

- 1. Method of joint
- 2. Method of section

Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



$$\tan \theta = 1$$

$$\Rightarrow \theta = 45^\circ$$

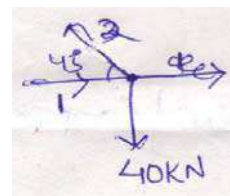
Joint C

$$S_1 = S_2 \cos 45$$

$$\Rightarrow S_1 = 40 \text{ kN (Compression)}$$

$$S_2 \sin 45 = 40$$

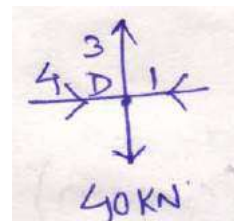
$$\Rightarrow S_2 = 56.56 \text{ kN (Tension)}$$



Joint D

$$S_3 = 40 \text{ kN (Tension)}$$

$$S_1 = S_4 = 40 \text{ kN (Compression)}$$

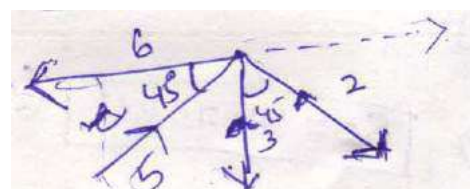


Joint B

Resolving vertically,

$$\sum V = 0$$

$$S_5 \sin 45 = S_3 + S_2 \sin 45$$



$$\Rightarrow S_5 = 113.137 \text{ kN (Compression)}$$

Resolving horizontally,

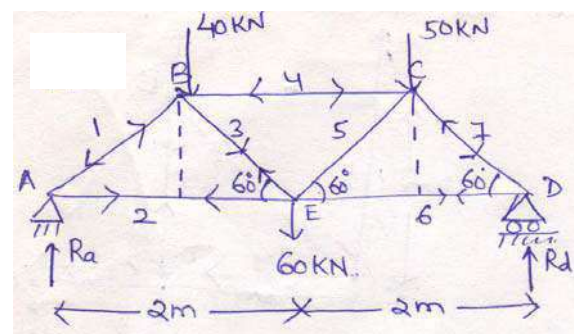
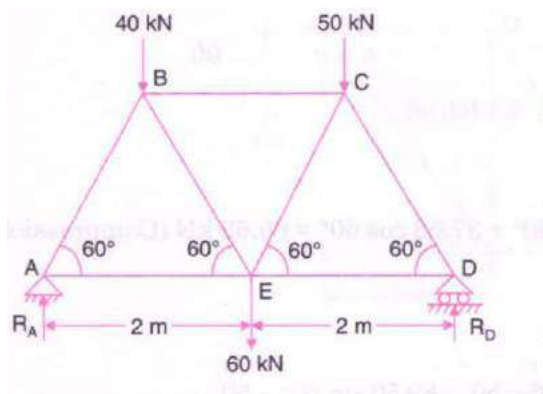
$$\sum H = 0$$

$$S_6 = S_5 \cos 45 + S_2 \cos 45$$

$$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$$

$$\Rightarrow S_6 = 120 \text{ kN (Tension)}$$

Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2m.



Taking moment at point A,

$$\sum M_A = 0$$

$$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$$

$$\Rightarrow R_d = 77.5 \text{ kN}$$

Now resolving all the forces in vertical direction,

$$\sum V = 0$$

$$R_a + R_d = 40 + 60 + 50$$

$$\Rightarrow R_a = 72.5 \text{ kN}$$

Joint A

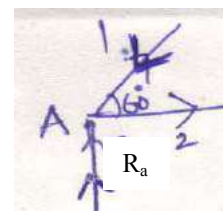
$$\sum V = 0$$

$$\Rightarrow R_a = S_1 \sin 60$$

$$\Rightarrow S_1 = 83.72 \text{ kN (Compression)}$$

$$\sum H = 0$$

$$\Rightarrow S_2 = S_1 \cos 60$$



$$\Rightarrow S_1 = 41.86 \text{ KN (Tension)}$$

Joint D

$$\sum V = 0$$

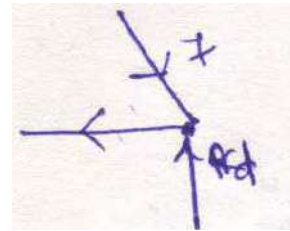
$$S_7 \sin 60 = 77.5$$

$$\Rightarrow S_7 = 89.5 \text{ KN (Compression)}$$

$$\sum H = 0$$

$$S_6 = S_7 \cos 60$$

$$\Rightarrow S_6 = 44.75 \text{ KN (Tension)}$$



Joint B

$$\sum V = 0$$

$$S_1 \sin 60 = S_3 \cos 60 + 40$$

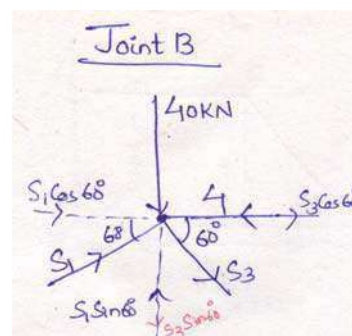
$$\Rightarrow S_3 = 37.532 \text{ KN (Tension)}$$

$$\sum H = 0$$

$$S_4 = S_1 \cos 60 + S_3 \cos 60$$

$$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$$

$$\Rightarrow S_4 = 60.626 \text{ KN (Compression)}$$

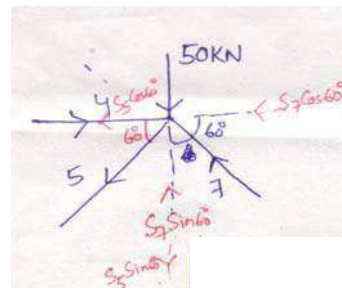


Joint C

$$\sum V = 0$$

$$S_5 \sin 60 + 50 = S_7 \sin 60$$

$$\Rightarrow S_5 = 31.76 \text{ KN (Tension)}$$



Plane Truss (Method of section)

In case of analysing a plane truss, using method of section, after determining the support reactions a section line is drawn passing through not more than three members in which forces are unknown, such that the entire frame is cut into two separate parts.

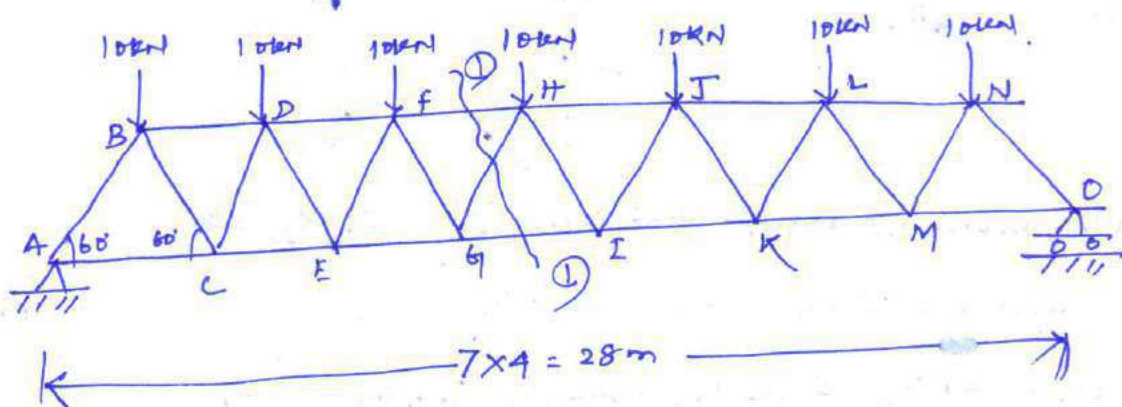
~~Each~~ Each part should be in equilibrium under the action of loads, reactions and the forces in the members.

Method of section is preferred for the following cases:

(i) analysis of large truss in which forces in only few members are required

(ii) If method of joint fails to start or proceed with analysis for not getting a joint with only two unknown forces.

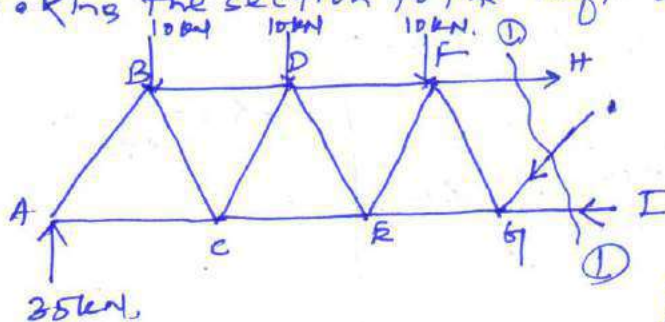
Example 1.



Determine the forces in the members FH, HG, and GL in the truss.
 Due to symmetry $R_A = R_O = \frac{1}{2} \times \text{total downward load}$

$$= \frac{1}{2} \times 70 = \boxed{35 \text{ kN}}$$

Taking the section to the left of the cut.



Taking moment about G

$$\sum M_G = 0$$

$$F_{FH} \times 4 \sin 60 + 35 \times 12$$

$$= 10 \times 2 + 10 \times 6 + 10 \times 10$$

$$\Rightarrow F_{FH} = \frac{(20 + 60 + 100) - 420}{4 \sin 60}$$

$$= -69.28 \text{ kN}$$

Negative sign indicates that direction should have opposite i.e. it's compressive in nature.

Now resolving all the forces vertically $\Sigma Y = 0$

$$10 + 10 + 10 + F_{GH} \sin 60 = 35$$

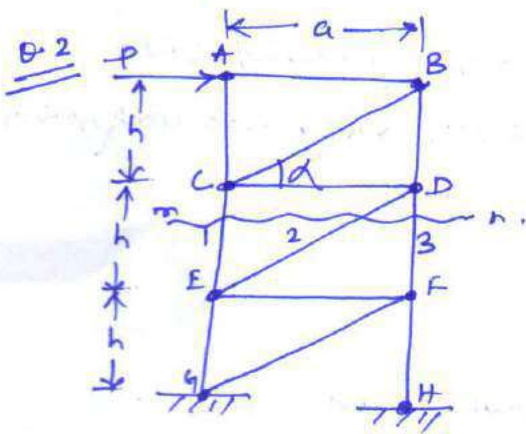
$$\Rightarrow F_{GH} = \frac{35 - 30}{\sin 60}$$

$$\Rightarrow \boxed{F_{GH} = 5.78 \text{ kN.}} \text{ (compressive)}$$

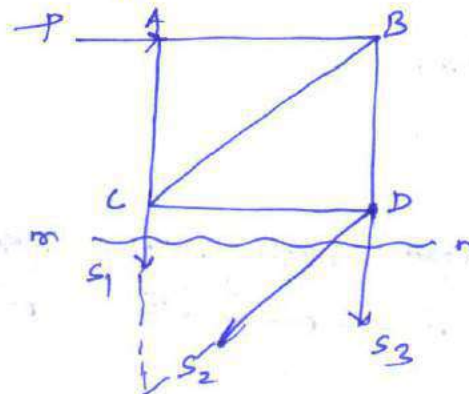
Resolving all the forces horizontally $\Sigma X = 0$.

$$F_{FH} + F_{GH} \cos 60 = F_{GI}$$

$$\Rightarrow F_{GI} = 69.28 + 5.78 \cos 60 = \boxed{72.17 \text{ kN.}} \text{ (tension)}$$



Using method of sections determine the axial forces in bars 1, 2 and 3.



Taking moment about ~~the~~ joint D $\Sigma M_D = 0$.

$$S_1 \times a = P \times h \Rightarrow \boxed{S_1 = \frac{Ph}{a}} \text{ --- (1) (tension)}$$

Similarly taking E as the moment centre $\Sigma M_E = 0$

$$S_2 \times a + P \times 2h = 0$$

$$\Rightarrow \boxed{S_2 = \frac{-2Ph}{a}}$$

(-ve sign indicates direction of force will be opposite and it will be compressive in nature)

Resolving all the forces horizontally $\Sigma X = 0$.

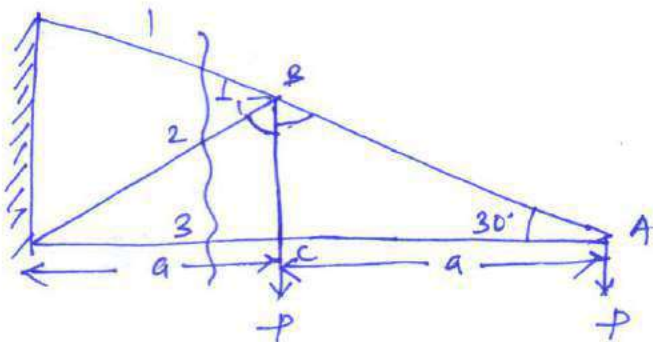
$$S_2 \cos \alpha = P$$

$$\Rightarrow S_2 = \frac{P}{\cos \alpha}$$

$$\boxed{\frac{P \sqrt{a^2 + h^2}}{a}} \text{ (Ans.)}$$

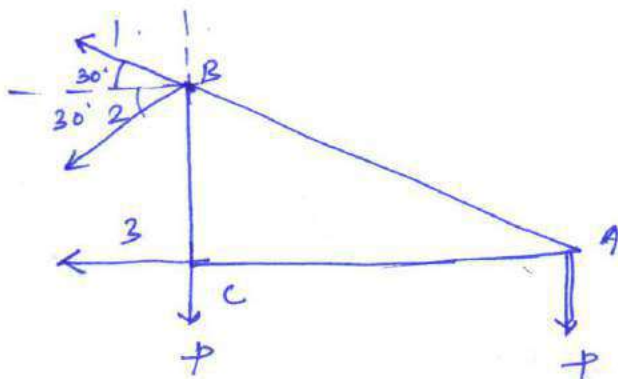
$$\cos \alpha = \frac{a}{\sqrt{a^2 + h^2}}$$

(2)



$$\frac{BC}{AC} = \tan 30^\circ$$

$$\Rightarrow BC = a \tan 30 = \boxed{0.578a}$$



$$\sum M_B = 0$$

$$S_3 \times 0.578a + P \times a = 0$$

$$\Rightarrow S_3 = \frac{-Pa}{0.578a} = -1.73P$$

(-ve sign indicates direction is opposite and it is compressive in nature)

Resolving vertically $\sum Y = 0$

$$S_1 \sin 30 = 2P + S_2 \sin 30$$

$$\Rightarrow S_1 = \frac{2P + S_2/2}{\sin 30} = (4P + S_2) \quad \text{--- (2)}$$

Now resolving horizontally $\sum X = 0$

$$S_1 \cos 30 + S_2 \cos 30 = 1.73P$$

$$\Rightarrow (4P + S_2) \times \frac{\sqrt{3}}{2} + S_2 \frac{\sqrt{3}}{2} = 1.73P$$

$$\Rightarrow 2\sqrt{3}P + \frac{\sqrt{3}}{2}S_2 + \frac{\sqrt{3}}{2}S_2 = 1.73P$$

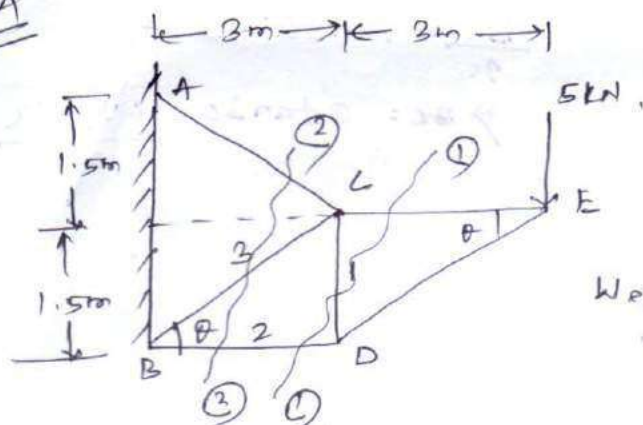
$$\Rightarrow \frac{\sqrt{3}}{2}S_2 = 1.73P - 2\sqrt{3}P$$

$$= -1.73P$$

$$\Rightarrow S_2 = \frac{-1.73P}{\sqrt{3}} = \boxed{-P} \quad \text{(-ve sign indicates the direction is opposite and it is compressive)}$$

$$\text{Now } S_1 = 4P - P = \boxed{3P} \quad \text{(tension)}$$

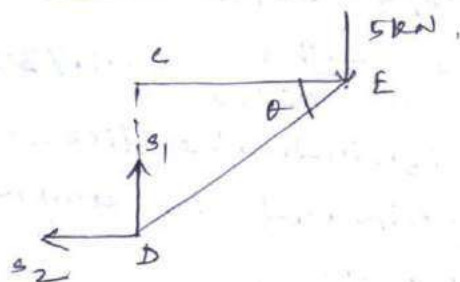
Q.4



using method of sections,
find axial forces in each bar
1, 2 and 3 of the plane
truss.

We have $\tan \theta = \left(\frac{1.5}{3}\right) \Rightarrow \theta = 26.56^\circ$

considering section 1-1



Resolving vertically, $\Sigma Y = 0$
 $S_1 = 5 \text{ kN}$ (tension)

Now taking moment about C

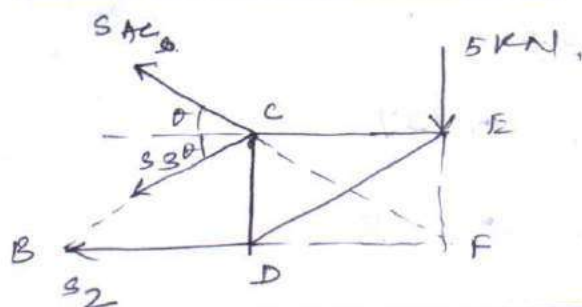
$$S_2 \times 1.5 + 5 \times 3 = 0$$

$$\Rightarrow S_2 = -10 \text{ kN}$$

-ve sign indicates direction should have been opposite

$S_2 = 10 \text{ kN}$ (compression)

considering section 2-2

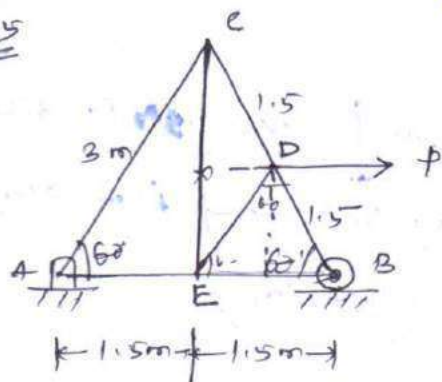


Taking moment about F

$$\Sigma M_F = 0$$

$$\Rightarrow S_3 = 0$$

Q.5



Assignment

Using method of joint and
method of section find the axial
force in the bar x.

Method of Joint

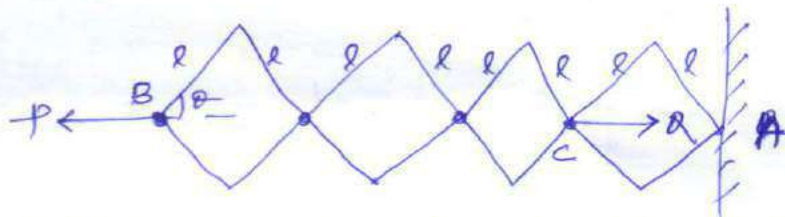
considering the whole structure and

taking moment about A $\Sigma M_A = 0$.

$$R_B \times 3 = P \times 1.5 \sin 60$$

$$\Rightarrow R_B = \frac{\sqrt{3}}{4} P$$

Q.1 (6.3) Calculate the relation betn active forces P and Q for equilibrium of system of bars. The bars are so arranged that they form identical rhombuses.



Let l = length of each side of bar.

θ = angle made by each side of the rhombus

Distance of P from fixed point A = $6l \cos \theta$
 " " " " " " " " = $2l \cos \theta$

Let the virtual displacement of P is $B-B'$

$$B-B' = dx_1 = \frac{d}{d\theta} (6l \cos \theta) d\theta = -6l \sin \theta d\theta$$

Similarly the virtual displacement of Q is $C-C'$
 $= dx_2 = -2l \sin \theta d\theta$

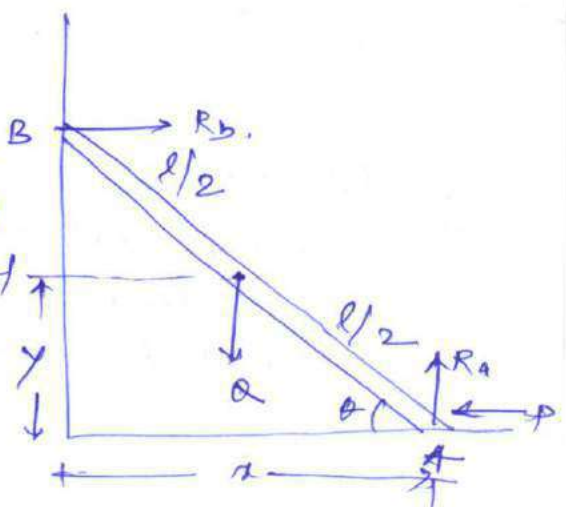
Applying principle of virtual work $\Sigma W = 0$

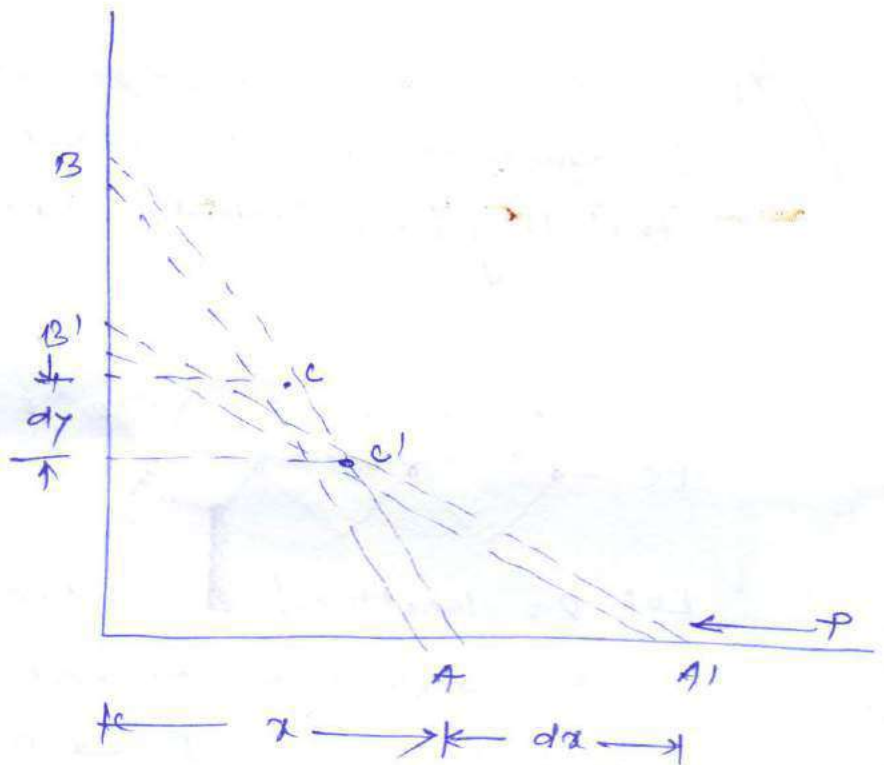
$$P \cdot dx_1 = Q \cdot dx_2$$

$$\Rightarrow P \cdot (6l \sin \theta d\theta) = Q \cdot (2l \sin \theta d\theta)$$

$$\Rightarrow \boxed{P = \frac{Q}{3}} \quad (\text{Ans})$$

Q.2 A prismatic bar AB of length l and wt. Q stands in a vertical plane and is supported by smooth surfaces at A and B . Using principle of virtual work find the magnitude of horizontal force P applied at A if the bar is in equilibrium.





Let the horizontal distance of P from D is x

$$x = l \cos \theta$$

$$AA' = dx = -l \sin \theta d\theta$$

Vertical distance of Q from D is y

$$y = \frac{l}{2} \sin \theta$$

$$CC' = dy = \frac{l}{2} \cos \theta d\theta$$

Normal reactions R_A and R_B have no work along the planes.

Applying principle of virtual work $\sum W = 0$

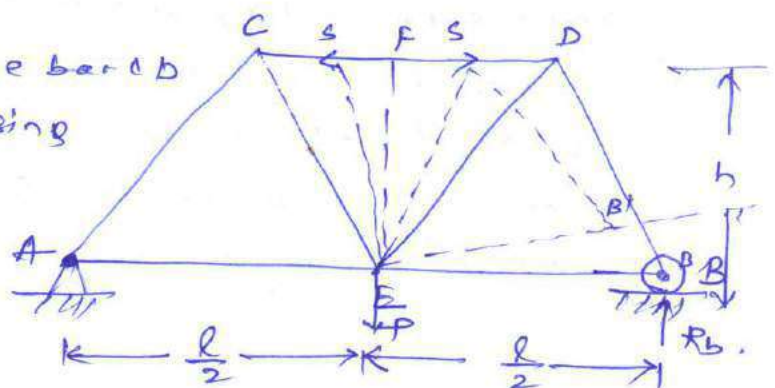
$$P dx = Q dy$$

$$P l \sin \theta d\theta = Q \frac{l}{2} \cos \theta d\theta$$

$$\Rightarrow \boxed{P = \frac{Q}{2} \cot \theta}$$

Q.3 (6.14)

Find axial forces in the bars of the simple truss by using method of virtual work.



Let S be the compressive force in bar CD .

consider the part EBD of the truss under the action of force R_b , P and S

Keeping E fixed and giving EB an angular displacement $d\alpha$

$$\sum W = 0,$$

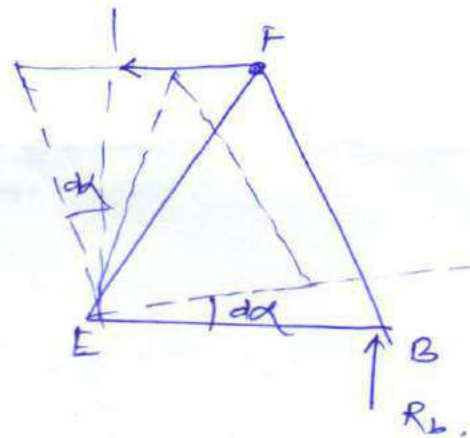
$$R_L \times BB' = S \times FF'$$

$$BB' = \frac{l}{2} d\alpha$$

$$f_{\beta 1} = h d q$$

$$R_b \times \frac{l}{2} d\alpha = s \times h d\alpha$$

$$\gamma \quad S = \frac{R_b l}{2h} \quad \text{--- c1)}$$



Now considering whole frame as equilibrium body $\Sigma Y = 0$.

$$R_a \vdash R_b = \top.$$

$$R_b \cdot l = \frac{P \cdot l}{2} \Rightarrow R_b = \frac{P}{2} \quad \text{--- (2)}$$

Substituting the value of R_B in eq. (1)

$$s = \frac{pl}{4h}$$

0.4 (0.15)

Using principle of virtual work
find reactions R_A for the truss.

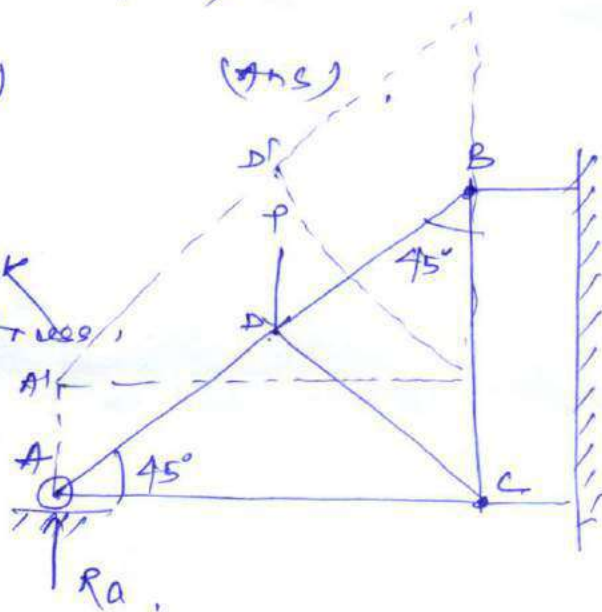
Let the truss is virtual displaced by an amount δy

$$\lim_{x \rightarrow 0} x = 0$$

$$R \times A \subseteq P \times D$$

where $AA' = DD' = dy$

$$\frac{1}{2} \boxed{R_a = p}$$



modipada to big bazar
~~but~~ near jashnath
mandir
right hand side

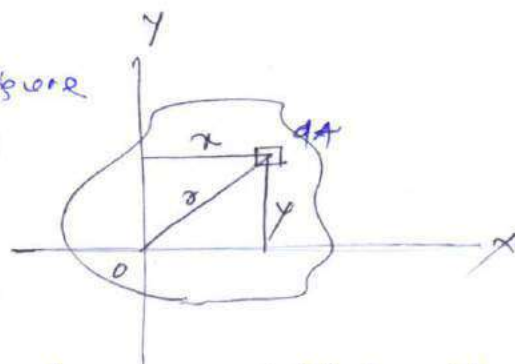
Moment of Inertia of Plane figures

02/12/19

(1)

The moment of inertia of any plane figure with respect to x and y axes in its plane are expressed as

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA$$



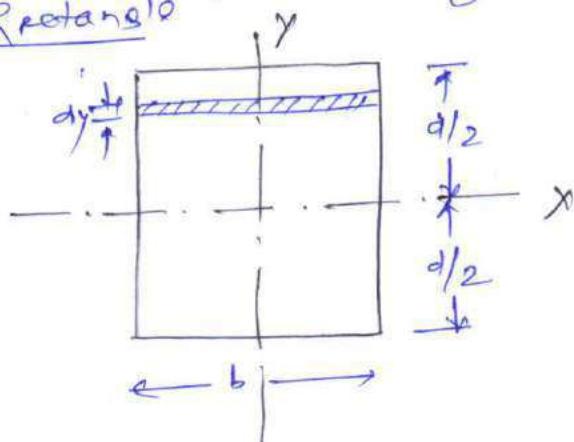
I_{xx} and I_{yy} are also known as second moment of inertia area about the axes as it is distance is squared from corresponding axis.

Unit

Unit of moment of inertia of area is expressed as m^4 or mm^4 .

Moment of Inertia of Plane figures:-

(i) Rectangle



Considering a rectangle of width b and depth d , Moment of inertia about centroidal axis xx parallel to the short side i.e. b

Now considering an elementary strip of width dy

Moment of inertia of the elemental strip about centroidal axis xx is

$$I_{xx} = y^2 dA \\ = y^2 b dy$$

So moment of inertia of entire ~~figure~~ area

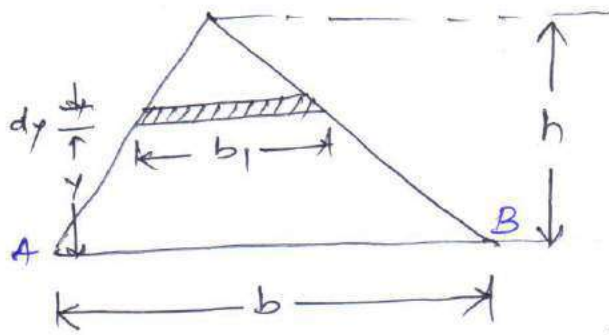
$$I_{xx} = \int_{-d/2}^{d/2} y^2 b dy = b \left[\frac{y^3}{3} \right]_{-d/2}^{d/2} = b \left[\frac{d^3}{24} + \frac{d^3}{24} \right]$$

$$\Rightarrow I_{xx} = \frac{bd^3}{12}$$

Similarly ~~moment~~

$$I_{yy} = \frac{db^3}{12}$$

Cii) Triangle :- (Moment of inertia of a triangle about its base)



Consider a small elementary strip, at a distance y from the base of thickness dy . Let dA is the area of strip

$$dA = b_1 dy$$

$$\text{And } b_1 = \frac{(h-y)}{h} \times b$$

Moment of inertia of strip about base AB

$$= y^2 dA = y^2 b_1 dy$$

$$= y^2 \frac{(h-y)}{h} \cdot b dy$$

$\therefore$ Moment of inertia of the triangle about AB

$$I_{AB} = \int_0^h \frac{y^2 (h-y)}{h} b dy = \int_0^h \left(y^2 - \frac{y^3}{h} \right) b dy$$

$$= b \left[\frac{y^3}{3} - \frac{y^4}{4h} \right]_0^h = b \left[\frac{h^3}{3} - \frac{h^4}{4h} \right]$$

$$= b \left[\frac{h^3}{3} - \frac{h^3}{4} \right] = \frac{bh^3}{12}$$

$$\therefore \boxed{I_{AB} = \frac{bh^3}{12}}$$

Ciii) Moment of inertia of a circle about its centroidal axis

Considering an elementary strip of thickness dr , the area of strip is dA .

Moment of inertia of strip about xx

$$= y^2 dA$$

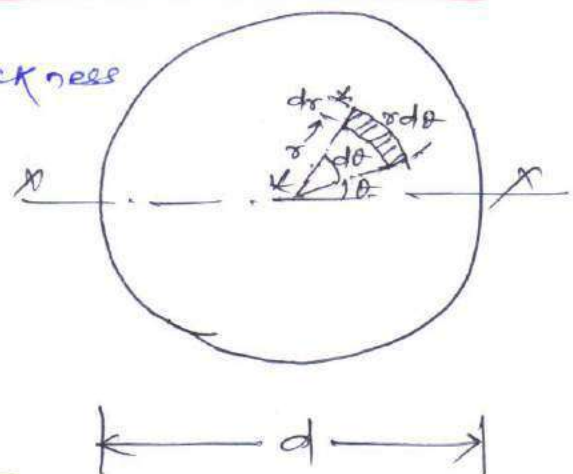
$$= (r \sin \theta)^2 r d\theta dr$$

$$= r^3 \sin^2 \theta d\theta dr$$

$\therefore$ Moment of inertia of circle about xx axis

$$I_{xx} = \int_0^R \int_0^{2\pi} r^3 \sin^2 \theta d\theta dr$$

$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1 - \cos 2\theta}{2} \right) d\theta dr$$



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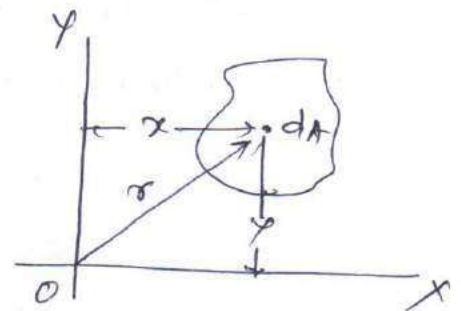
(2)

$$\begin{aligned}
 &= \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} dr \\
 &= \int_0^R \frac{r^3}{2} \left(2\pi - \frac{\sin 4\pi}{2} \right) dr \\
 &= \left[\frac{r^4}{8} \right]_0^R [2\pi - 0] \\
 &= \frac{R^4}{8} 2\pi = \frac{\pi R^4}{4} \\
 \Rightarrow \boxed{I_{xx} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}}
 \end{aligned}$$

$$(\because R = \frac{D}{2})$$

Polar moment of inertia:-

Moment of inertia about an axis perpendicular to the plane of area is called polar moment of inertia. It may be denoted as J or I_{xx} .



$$\boxed{I_{xx} = \int r^2 dA}$$

Radius of Gyration:-

Radius of gyration may be defined by a relation

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

where K = radius of gyration

I = moment of inertia

A = cross-sectional area

So, we can have the following relations

$$\begin{aligned}
 K_{xx} &= \sqrt{\frac{I_{xx}}{A}} \\
 K_{yy} &= \sqrt{\frac{I_{yy}}{A}} \\
 K_{AB} &= \sqrt{\frac{I_{AB}}{A}}
 \end{aligned}$$

Theorems of Moment of inertia

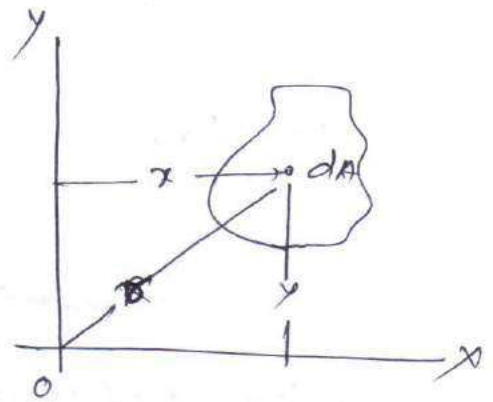
There are two theorems of moment of inertia

(a) perpendicular axis theorem

(b) parallel axis theorem.

perpendicular axis theorem:-

Moment of inertia of an area about an axis $\perp$ to its plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of area.



$$I_{xx} = I_{xx} + I_{yy}$$

$$I_{xx} = \sum r^2 dA$$

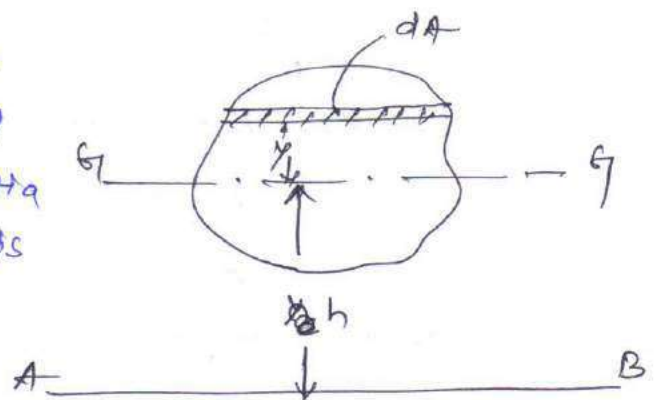
$$= \sum (x^2 + y^2) dA$$

$$= \sum x^2 dA + \sum y^2 dA$$

$$\Rightarrow \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Parallel axis theorem:-

Moment of inertia about an axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axes.



$$\boxed{I_{AB} = I_{GG} + Ah^2}$$

Moment of inertia of standard sections:-

02/12/19

3

i) Moment of inertia of a rectangle about its centroidal axis xx

$$I_{xx} = \frac{bd^3}{12}$$

Similarly moment of inertia about its centroidal axis yy

$$I_{yy} = \frac{db^3}{12}$$

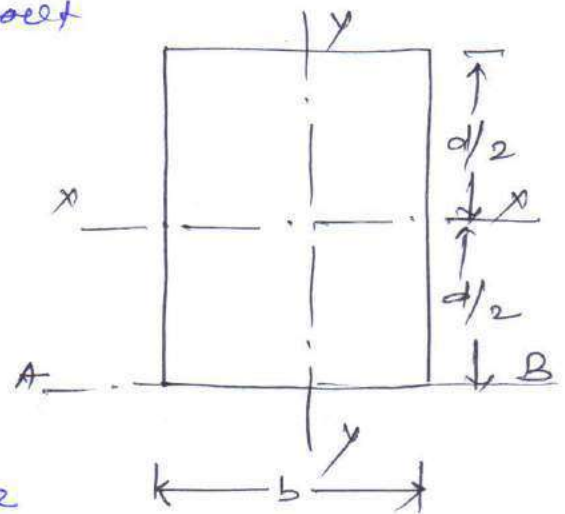
Now moment of inertia of rectangle about its base AB can be obtained by applying parallel axis theorem

$$I_{AB} = I_{xx} + Ah^2$$
$$= \frac{bd^3}{12} + (bd)\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$= \frac{3bd^3 + bd^3}{12} = \frac{4bd^3}{12} = \frac{bd^3}{3}$$

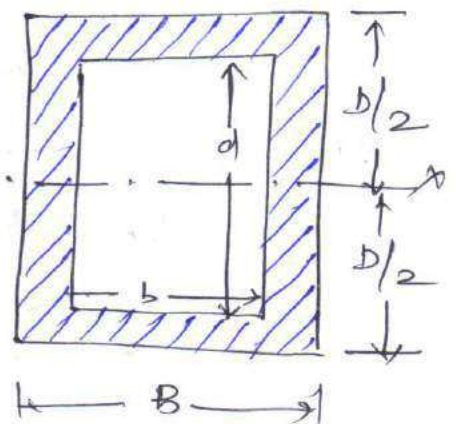
$$\Rightarrow \boxed{I_{AB} = \frac{bd^3}{3}}$$



cii) Moment of inertia of a hollow rectangular section:-

Moment of inertia of hollow rectangular section

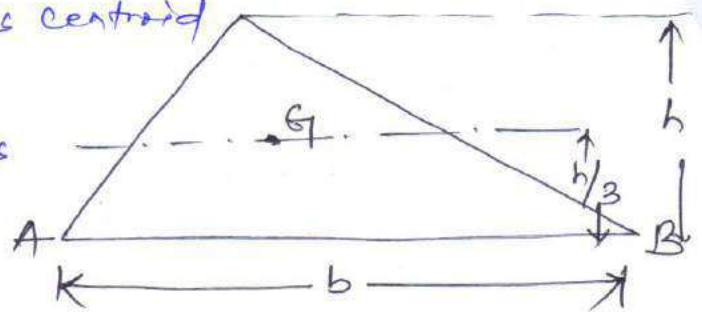
$$\boxed{I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12} (BD^3 - bd^3)}$$



ciii) Moment of inertia of triangle about its ^{centroidal} base

Moment of inertia of triangle about its base
 = moment of inertia about its centroid
 + $A h^2$

(using parallel axis theorem)



$$\Rightarrow I_{AB} = I_{xx} + A h^2$$

$$\Rightarrow \frac{b h^3}{12} = I_{xx} + \frac{1}{2} b h \times \left(\frac{h}{3}\right)^2$$

$$= I_{xx} + \frac{b h^3}{18}$$

$$\Rightarrow I_{xx} = \frac{b h^3}{12} - \frac{b h^3}{18} = \frac{6 b h^3 - 4 b h^3}{36}$$

$$= \frac{2 b h^3}{36} = \frac{b h^3}{18}$$

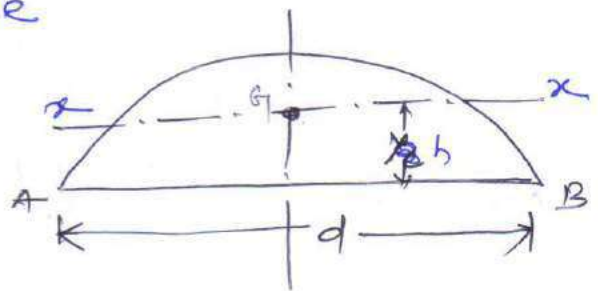
$$\Rightarrow \boxed{I_{xx} = \frac{b h^3}{36}}$$

civ) Moment of inertia of semicircle
 (a) about diametral axis

Moment of inertia of semicircle

$$\text{about AB} = \frac{1}{2} \frac{\pi d^4}{64}$$

$$= \boxed{\frac{\pi d^4}{128}}$$



(b) about centroidal axis xx

$$\boxed{R_h = \frac{4R}{3\pi} = \frac{2d}{3\pi}}$$

$$\text{area } A = \frac{1}{2} \frac{\pi d^2}{4} = \frac{\pi d^2}{8}$$

Using parallel axis theorem

$$I_{AB} = I_{xx} + A h^2$$

$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2 = \cancel{\frac{\pi d^4}{128}} + \frac{\pi d^4}{18\pi}$$

$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \frac{4d^2}{9\pi^2}$$

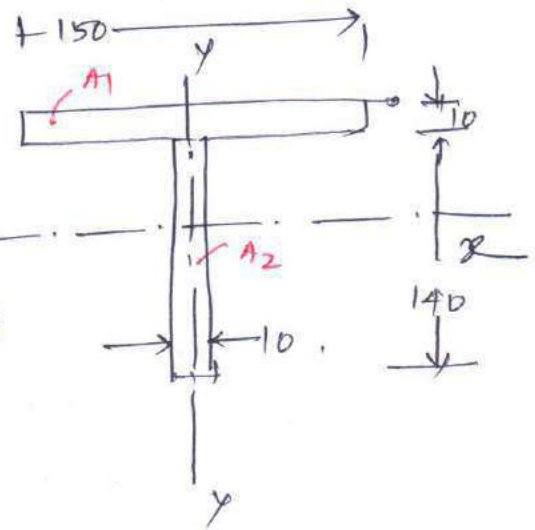
$$= I_{xx} + \frac{\pi d^4}{18\pi}$$

$$\Rightarrow I_{xx} = \left(\frac{\pi d^4}{128} - \frac{d^4}{18\pi} \right)$$

Moment of inertia of composite figures:-

Q.1 Determine the moment of inertia of the composite section about an axis passing through the centroidal axis. Also determine MS about axis of symmetry and radius of gyration.

Soln Dividing the composite area into A_1 and A_2



$$A_1 = 150 \times 10 = 1500 \text{ mm}^2$$

$$A_2 = 140 \times 10 = 1400 \text{ mm}^2$$

Distance of centroid from base of the composite figure

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{(A_1 + A_2)} = \frac{1500 \times 145 + 1400 \times 70}{2900}$$

$$= 108.79 \text{ mm}$$

Moment of inertia of the area about xx axis

$$I_{xx} = \left\{ \frac{150 \times 10^3}{12} + 1500 \times (145 - 108.79)^2 \right\}$$

$$+ \left\{ \frac{10 \times 140^3}{12} + 1400 \times (108.79 - 70)^2 \right\}$$

$$= (12500 + 1966746.15) + (2286666.667 + 2106529.74)$$

$$= 6372442.557 \text{ mm}^4$$

Similarly

$$I_{yy} = \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12} = 2812500 + 11666.66667$$

$$= 2824166.667 \text{ mm}^4$$

Radius of gyration $k = \sqrt{\frac{I}{A}}$

so $k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{6372442.5}{2900}} = 46.87 \text{ mm}$

Similarly $k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{2824166.667}{2900}} = 31.206 \text{ mm} \quad (\text{Ans})$

Q.2 Determine the ME of L-section about its centroidal axes parallel to the legs. Also find the polar moment of inertia.

We have $A_1 = 125 \times 10 = 1250 \text{ mm}^2$

$A_2 = 75 \times 10 = 750 \text{ mm}^2$

Total area $A_1 + A_2 = 2000 \text{ mm}^2$

Distance of centroid from 1-1 axis

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1250 \times 62.5 + 750 \times 5}{2000} = 40.9375 \text{ mm}$$

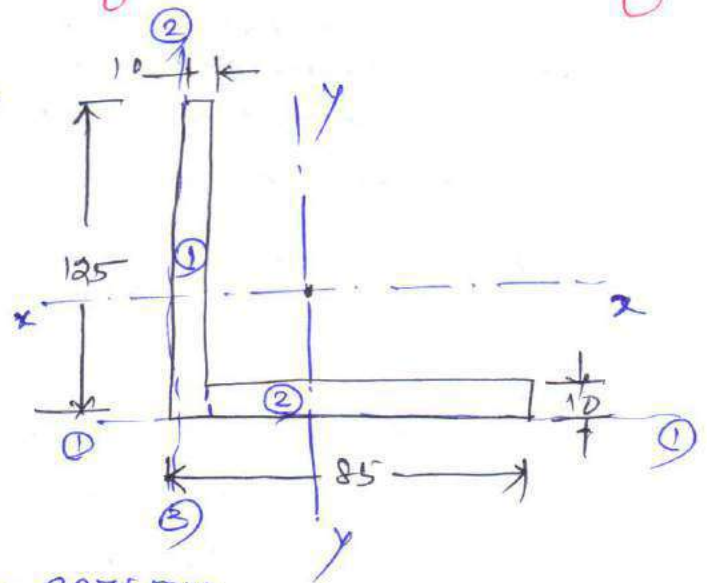
Distance of centroidal axis yy from 2-2 axis

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{1250 \times 5 + 750 \times (\frac{75}{2} + 10)}{2000} = \frac{1250 \times 5 + 750 \times 47.5}{2000} = 20.93 \text{ mm}$$

Moment of inertia about xx axis

$$I_{xx} = \left\{ \frac{10 \times 125^3}{12} + 1250 \times (62.5 - 40.9375)^2 \right\} + \left\{ \frac{75 \times 10^3}{12} + 750 \times (40.9375 - 5)^2 \right\}$$

$$= (1627604.167 + 581176.7578) + (6250 + 968627.9297) = 3183658.854 \text{ mm}^4$$



Similarly MI about yy centroidal axis

$$\begin{aligned}
 I_{yy} &= \left\{ \frac{125 \times 10^3}{12} + 1250 \times (20.93 - 5)^2 \right\} \\
 &+ \left\{ \frac{10 \times 75^3}{12} + 750 \times (47.5 - 20.93)^2 \right\} \\
 &= (10416.66667 + 317206.125) + (351562.5 + 529473.675) \\
 &= \boxed{1208658.967 \text{ mm}^4}
 \end{aligned}$$

Polar moment of inertia $I_{xx} = I_{xx} + I_{yy}$

$$= \boxed{4392317.821 \text{ mm}^4} \quad (\text{Ans})$$

Q.3 Determine the MI of the asymmetrical I section about its centroidal axes x-x and y-y. Also determine the polar moment of inertia of the section.

We have from the figure

$$A_1 = 200 \times 9 = 1800 \text{ mm}^2$$

$$A_2 = \phi 232 \times 6.7 = 1554.4 \text{ mm}^2$$

$$A_3 = 200 \times 9 = 1800 \text{ mm}^2$$

Position of centroidal axis x-x from base

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

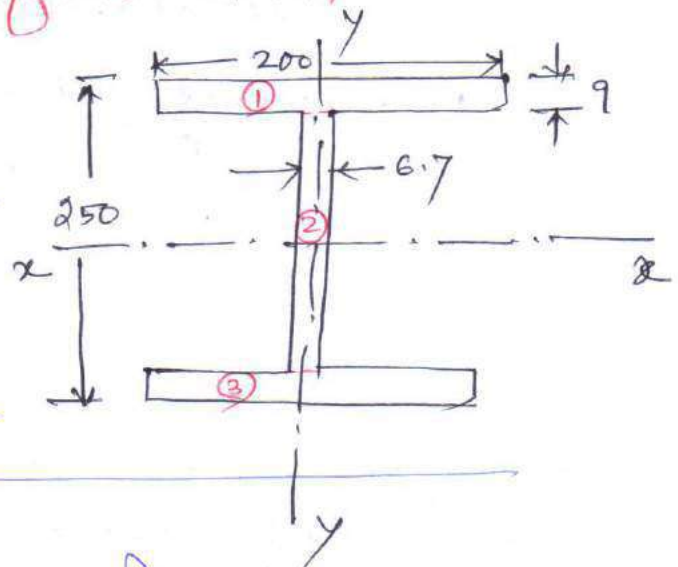
$$= \frac{1800 \times (4.5 + 232 + 9) + 1554.4 \times \left(\frac{232}{2} + 9\right) + 1800 \times 4.5}{(1800 + 1554.4 + 1800)}$$

$$= \frac{1800 \times 245.5 + 1554.4 \times 125 + 1800 \times 4.5}{(1800 + 1554.4 + 1800)}$$

$$= 125 \text{ mm}$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{1800 \times 100 + 1554.4 \times 96.65 + 1800 \times 100}{(1800 + 1554.4 + 1800)} = 98.98$$



ML about xx axis

$$\begin{aligned}
 L_{xx} &= \left\{ \frac{200 \times 9^3}{12} + 1800 \times (125 - 4.5)^2 \right\} + \left\{ \frac{6.7 \times 232^3}{12} + 1554.4 \times (125 - 4.5)^2 \right\} \\
 &+ \left\{ \frac{200 \times 9^3}{12} + 1800 \times (125 - 4.5)^2 \right\} \\
 &= (12150 + 26136450) + (6972002.133 + 0) \\
 &+ (12150 + 26136450) \\
 &= 26148600 + 6972002.133 + 26148600 \\
 &= \boxed{59269202.13 \text{ mm}^4}
 \end{aligned}$$

ML about yy axis

$$\begin{aligned}
 L_{yy} &= \frac{9 \times 200^3}{12} + \frac{232 \times 6.7^3}{12} + \frac{9 \times 200^3}{12} \\
 &= 6000000 + 5814.75 + 6000000 \\
 &= \boxed{12005814.75 \text{ mm}^4}
 \end{aligned}$$

Polar moment of inertia $I_{xx} = L_{xx} + L_{yy}$

$$= \boxed{71275016.88 \text{ mm}^4}$$

Q.4

Calculate the moment of inertia of the shaded area about xx axis.

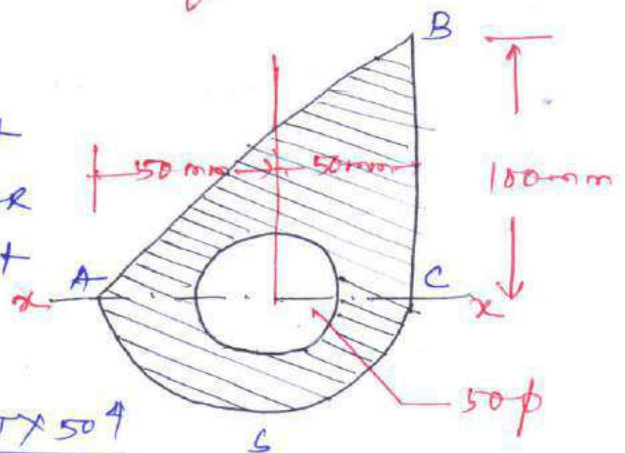
ML of the shaded section about xx = ML of triangle ABC about xx + ML of semicircle ACS about xx - ML of circle

$$= \frac{100 \times 100^3}{12} + \frac{\pi \times 100^4}{128} - \frac{\pi \times 50^4}{64}$$

$$= 8333333.333 + 2454369.261 - 306796.1576$$

$$= 10480906.44 \text{ mm}^4$$

$$= \boxed{1.048 \times 10^7 \text{ mm}^4}$$



- Rectilinear Translation :-

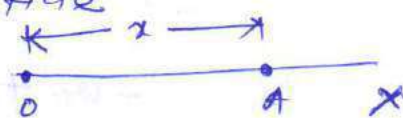
In statics, it was considered that the rigid bodies are at rest. In dynamics, it is considered that they are in motion. Dynamics is commonly divided into two branches. Kinematics and kinetics.

In, kinematics, we are concerned with space time relationship of a given motion of a body and not at all with the forces that cause the motion.

- In kinetics we are concerned with finding the kind of motion that a given body or system of bodies will have under the action of given forces or with what forces must be applied to produce a desired motion.

Displacement

From the fig. displacement of a particle can be defined by its x -coordinate, measured from the fixed reference point O .



- When the particle is to the right of fixed point O , this displacement can be considered positive and when it is towards the ~~right~~ left hand side it is considered as negative.

General displacement time equation

$$x = f(t) \quad \text{--- (1)}$$

where $f(t)$ = function of time

for example

$$x = c + bt$$

In the above equation c , represents the initial displacement at $t = 0$, while the constant b shows the rate at which displacement increases. It is called uniform rectilinear motion.

Second example is $x = \frac{1}{2} at^2$

where x is proportional to the square of time.

velocity

Acceleration

Example The rectilinear motion of a particle is defined by the displacement-time equation $x = x_0 - v_0 t + \frac{1}{2} at^2$. Construct displacement-time and velocity diagrams for this motion and find the displacement and velocity at time $t = 2$ s. $x_0 = 750$ mm, $v_0 = 500$ mm/s, $a = 0.125$ m/s²

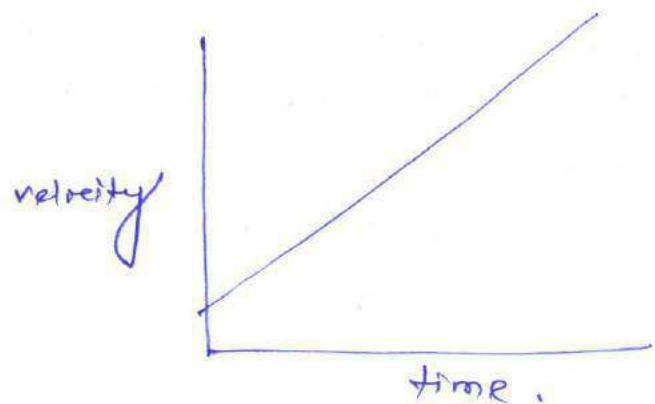
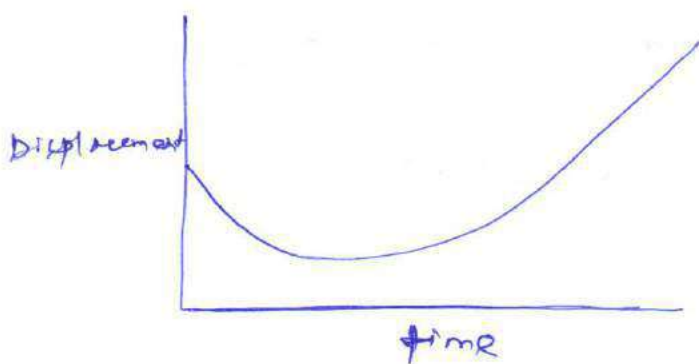
The equation of motion is

$$x = x_0 - v_0 t + \frac{1}{2} at^2 \quad \text{--- (1)}$$

$$v = \frac{dx}{dt} = -v_0 + at \quad \text{--- (2)}$$

substituting x_0 , v_0 and a in equation (1)

$$x = 750 - 500t + 0.0625t^2$$



2

Q-1 A bullet leaves the muzzle of a gun with velocity $u = 750 \text{ m/s}$. Assuming constant acceleration from breech to muzzle find time t occupied by the bullet in travelling through gun barrel which is 750 mm long.

initial velocity of bullet $u = 0$
final velocity of bullet $v = 750 \text{ m/s}$,
total distance $s = 0.75 \text{ m}$,
 $t = ?$

We have $v^2 - u^2 = 2as$,
 $\Rightarrow v^2 = 2as \Rightarrow a = \frac{v^2}{2s} = \frac{750^2}{2 \times 0.75}$
 $= 375000 \text{ m/sec}^2$

Again $v = u + at$
 $\Rightarrow 750 = 375000 \times t$
 $\Rightarrow t = \frac{750}{375000} = \boxed{0.002 \text{ sec.}}$

Q-2 A stone is dropped into well and falls vertically with constant acceleration $g = 9.8 \text{ m/sec}^2$. The sound of impact of stone in the bottom of well is heard after 6.5 sec . If velocity of sound is 336 m/s . How deep is the well?

$v = 336 \text{ m/sec}$

Let $s =$ depth of well

$t_1 =$ time taken by the stone into the well

$t_2 =$ time taken by the sound to be heard.

total time $t = (t_1 + t_2) = 6.5 \text{ sec}$.

Now $s = ut + \frac{1}{2}gt^2$

$\Rightarrow s = 0 + \frac{1}{2}gt^2$

$\Rightarrow t_1 = \sqrt{\frac{2s}{g}}$

When the sound travels with uniform velocity
 $s = vt_2$ or $t_2 = \frac{s}{v}$

$$\sqrt{\frac{2s}{g}} + \frac{s}{v} = 6.5$$

$$\Rightarrow \frac{2s}{g} = \left(6.5 - \frac{s}{336}\right)^2$$

$$\Rightarrow 2s = 9.81 \left(6.5 - \frac{s}{336}\right)^2$$

$$= 9.81 \left(\frac{2184 - s}{336}\right)^2$$

$$= 0.0291 (2184 - s)^2$$

$$= 0.0291 (4769856 + s^2 - 4368s)$$

$$= 138802.809 + 0.0291s^2 - 127.10588s$$

$$\Rightarrow 0.0291s^2 - 127.10588s + 138802.809 = 0$$

$$\Rightarrow s =$$

$$0.2038s = 42.25 + 0.0000885s^2 - 0.0386s$$

$$\cancel{5.2174}$$

$$0.0000885s^2 - 0.1658s + 42.25 = 0$$

$$s = 17.31 \text{ m.}$$

A2

A rope AB is attached at B to a small block of negligible dimensions and passes over a pulley C so that its free end A hangs 1.5 m above ground when the block rests on the floor. The end A of the rope is moved horizontally in a straight line by a man walking with a uniform velocity $v_0 = 3 \text{ m/s}$. Plot the velocity-time diagram (b) find the time t required for the block to reach the pulley if $h = 4.5 \text{ m}$, pulley dimensions are negligible.

A3

A particle starts from rest and moves along a straight line with constant acceleration a . If it acquires a velocity $v = 3 \text{ m/s}$ after having travelled a distance $s = 7.5 \text{ m}$, find magnitude of acceleration.

20/11/2019

①

Principles of Dynamics:Newton's law of motion:

First law: Every body continues in its state of rest or of uniform motion in a straight line except in so far as it may be compelled by force to change that state.

Second Law:

The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Third law To every action there is always an equal and contrary reaction or the mutual actions of any two bodies are always equal and oppositely directed.

General Equation of Motion of a Particle:

$$ma = f$$

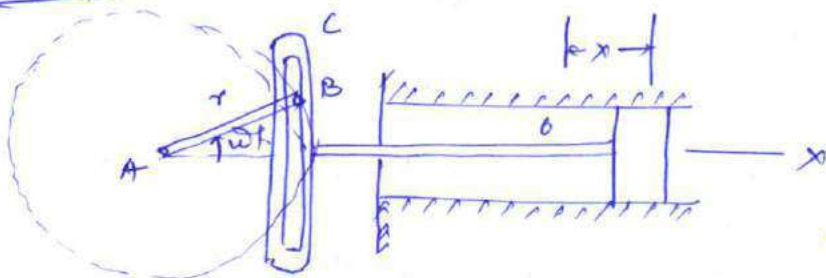
Differential equation of Rectilinear motion:

Differential form of equation for rectilinear motion can be expressed as

$$\frac{W}{g} \ddot{x} = X$$

where $\ddot{x}$ = acceleration

X = Resultant acting force.

Example

For the engine shown in fig., the combined wt. of piston and piston rod $W = 450 \text{ N}$, crank radius $r = 250 \text{ mm}$ and uniform

speed of rotation $n = 120 \text{ rpm}$. Determine the magnitude of resultant force acting in piston (a) at extreme position and at the middle position.

piston has a simple harmonic motion represented by displacement-time equation

$$x = r \cos \omega t \quad \text{--- (1)}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s.}$$

$$\dot{x} = -r\omega \sin \omega t$$

$$\ddot{x} = -r\omega^2 \cos \omega t \quad \text{--- (2)}$$

Differential equation of motion

$$\frac{W}{g} \ddot{x} = X$$

$$\Rightarrow -\frac{W}{g} r\omega^2 \cos \omega t = X$$

$$\Rightarrow X = -\frac{450}{9.81} \times 0.25 (4\pi)^2 \cos(4\pi t)$$

for extreme position

$$\cos \omega t = -1$$

$$\text{so } |X| = 1810 \text{ N.}$$

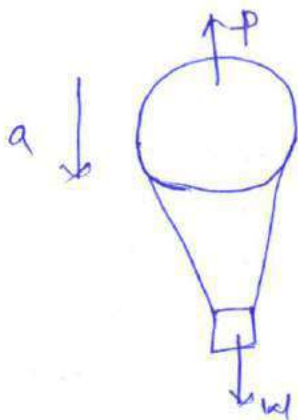
For ~~extreme~~ middle position $\cos \omega t = 0$.

so Resultant force = 0.

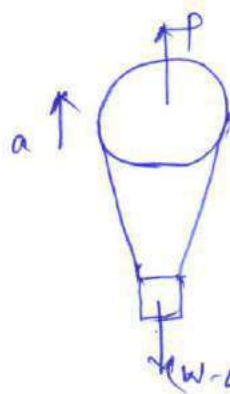
E-2

A balloon of mass m is falling vertically down with constant acceleration a . What amount of ballast Q must be thrown out in order to give balloon an equal upward acceleration a .

P = buoyant force.



(i)



(ii)

(i) considering 1st case when balloon is falling,

$$\frac{W}{g} a = W - P \quad \text{--- (1)}$$

$$\text{(ii)} \quad \frac{W-Q}{g} a = P - (W-Q) \quad \text{--- (2)}$$

$$\text{Eq (1) + Eq (2)}$$

$$\frac{Q}{g} a = W + W - Q = 2W - Q$$

$$\Rightarrow Q \left(\frac{a}{g} + 1 \right) = 2W$$

$$\Rightarrow Q = \frac{2Wg}{(a+g)}$$

$$\frac{W a}{g} = (W - P)$$

$$\frac{(W - R) a}{g} = P - (W - R)$$

$$\frac{W a + (W - R) a}{g} = W - P + P - (W - R) = R$$

$$\Rightarrow \frac{W a + W a - R a}{g} = R$$

$$\Rightarrow 2 W a = R g + R a$$

$$\Rightarrow R = \frac{2 W a}{(g + a)}$$

Q.1

A wt. $W = 4450 \text{ N}$ is supported in a vertical plane by strings and pulleys arranged shown in fig. If the free end A of the string is pulled vertically downward with constant acceleration $a = 18 \text{ m/s}^2$ find tension S in the string.

Differential equation of motion for the system is

$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

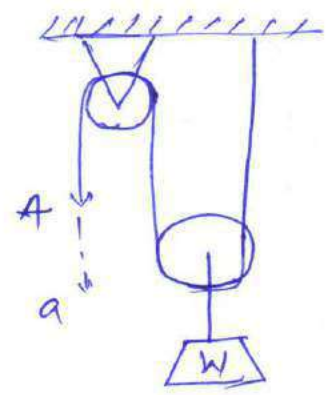
$$\Rightarrow 2S = W + \frac{W a}{2g}$$

$$= \frac{W}{2} \left(2 + \frac{a}{g} \right)$$

$$= W \left(1 + \frac{a}{2g} \right)$$

$$\Rightarrow S = \frac{W}{2} \left(1 + \frac{a}{2g} \right)$$

$$= \frac{4450}{2} \left(1 + \frac{18}{2 \times 9.81} \right) = \boxed{4266.28 \text{ N}}$$



$$\frac{Wa}{g} = (W - P)$$

$$\frac{(W - Q)a}{g} = P - (W - Q)$$

$$\frac{Wa + (W - Q)a}{g} = W - P + P - (W - Q) = Q$$

$$\Rightarrow \frac{Wa + Wa - Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg + Qa$$

$$\Rightarrow Q = \frac{2Wa}{(g + a)}$$

Ex 1

A wt. $W = 4450\text{N}$ is supported in a vertical plane by strings and pulleys arranged shown in fig. If the free end A of the string is pulled vertically downward with constant acceleration $a = 18\text{m/s}^2$ find tension S in the string.

Differential equation of motion for the system is

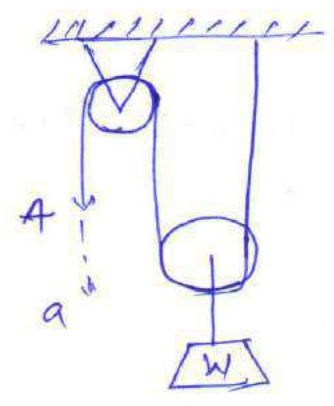
$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

$$= \frac{W}{2} \left(2 + \frac{a}{g} \right)$$
$$= W \left(1 + \frac{a}{2g} \right)$$

$$\Rightarrow S = \frac{W}{2} \left(1 + \frac{a}{2g} \right)$$

$$= \frac{4450}{2} \left(1 + \frac{18}{2 \times 9.81} \right) = \boxed{4266.28 \text{ N}}$$



Q.2

An elevator of gross wt $W = 4450 \text{ N}$ starts to move upward direction with a constant acceleration and acquires a velocity $v = 18 \text{ m/s}$; after travelling a distance $= 1.8 \text{ m}$, find tensile force S in the cable during its motion. $v = 18 \text{ m/s}$

$W = 4450 \text{ N}$

$V = 18 \text{ m/s}$

initial velocity $u = 0$

distance travelled $x = 1.8 \text{ m}$,

$S - W = \frac{W}{g} \cdot a$

$\Rightarrow S = W + \frac{W}{g} a = W \left(1 + \frac{a}{g} \right) \quad \text{--- (1)}$

Now applying equation of kinematics

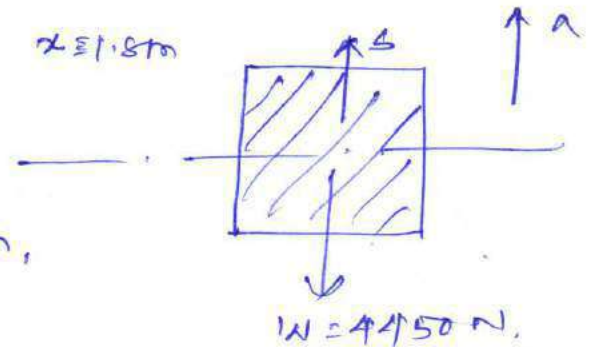
$v^2 - u^2 = 2as$

$\Rightarrow 18^2 - 0 = 2a \times 1.8$

$\Rightarrow a = \frac{18^2}{2 \times 1.8} = \boxed{90 \text{ m/s}^2}$

substituting the value of a in eq. (1)

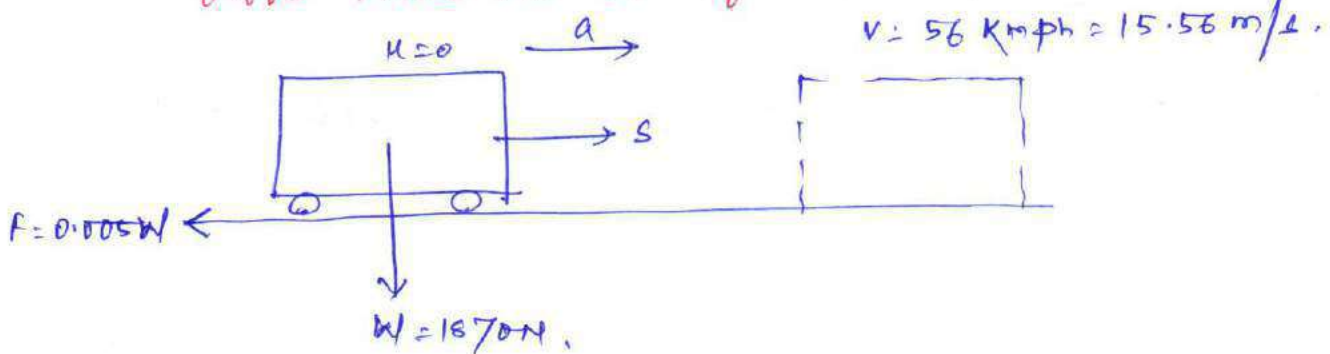
$S = 4450 \left(1 + \frac{90}{9.81} \right) = \boxed{45275.7 \text{ N}}$



A-1

Q.3

A train weighing 1870 N without the locomotive starts to move with constant acceleration along a straight track and in first 60 s acquires a velocity of 56 kmph . Determine the tension S in draw bar between locomotive and train if the air resistance is 0.005 times the wt. of the train.



$$S - F = \frac{W}{g} \cdot a$$

$$\Rightarrow S = 0.005W + \frac{W a}{g} \quad \text{--- (1)}$$

from eq. of kinematics,

$$v = u + at$$

$$\Rightarrow a = \left(\frac{15.56 - 0}{60} \right) = 0.26 \text{ m/sec}^2$$

substituting the value of a in eq. (1)

$$S = W \left(0.005 + \frac{a}{g} \right)$$

$$= 1870 \left(0.005 + \frac{0.26}{9.81} \right) = \boxed{58.9 \text{ kN}}$$

A-2
Q-4
A wt. W is attached to the end of a small flexible rope of dia. $d = 6.25 \text{ mm}$, and is raised vertically by winding the rope on a reel. If the reel is turned uniformly at a rate of 2 rps. What will be the tension in rope.

dia of rope $d = 6.25 \text{ mm} = 0.00625 \text{ m}$

No. of revolutions $N = 2 \text{ rps}$.

let x = initial radius of reel.

t = time taken for N revolutions.

R = radius after t sec.

$$R = [x + (N \cdot d)]$$

Now mean velocity $v = R \omega$

$$\omega = 2\pi N$$

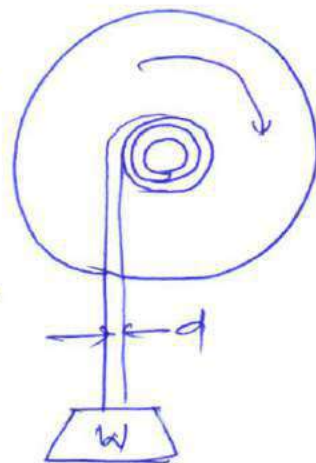
$$\therefore v = (x + N \cdot d) 2\pi N$$

acceleration of rope $a = \frac{dv}{dt}$

$$a = \frac{d}{dt} [2\pi N x + 2\pi N^2 d] = 2\pi N^2 d$$

$$S - W = \frac{W}{g} \cdot a \Rightarrow S = W + \frac{W a}{g} = W \left(1 + \frac{a}{g} \right)$$

$$\Rightarrow S = W \left(1 + \frac{2\pi N^2 d}{g} \right)$$



$$\Rightarrow S = W \left(1 + \frac{2\pi \times 2^2 \times 0.00625}{9.81} \right)$$

=

Ass-3

Q.5

A mine cage of wt $W = 8.9 \text{ kN}$ starts from rest and moves downwards with constant acceleration travelling a distance $S = 30 \text{ m}$ in 10 sec . Find the tensile force in the cable.

Wt. of cage $W = 8.9 \text{ kN}$.

initial velocity $u = 0$.

distance travelled $S = 30 \text{ m}$

time $t = 10 \text{ sec}$.

$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow 30 = \frac{1}{2} a \times 10^2$$

$$\Rightarrow t = \frac{60}{10^2} = 0.6 \text{ m/sec}^2$$

Differential equation of rectilinear motion

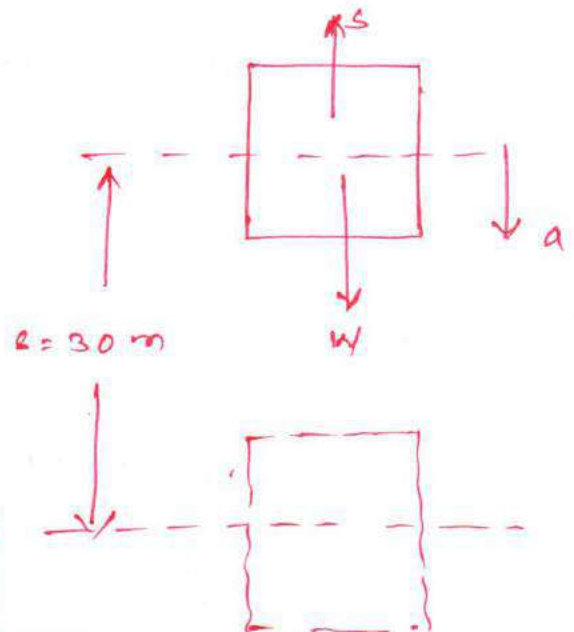
$$W - S = \frac{W}{g} \cdot a$$

$$\Rightarrow S = W - \frac{W}{g} a = W \left(1 - \frac{a}{g} \right)$$

$$= 8.9 \left(1 - \frac{0.6}{9.81} \right)$$

$$\Rightarrow S = 8.35 \text{ kN.}$$

(Ans)



D'Alembert's Principle

25/11/14

①

Differential equation of motion (rectilinear) can be written as

$$X - m\ddot{x} = 0 \quad \text{--- (1)}$$

Where X = Resultant of all applied force in the direction of motion

m = mass of the particle

The above equation may be treated as equation of dynamic equilibrium. To represent this equation, in addition to the real forces acting on the particle a fictitious force $m\ddot{x}$ is required to be considered. This force is equal to the product of mass of the particle and its acceleration and directed ⁱⁿ opposite direction, and is called the inertia force of the particle.

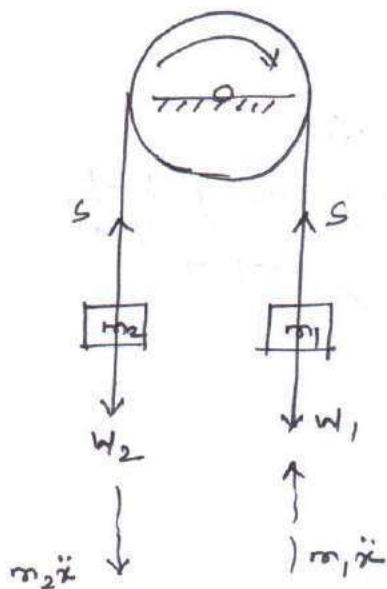
$$- \sum m\ddot{x} = -\ddot{x} \sum m = -\frac{W}{g} \ddot{x}$$

Where W = total weight of the body

so the equation of dynamic equilibrium can be expressed as:

$$\sum X_i + \left(-\frac{W}{g} \ddot{x} \right) = 0 \quad \text{--- (2)}$$

Example 1



for the example shown considering the motion of pulley as shown by the arrow mark. we have upward acceleration $\ddot{x}_2$ for W_2 and downward acceleration $\ddot{x}_1$ for W_1 .

- corresponding inertia forces and their direction are indicated by dotted line.

- By adding inertia forces to the real forces (such as W_1, W_2 and tension in strings) we obtain, for each particle, a system of

forces in equilibrium.

The equilibrium equation for the entire system without S

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

$$\Rightarrow (m_1 + m_2)\ddot{x} = (W_1 - W_2) \Rightarrow \ddot{x} = \frac{W_1 - W_2}{W_1 + W_2} \cdot g$$

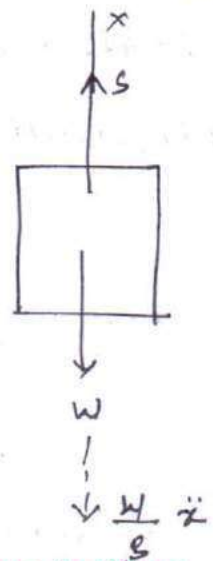
Example 2

A body is moving in upward direction by a rope.

So the equation of dynamic equilibrium considering the real and inertia forces.

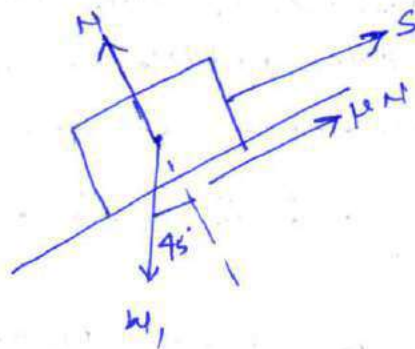
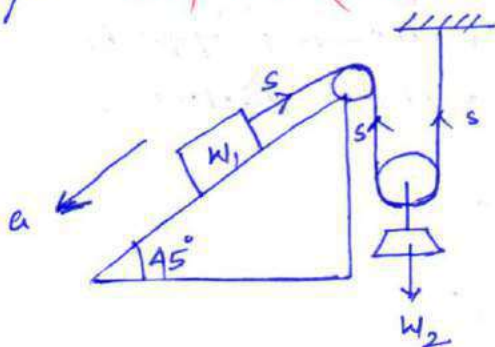
$$S - W - \frac{W}{g} a = 0, \text{ so tensile force in rope}$$

$$\Rightarrow S = W \left(1 + \frac{a}{g} \right)$$



Q.1

Find tension S in the string during motion of the system
 if $W_1 = 900 \text{ N}$, $W_2 = 450 \text{ N}$. Take μ between the inclined plane and block $W_1 = 0.2$



When W_1 moves downward in the inclined plane with an acceleration a , then acceleration of $W_2 = \frac{a}{2}$

Considering dynamic equilibrium of W_1 , from D'Alembert's principle

$$(W_1 \sin 45^\circ - \mu N - S) - \frac{W_1}{g} a = 0$$

$$\Rightarrow \frac{W_1}{g} a = W_1 \sin 45^\circ - \mu N - S$$

$$= W_1 \sin 45^\circ - \mu W_1 \cos 45^\circ - S$$

$$\Rightarrow a = \left(900 \times \frac{1}{\sqrt{2}} - 0.2 \times 900 \times \frac{1}{\sqrt{2}} - S \right) \frac{9.81}{900}$$

$$= (636.4 - 127.28 - S) \times 0.0109$$

$$\Rightarrow a = \frac{509.12 - 1.987352 S}{0.0109} \quad \text{--- (1)}$$

Similarly for weight W_2

$$2S - W_2 - \frac{W_2}{g} \frac{a}{2} = 0$$

$$\Rightarrow \frac{W_2 a}{2g} = 2S - W_2 \left(1 + \frac{a}{2g} \right) = 2S$$

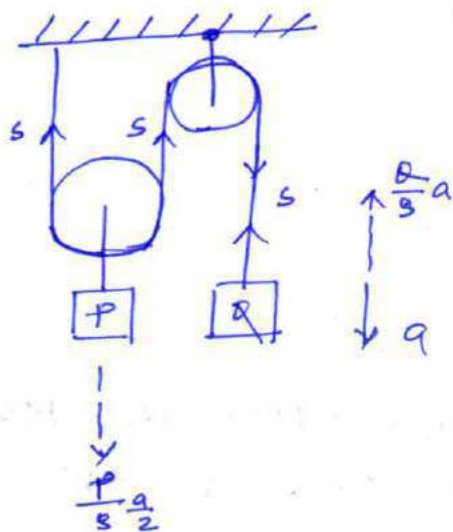
$$\Rightarrow 2S = \frac{450}{2} \left(1 + \frac{a}{19.62} \right) = 225 + 11.46 a \quad \text{--- (2)}$$

substituting the value of S in eq. (1)

$$a = 5.549 - 0.24525 - 0.125 a \Rightarrow a =$$

$$\begin{aligned}
 a &= 6.93676 - 1.387352 - 0.0109(225 + 11.46a) \\
 &= 6.93676 - 1.387352 - 2.4525 - 0.124914a \\
 &= 3.096908 - 0.124914a \\
 \Rightarrow \boxed{a &= 2.75 \text{ m/s}^2}
 \end{aligned}$$

Q.2 Two weights P and Q are connected by the arrangement shown in fig. Neglecting friction and inertia of pulley and cord find the acceleration a of wt. Q. Assume $P = 178 \text{ N}$, $Q = 133.5 \text{ N}$.



Applying D'Alembert's principle for Q

$$Q - s - \frac{Q}{g}a = 0$$

$$\Rightarrow s = \frac{Q}{g} \left(1 - \frac{a}{g} \right) \quad \text{--- (1)}$$

Applying D'Alembert's principle to P

$$2s - P - \frac{P}{2g}a = 0$$

$$\Rightarrow 2s = P \left(1 + \frac{a}{2g} \right)$$

$$\Rightarrow s = \frac{P}{2} \left(1 + \frac{a}{2g} \right) \quad \text{--- (2)}$$

$$= \frac{178}{2} \left(1 + \frac{a}{19.62} \right)$$

$$133.5 \left(1 - \frac{a}{9.81} \right) = 89 \left(1 + \frac{a}{19.62} \right)$$

$$\Rightarrow 133.5 - 13.608a = 89 + 4.536a$$

$$\Rightarrow 18.144a = 44.5$$

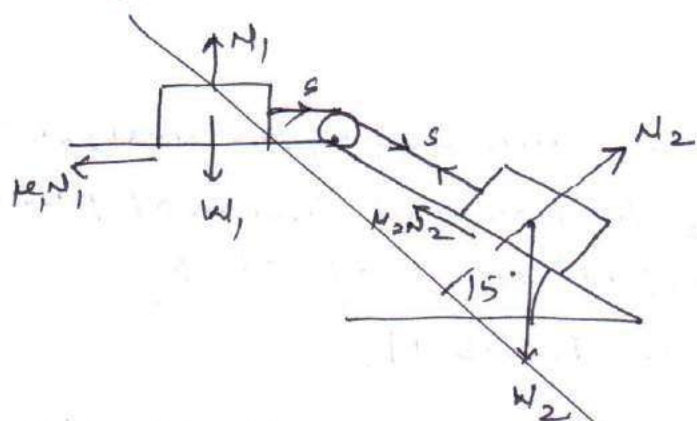
$$\Rightarrow \boxed{a = 2.45 \text{ m/s}^2}$$

(Ans)

Q.3

Assuming the car in the fig. to have a velocity of 6 m/s find shortest distance s in which it can be stopped with constant deceleration without disturbing the block. Data: $c = 0.6 \text{ m}$, $h = 0.9 \text{ m}$, $\mu = 0.5$

Q.3 Two blocks of wt $W_1 = 150\text{ N}$ and $W_2 = 500\text{ N}$ are connected by an inextensible string. Find the acceleration of the blocks and tension in the string. $\mu_1 = 0.1$, $\mu_2 = 0$.



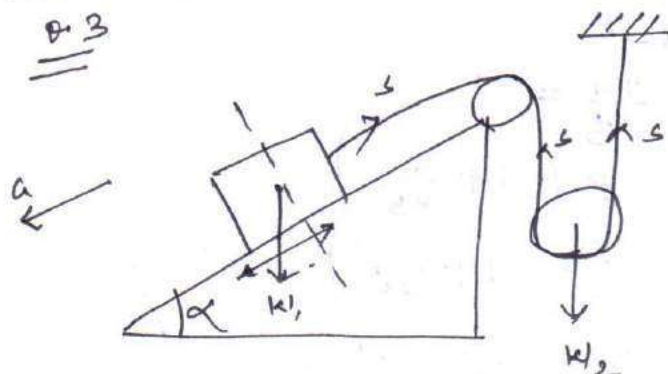
for block 1

$$S - \mu N_1 = 0$$

$$\Rightarrow S = \mu W_1 = 0.1 \times 150 = 15\text{ N}$$

for block 2

Q.3



$$W_1 = 890\text{ N} \quad W_2 = 445\text{ N}$$

$$\mu = 0.2 \quad \alpha = 45^\circ$$

find S .

considering equilibrium of W_1 and applying D'Alembert's principle

$$W_1 \sin 45^\circ - \mu N_1 - S - \frac{W_1}{g} a = 0$$

$$\begin{aligned} \Rightarrow S &= W_1 \sin 45^\circ - \mu N_1 - \frac{W_1}{g} a \\ &= \frac{890}{\sqrt{2}} - 0.2 \times 890 \times \frac{1}{\sqrt{2}} - \frac{890}{9.81} a \\ &= 629.32 - 125.865 - 90.72a \end{aligned}$$

$$\boxed{S = 503.455 - 90.72a} \quad \text{--- (1)}$$

Applying D'Alembert's principle for W_2

$$2S - W_2 - \frac{W_2}{g} \frac{a}{2} = 0$$

$$\Rightarrow 2S = W_2 \left(1 + \frac{a}{2g} \right)$$

$$\Rightarrow S = \frac{W_2}{2} \left(1 + \frac{a}{2g} \right) = \frac{445}{2} \left(1 + \frac{a}{19.62} \right) = 222.5 + 11.34a \quad \text{--- (2)}$$

equating (1) and (2)

25/11/14 (3)

$$503.455 - 90.72a = 222.5 + 11.34a$$

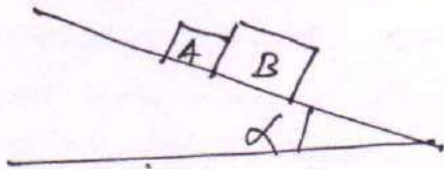
$$\Rightarrow 102.6604a = 280.955$$

$$\Rightarrow \boxed{a = 2.75 \text{ m/s}^2}$$

$$\text{so } S = 222.5 + 11.34 \times 2.75$$

$$= \boxed{253.71 \text{ N.}}$$

Q4

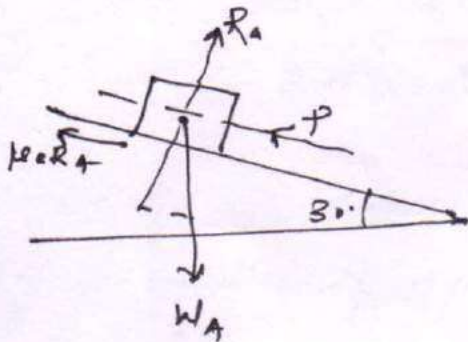


$$W_A = 44.5 \text{ N} \quad W_B = 89 \text{ N}$$

$$\alpha = 30^\circ \quad \mu_a = 0.15$$

$$\mu_B = 0.3$$

find pressure P between blocks.



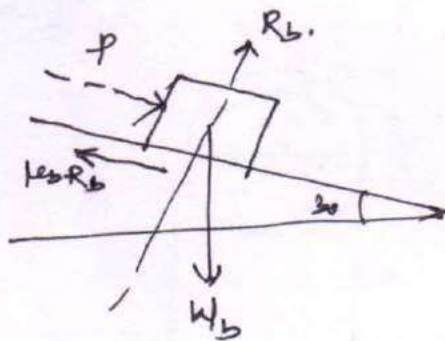
$$W_A \sin 30 - P - \mu_a R_A - \frac{W_A}{g} a = 0$$

$$\Rightarrow P = W_A \sin 30 - \mu_a R_A - \frac{W_A}{g} a$$

$$= 44.5 \times \frac{1}{2} - 0.15 \times 44.5 \times \cos 30 - \frac{44.5}{9.81} a$$

$$= 22.25 - 5.78 - 4.53a \quad \text{--- (1)}$$

$$= 16.47 - 4.53a \quad \text{--- (1)}$$



$$P + W_B \sin 30 - \mu_B R_B - \frac{W_B}{g} a = 0$$

$$\Rightarrow P = -\frac{W_B}{2} + 0.3 \times 89 \cos 30 + \frac{89}{9.81} a$$

$$= -\frac{89}{2} + 23.122 + 9.07a$$

$$= -21.378 + 9.07a \quad \text{--- (2)}$$

$$16.47 - 4.53a = -21.378 + 9.07a$$

$$\Rightarrow 13.6a = 37.848$$

$$\Rightarrow a = 2.78 \text{ m/s}^2$$

$$P = 3.87 \text{ N.}$$

Momentum and Impulse

27/11/2019 (1)

We have the differential equation of rectilinear motion of a particle

$$\frac{W}{g} \ddot{x} = X$$

Above equation may be written as

$$\frac{W}{g} \frac{d\dot{x}}{dt} = X$$

$$\text{or } \boxed{d\left(\frac{W}{g} \dot{x}\right) = X dt} \quad \text{--- (1)}$$

In the above equation we will assume force X as a function of time represented by a force time diagram.

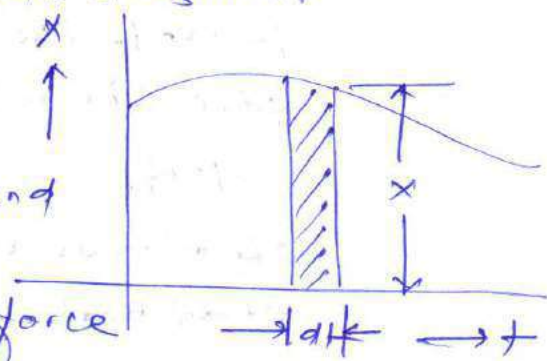
The righthand side of eq. (1)

is then represented by the area of shaded elemental strip of height X and width dt . This quantity i.e.

$(X dt)$ is called impulse of the force

X in time dt . The expression on the left hand side

of the expression $\left(\frac{W}{g} \dot{x}\right)$ is called momentum of particle.



so the eq. (1) represents the differential change in momentum of a particle in time dt .

Integrating eq. (1) we have

$$\boxed{\frac{W}{g} \dot{x} + C = \int_0^t X dt} \quad \text{--- (2)}$$

where C is a constant of integration

Now assuming an initial momentum, $t=0$, the particle has an initial velocity $\dot{x}_0$

$$\text{so } \boxed{C = -\frac{W}{g} \dot{x}_0} \quad \text{--- (3)}$$

so equation (2) becomes

$$\boxed{\frac{W}{g} \dot{x} - \frac{W}{g} \dot{x}_0 = \int_0^t X dt} \quad \text{--- (4)}$$

From equation (4) it is clear that the total change in momentum of a particle during a finite interval of time is equal to the impulse of acting force.

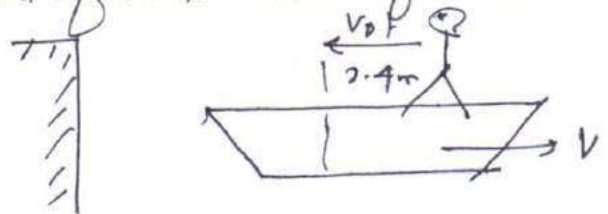
~~Example~~ in other words

$$\boxed{F \cdot dt = d(mv)}$$

where $m \times v = \text{momentum}$

Example given

Q-1 A man of wt 712 N stands in a boat so that he is 4.5 m from a pier on the shore. He walks 2.4 m in the boat towards the pier and then stops. How far from the pier will he be at the end of time. Wt. of boat is 890 N.



wt of man $w_1 = 712 \text{ N}$

wt of boat $w_2 = 890 \text{ N}$

Let v_0 is the initial velocity of man and t is time

then $v_0 t = x$

$$\Rightarrow v_0 t = 2.4 \text{ m}$$

$$\Rightarrow v_0 = \left(\frac{2.4}{t} \right) \text{ m/s.}$$

Let $V = \text{velocity of boat towards right}$
according to conservation of momentum

$$w_1 v_0 = (w_1 + w_2) V$$

$$\Rightarrow V = \frac{w_1 v_0}{(w_1 + w_2)}$$

distance covered by boat

$$s = V \cdot t = \frac{w_1 v_0}{(w_1 + w_2)} \cdot t$$

$$\Rightarrow s = \frac{712 \times 2.4}{712 + 890} = \boxed{1.067 \text{ m}}$$

position of man from pier

$$= 4.5 + s - x$$

$$= 4.5 + 1.567 - 2.4 = \boxed{3.167 \text{ m}} \quad (\text{Ans}).$$

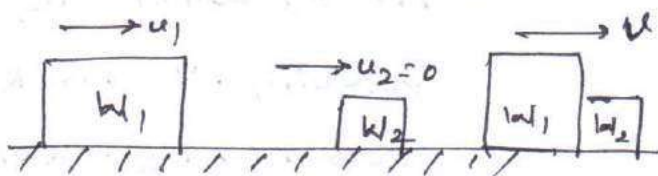
0.2

A locomotive wt 534 kN has a velocity of 16 kmph and backs into a freight car of wt 86 kN that is at rest on a track. after coupling, at what velocity v the entire system continues to move. Neglect friction.

conservation of momentum

$$W_1 u_1 + W_2 u_2 = (W_1 + W_2) v$$

$$\Rightarrow v = \frac{534 \times 4.45}{(534 + 86)} = \boxed{3.82 \text{ m/s.}}$$



0.3

A 667.5 man sits in a 333.75 N canoe and fire a rifle bullet horizontally. ~~directed over~~ find velocity v with which the canoe will move after the shot. the rifle has a muzzle velocity 660 m/s and wt. of bullet is 0.28 N.

$$\text{Wt. of man } W_1 = 667.5 \text{ N.}$$

$$\text{Wt. of canoe } W_2 = 333.75 \text{ N.}$$

$$\text{Wt. of bullet } W_3 = 0.28 \text{ N.}$$

$$\text{velocity of muzzle } u = 660 \text{ m/s.}$$

$$V = \text{final velocity of canoe.}$$

According to conservation of momentum

$$W_3 u = (W_1 + W_2) V$$

$$\Rightarrow V = \frac{0.28 \times 660}{(667.5 + 333.75)} = \boxed{0.182 \text{ m/s.}}$$

Q.4

A wood block wt 22.25 N rests on a smooth horizontal surface. A revolver bullet weighing 0.14 N is shot horizontally into the side of block. If the block attains a velocity of 3 m/s what is muzzle velocity.

Wt. of wood block $W_1 = 22.25 \text{ N}$.

Wt. of bullet $W_2 = 0.14 \text{ N}$.

velocity of block $V = 3 \text{ m/s}$.

velocity of muzzle $= u$

According to conservation of momentum

$$\cancel{W_1 V} = W_2 u = (W_1 + W_2) V$$

$$\Rightarrow u = \frac{(22.25 + 0.14) 3}{0.14}$$

$$= \boxed{479.98 \text{ m/s.}}$$

Conservation of momentum

When the sum of impulses due to external force is zero the momentum of the system remain conserved

$$\text{When } \sum \int F dt = 0$$

$$\boxed{\sum \left(\frac{W}{g} \right) x_2 = \sum \left(\frac{W}{g} \right) x_1}$$

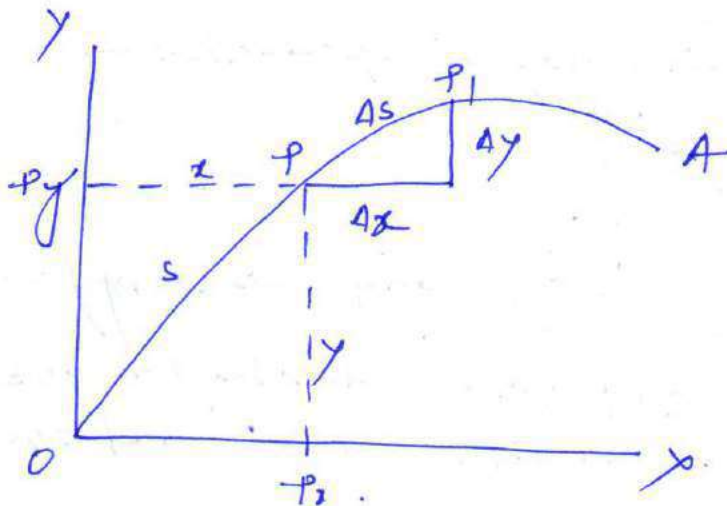
$\therefore$ final momentum = initial momentum.

Curvilinear Translation

①

When a moving particle describes a curved path it is said to have curvilinear motion.

Displacement



Consider a particle P in a plane on a curved path.

To define the particle we need two coordinates x and y .

As the particle moves, these coordinates ~~move~~ change.

change with time and the displacement time equations are

$$x = f_1(t) \quad y = f_2(t) \quad \text{--- (1)}$$

The motion of particle can also be expressed as

$$y = f(x) \quad s = f_3(t)$$

where $y = f(x)$ represents the equation of path of A and $s = f_3(t)$ gives displacement s measured along the path as a function of time.

Velocity:-

Considering an infinitesimal time difference from t to $t + \Delta t$ during which the particle moves from P to P1 along its path.

then velocity of particle may be expressed as

$$\boxed{\bar{V}_{av} = \frac{\Delta \vec{s}}{\Delta t}}$$

$$\boxed{\begin{aligned} (V_{av})_x &= \frac{\Delta x}{\Delta t} \\ (V_{av})_y &= \frac{\Delta y}{\Delta t} \end{aligned}}$$

(average velocity along x and y coordinates)

It can also be represented as

$$v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

so the total velocity may be represented by

$$v = \sqrt{\dot{x}^2 + \dot{y}^2}$$

and $\cos(\theta, x) = \frac{\dot{x}}{v}$ and $\cos(\theta, y) = \frac{\dot{y}}{v}$

where $\theta(\theta, x)$ and (θ, y) denotes the angles betⁿ the direction of velocity vector $\vec{v}$ and the coordinate axes.

Acceleration :-

The acceleration particles may be described as

$$\begin{aligned} a_x &= \frac{dv_x}{dt} = \ddot{x} \\ a_y &= \frac{dv_y}{dt} = \ddot{y} \end{aligned}$$

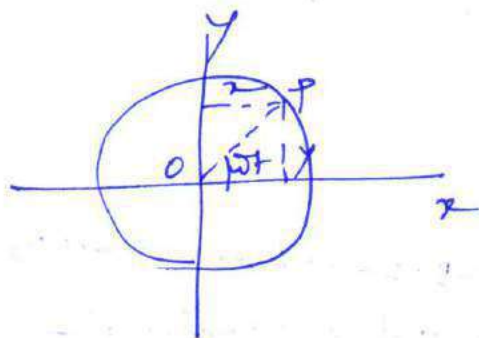
It is also known as instantaneous acceleration

Total acceleration $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

Considering particular path for above case.

$$x = r \cos \omega t \quad y = r \sin \omega t$$

$$x^2 + y^2 = r^2$$



$$\dot{x} = -r\omega \sin \omega t \quad \dot{y} = r\omega \cos \omega t$$

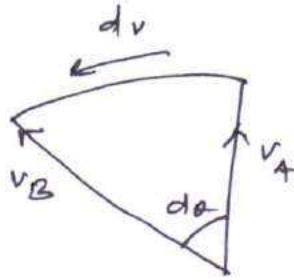
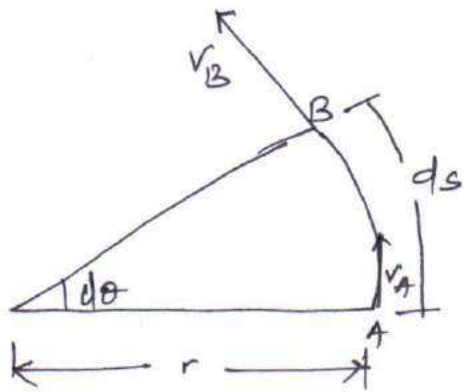
$$v = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$\ddot{x} = -r\omega^2 \cos \omega t \quad \ddot{y} = -r\omega^2 \sin \omega t$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$$

D'Alembert's Principle in Curvilinear Motion

Acceleration during circular motion



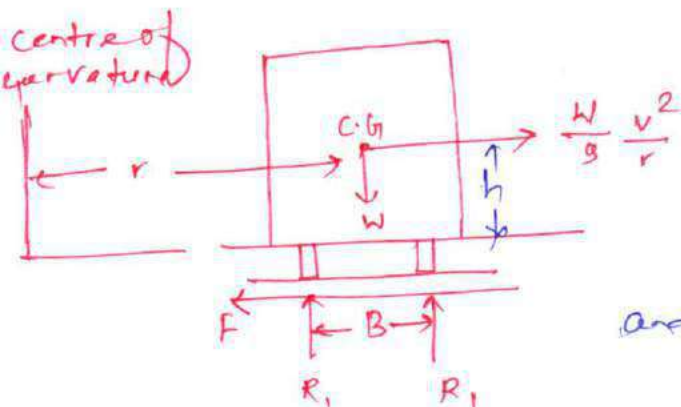
$$\begin{aligned} v_A &= \text{tangential velocity at A} \\ &= \text{tangential velocity at B} \\ &= v_B = v \end{aligned}$$

$$\begin{aligned} \text{Now } dv &= v d\theta = v \frac{ds}{r} = \frac{v}{r} ds \\ \text{acceleration} &= \frac{dv}{dt} = \boxed{\frac{v^2}{r}} \end{aligned}$$

so when a body moves with uniform velocity v along a curved path of radius r , it has a radial inward acceleration of magnitude $\frac{v^2}{r}$

Applying D'Alembert's principle to set equilibrium condition an inertia force of magnitude $\frac{W}{g} a$ $= \frac{W}{g} \frac{v^2}{r}$ must be applied in outward direction it is known as centrifugal force.

Motion on a level road



Consider a body is moving with uniform velocity on a curvilinear curve of radius r . Let the road is flat.

Let W = wt. of the body
and inertia force is given by

$$\frac{W}{g} a = \frac{W}{g} \frac{v^2}{r}$$

Condition for skidding :-

Let W = wt. of vehicle

R_1, R_2 = reactions at wheel

F = frictional force.

$\frac{W}{g} \cdot \frac{v^2}{r}$ = inertia force

Skidding takes place when the frictional force reaches limiting value i.e.

$$F = \mu R$$

Then max permissible speed to avoid skidding

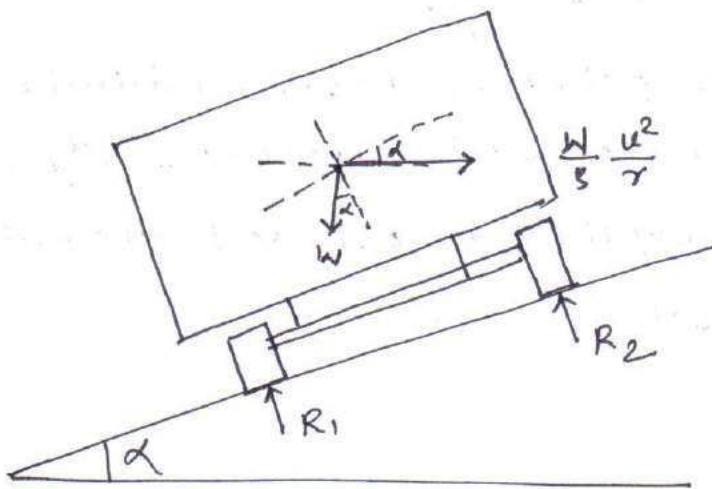
$$v = \sqrt{\frac{gr}{2} \frac{B}{h}}$$

The distance betn inner and outer wheel is equal to the gauge of railway track and represented as B .

so

$$v = \sqrt{\frac{gr}{2} \frac{G}{h}}$$

Designed speed and angle of Banking



Σ of all the forces in the inclined plane

$$\frac{W}{g} \frac{v^2}{r} \cos \alpha - W \sin \alpha = 0$$

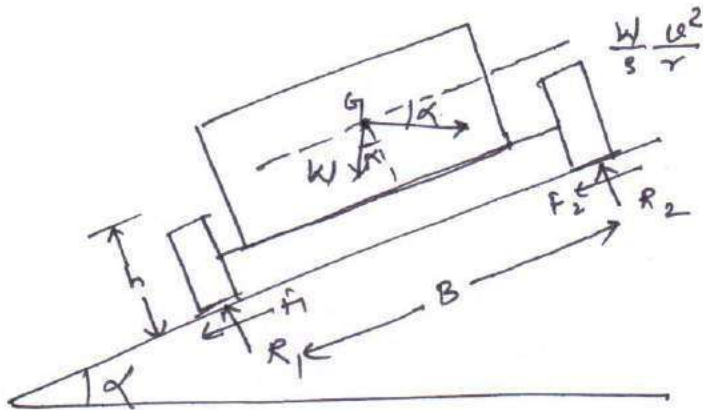
$$\Rightarrow \tan \alpha = \frac{v^2}{gr}$$

Relation betn the angle of banking and designed speed

is $\tan \alpha = \frac{v^2}{gr}$

condition for skidding and overturning:-

(2)



(a) condition for skidding

$$v = \sqrt{\tan(\alpha + \phi) \times g r}$$

where α = angle of inclination

$$\tan \phi = \mu$$

g = ~~effective~~ gravitational acceleration

r = radius of curve

then the vehicle will skid if the velocity is more than this value.

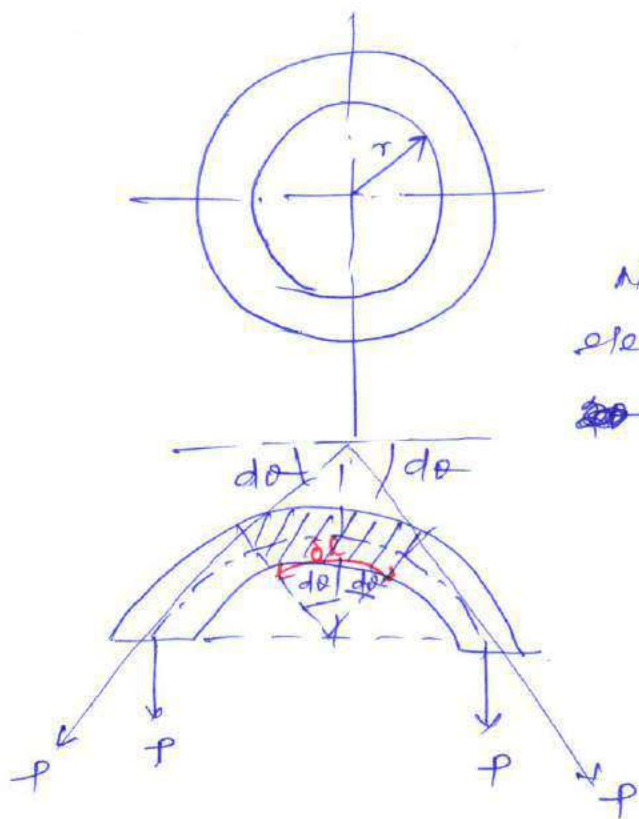
(b) condition for overturning:

Limiting speed from consideration of overturning

$$v = \sqrt{g \frac{G + (2he/G)}{2h - e}}$$

Q.1

A circular ring has a mean radius $r = 500 \text{ mm}$ and is made of steel for which $w = 77.12 \text{ kN/m}^3$ and for which ultimate strength in tension is 413.25 MPa . Find the uniform speed of rotation about its geometrical axis perpendicular to the plane of the ring at which it will burst?



mean radius $r = 500 \text{ mm} = 0.5 \text{ m}$,

density of the wheel $w = 77.12 \text{ kN/m}^3$

$\sigma_t =$ ultimate strength $= 413.85 \times 10^6 \text{ Pa}$

Now considering an infinitesimal small elementary ring subtended at an angle of $2d\theta$

centrifugal force acting

$$F_c = \frac{dw}{g} \cdot \frac{v^2}{r}$$

Let $P =$ tension on the ring

$A =$ cross-sectional area of ring,

$dw =$ wt. of the element

$$= w \times \text{volume}$$

$$= w \times A \times dr$$

$$= w A \times r \times 2d\theta$$

Now centrifugal force

$$\frac{w}{g} (A dr) \times \frac{v^2}{r} = \frac{w}{g} \times A \times r \times 2d\theta \times \frac{v^2}{r} = \frac{2w A dr v^2}{g}$$

Balancing forces along the radius $= 2P \sin d\theta$

$$= \frac{2w A dr v^2}{g} \quad \text{--- (1)}$$

as $d\theta$ is very small $\sin d\theta \approx d\theta$

Eq. (1) may be written as

$$2P d\theta = \frac{2w A dr v^2}{g}$$

$$\Rightarrow \left[P = \frac{w A v^2}{g} \right] \quad \text{--- (2)}$$

Tensile stress on the ring $\sigma_t = \frac{P}{A} = \frac{w v^2}{g}$

Now substituting the values

$$413.85 \times 10^6 = \frac{77.12 \times 10^3 \times v^2}{9.81} \Rightarrow v = 229.45 \text{ m/s.}$$

$$\text{Now } v = \frac{\pi D N}{60} \Rightarrow N = \frac{60 \times 229.45}{\pi \times 1} = \boxed{4382.0 \text{ rpm}}$$

D'Alembert's Principle in Curvilinear Motion

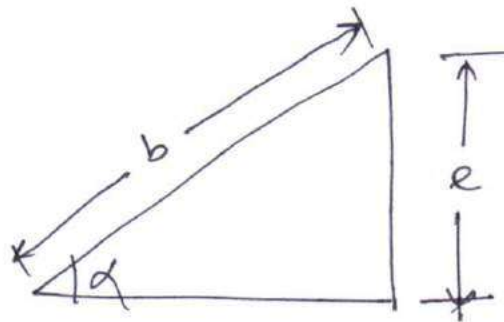
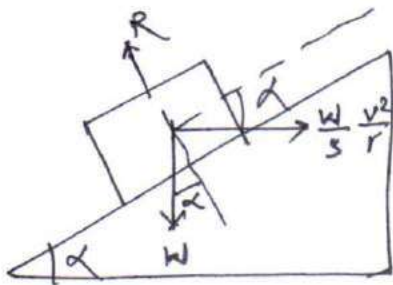
3

Equation of motion of a particle may be written as

$$\left. \begin{aligned} X - m\ddot{x} &= 0 \\ Y - m\ddot{y} &= 0 \end{aligned} \right\} \text{--- (1)}$$

Ex

Find the proper super elevation 'e' for 7.2 m highway curve of radius $r = 600\text{m}$ in order that a car travelling with a speed of 80 kmph will have no tendency to skid sideways.



$$b = 7.2\text{m} \quad r = 600\text{m} \quad v = 80\text{kmph} = 22.23\text{ m/s}$$

Resolving along the inclined plane

$$W \sin \alpha = \frac{W}{g} \cdot \frac{v^2}{r} \cos \alpha$$
$$\Rightarrow \tan \alpha = \frac{v^2}{rg}$$

from the geometry $\sin \alpha = \frac{e}{b}$, since α is very small

let $\sin \alpha \approx \tan \alpha$

$$\frac{v^2}{rg} = \frac{e}{b} \Rightarrow e = \frac{bv^2}{rg} = \frac{7.2 \times 22.23^2}{600 \times 9.81}$$
$$= 0.604\text{m (Ans)}$$

Q.2

A racing car travels around a circular track of 300m radius with a speed of 884 kmph. What angle α should the floor of the track make with horizontal in order to safeguard against skidding.

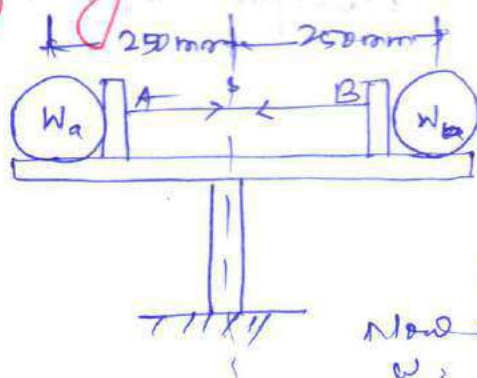
$$\text{Velocity } v = 884 \text{ kmph} \quad r = 300 \text{ m} \\ = 106.67 \text{ m/s.}$$

We have angle of banking $\tan \alpha = \frac{v^2}{rg}$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{106.67^2}{300 \times 9.81} \right) = \boxed{75.5^\circ} \quad (\text{Ans})$$

Q.4

Two balls of wt $W_A = 44.5 \text{ N}$ and $W_B = 66.75 \text{ N}$ are connected by an elastic string and supported on a turntable as shown. When the turntable is at rest, the tension in the string is $S = 222.5 \text{ N}$ and the balls exert this same force on each of the stops A and B. What forces will they exert on the stops when the turntable is rotating uniformly about the vertical axis CD at 60 rpm?



We have;

$$W_A = 44.5 \text{ N} \quad W_B = 66.75 \text{ N}$$

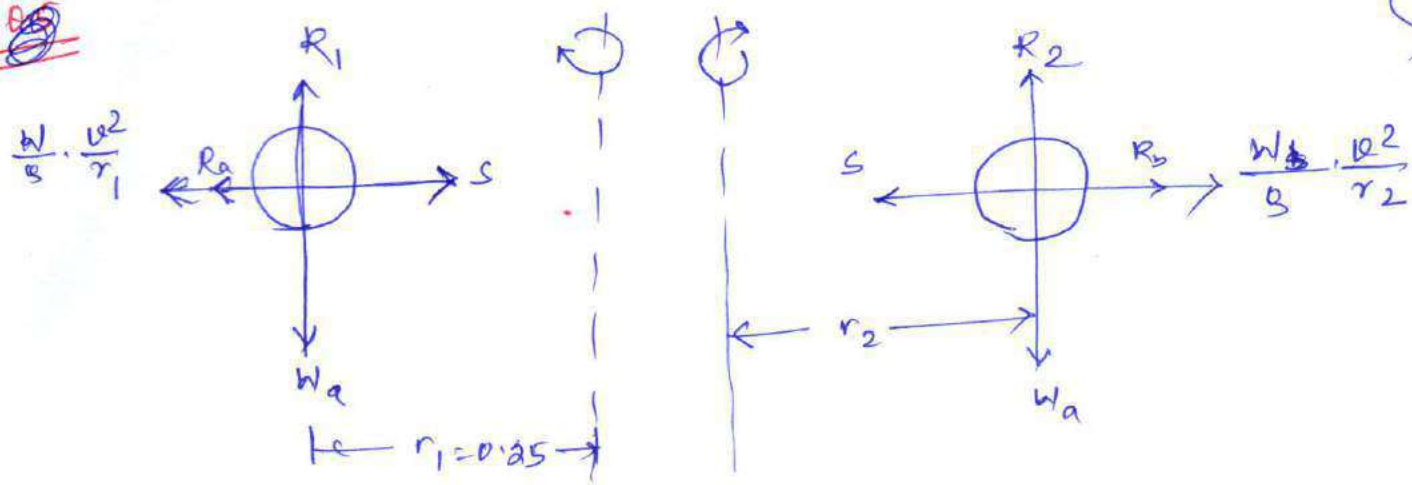
$$S = 222.5 \text{ N}$$

$$N = 60 \text{ rpm,}$$

$$\text{radius of rotation } r_1, r_2 = 0.25 \text{ m}$$

$$\text{Now angular velocity } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi \text{ rad/s}$$

~~85~~



~~considering~~ considering the left hand side ball

$$R_a + \frac{W_a}{g} \cdot r_1 \omega^2 = S$$

$$\Rightarrow R_a = 222.5 - \frac{11.5}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{177.72 \text{ N.}}$$

Considering the ball on right hand side

$$R_b + \frac{W_b}{g} \times r_2 \times \omega^2 = S$$

$$\Rightarrow R_b = 222.5 - \frac{66.75}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{155.39 \text{ N.}}$$

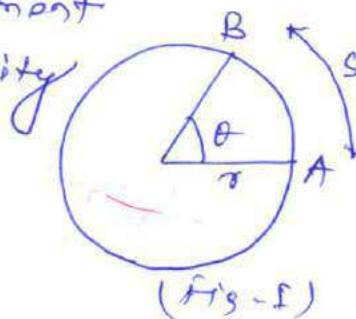
Rotation of Rigid Bodies:-

(1) (2)

Angular motion:-

The rate of change of angular displacement with time is called angular velocity and denoted by ω .

$$\boxed{\omega = \frac{d\theta}{dt}} \quad \text{--- (1)}$$



The rate of change of angular velocity with time is called angular acceleration and denoted by α .

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}} \quad \text{--- (2)}$$

Angular acceleration may also be represented as:

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}} \quad \text{--- (3)} \quad \left(\because \frac{d\theta}{dt} = \omega \right)$$

Relationship between angular motion and linear motion

from fig-1 $s = r\theta$

tangential velocity (linear) of the particle is

$$\boxed{v = \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}} \quad \text{--- (4)}$$

$$\text{linear acceleration} \quad \boxed{a_t = \frac{dv}{dt} = r \frac{d^2\theta}{dt^2}} \quad \text{--- (5)}$$

if $\frac{v^2}{r} = \text{radial acceleration}$

$$\text{Then } \boxed{a_n = \frac{v^2}{r} = r\omega^2} \quad \text{--- (6)} \quad (\text{where } a_n = \text{radial acceleration})$$

uniform angular velocity (ω)

$$\boxed{\omega = \frac{2\pi N}{60} \text{ rad/sec}} \quad \text{--- (7)}$$

(2)

resulting force on this element

$$\delta p = \delta m \times a \quad (a = \text{tangential acceleration})$$

$$\text{but } a = r \times \alpha \quad (\alpha = \text{angular acceleration})$$

$$\therefore \boxed{\delta p = \delta m r \alpha}$$

$$\text{Rotational moment } \delta M_t = \delta p \times r$$

$$= \delta m r^2 \alpha$$

$$M_t = \sum \delta M_t = \sum \delta m r^2 \alpha$$

$$= \alpha \sum \delta m r^2$$

$$= \alpha I$$

$$\Rightarrow \boxed{M_t = \alpha I} \quad (I = \text{mass moment of inertia})$$

Product of mass moment of inertia and angular velocity of rotating body is called angular momentum

$$\text{so } \boxed{\text{Angular momentum} = I \omega}$$

Kinetic energy of rotating bodies

$$\boxed{K.E = \frac{1}{2} I \omega^2}$$

Ex 2

A flywheel weighing 50 kN and having radius of gyration 1 m loses its speed from 400 rpm to 280 rpm in 2 min. Calculate

(a) retarding torque, (b) change in KE during the period, (c) change in angular momentum.

$$\text{we have } \omega_0 = 400 \text{ rpm} = \frac{2\pi \times 400}{60} = 41.89 \text{ rad/s}$$

$$\omega = 280 \text{ rpm} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

$$t = 2 \text{ min} = 120 \text{ sec}$$

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \alpha = \frac{\omega - \omega_0}{t} = \boxed{-1.047 \text{ rad/s}^2}$$

$$W \text{ of flywheel} = 50000 \text{ N}$$

$$\text{mass of " } = \frac{50000}{9.81} = 5096.84 \text{ kg}$$

$$\text{Radius of gyration } k = 1 \text{ m}$$

$$I = mk^2$$

$$= 5096.84 \times 1 = 5096.84$$

(a) Retarding torque

$$L\alpha = 5096.84 \times 0.1047$$

$$= \boxed{533.64 \text{ Nm}}$$

(b) change in KE

$$= \text{initial KE} - \text{final KE}$$

$$= \frac{1}{2} I \omega_0^2 - \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \times 5096.84 (41.89^2 - 29.32^2)$$

$$= \boxed{\cancel{2280442.9} \text{ Nm}} \quad \boxed{2281115.462 \text{ Nm}}$$

(c) change in angular momentum

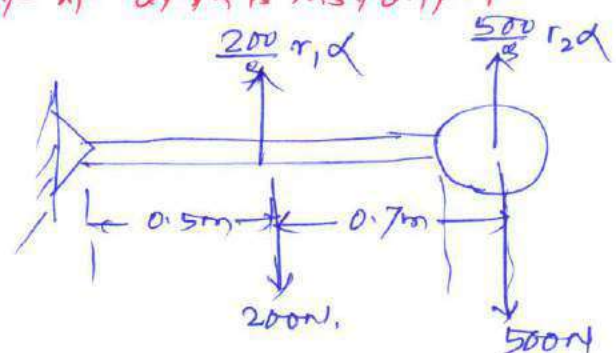
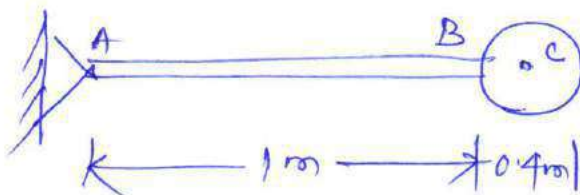
$$I\omega_0 - I\omega$$

$$= 5096.84 (41.89 - 29.32)$$

$$= \boxed{64067.298 \text{ Nm}}$$

Q3

A cylinder weighing 500N is welded to a 1m long uniform bar of 200N. Determine the acceleration with which the assembly will rotate about point A, if released from rest in horizontal position. Determine the reactions at A at this instant.



Let α = angular acceleration of the assembly (3)

I = mass moment of inertia of the assembly

$$I = I_G + Md^2 \quad (\text{transfer formula})$$

$$\text{mass } ML \text{ of bar about } A = \frac{1}{2} \times \frac{200}{9.81} \times 1^2 + \frac{200}{9.81} \times (0.5)^2$$
$$= 6.7968$$

mass ML of cylinder about A

$$= \frac{1}{2} \times \frac{500}{9.81} \times 0.2^2 + \frac{500}{9.81} \times 1.2^2$$
$$= 74.4$$

$$ML \text{ of the system} = 6.7968 + 74.4 = 81.2097$$

Rotational moment about A

$$M_A = 200 \times 0.5 + 500 \times 1.2 = 700 \text{ Nm}$$

$$M_A = I\alpha$$

$$\Rightarrow \alpha = \frac{700}{81.2097} = \boxed{8.6197} \text{ rad/sec}$$

Instantaneous acceleration of rod AB is vertical and $= r_1 \alpha = 0.5 \times 8.6197$
 $= 4.31 \text{ m/s}^2$

Similarly instantaneous acceleration of cylinder $= r_2 \alpha = 1.2 \times 8.6197$
 $= 10.34 \text{ m/s}^2$

Applying D'Alembert's dynamic equilibrium

$$R_A = 200 + 500 - \frac{200}{9.81} \times 4.31 - \frac{500}{9.81} \times 10.34$$

$$\Rightarrow \boxed{R_A = 84.93 \text{ N}} \quad (\text{Ans})$$

SAMPLE QUESTION PAPER

Th. 4(a)-ENGINEERING MECHANICS

Full Marks: 80

Time : 3 Hours

Answer any FIVE Questions including Q No. 1 & 2

Figures in the right hand margin indicates marks

1. Answer ALL the Questions:

2X10

- ☒ a) Define Force. State its Unit.
- ☒ b) State Lami's Theorem.
- ☒ c) State Varignon's Theorem.
- ☒ d) State Parallel Axis Theorem.
- ☒ e) Define Co-efficient of Friction.
- ☒ f) What is law of Machine ?
- g) Define Centre of Gravity and Centroid.
- h) Define Impulse and Momentum.
- ☒ i) State Newton's law of Collision.
- j) What is the C.G of a semi-circular lamina ?

2. Answer any SIX Questions:

5X6

- a) State and prove Parallelogram law of Force.
- b) Forces of 10N, 20N, 30N, 40N, 50N, 60N are acting along the side of a regular hexagon. Find out the resultant force.
- ☒ c) A body of weight 200N is placed on a horizontal plane. It is acted upon by a force 'P' applied at an angle 30° to the horizontal. The co-efficient of friction is 0.3. What force is required to just make the body slide?
- d) Calculate the C.G of a semi-circular lamina.
- e) Derive the velocity ratio of compound gear train.
- ☒ f) Define Mechanical Advantage, Velocity Ratio and Efficiency of a lifting machine and deduce the relationship between them.
- ☒ g) Determine the centroid of a T-slot section as shown in the Figure 1.

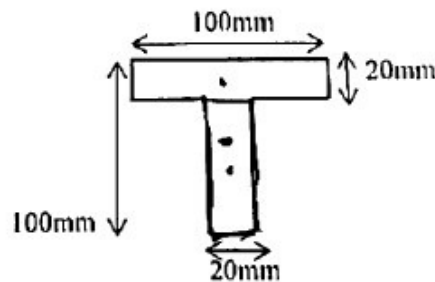


Figure 1

- 3 A smooth circular cylinder of radius 2m is lying in a triangular groove. One side of which makes 20° angle and the other 30° angle with the horizontal. Find the reaction at the surface of contact, if there is no friction and the cylinder weights 200N. 10

- 4 Find the moment of inertia about centroidal X-X axis and Y-Y axis of the section as shown in the Figure 2. 10

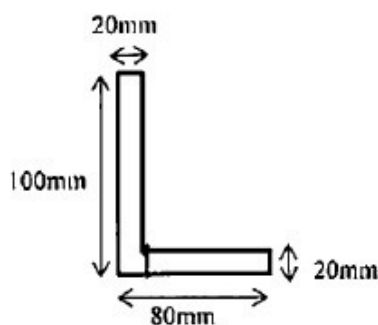


Figure 2

- 5 In a lifting machine, efforts of 10N and 16N lift loads of 48N and 78N respectively. If the velocity ratio of the machine is 6, find: 10

- Law of machine
- Maximum Mechanical Advantage
- Maximum Efficiency

- 6 Two bodies of mass 100kg and 40kg are connected by a thread and move along a horizontal plane under the action of a force of 450N applied to the first body of mass 100kg as shown in the Figure 3. The co-efficient of friction between sliding surface of the bodies and the plane is 0.3. Determine the acceleration of the two bodies and the tension in the thread using D'Alembert's principle. 10

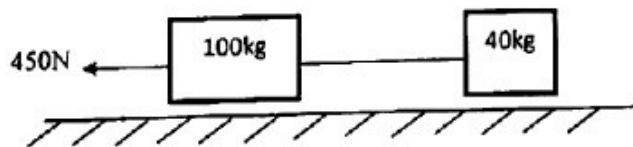


Figure 3

- 7 Write short note on any TWO: 10

- Coefficient of Restitution
- D'Alembert's principle
- Reversibility of machine & Self Locking machine

SAMPLE QUESTION PAPER

ENGINEERING MECHANICS

(Theory : 4)

Full Marks : 80

Time : 3 hours

Answer any five questions including Q. Nos. 1 & 2

Figures in the right-hand margin indicate marks

1. Answer the following :

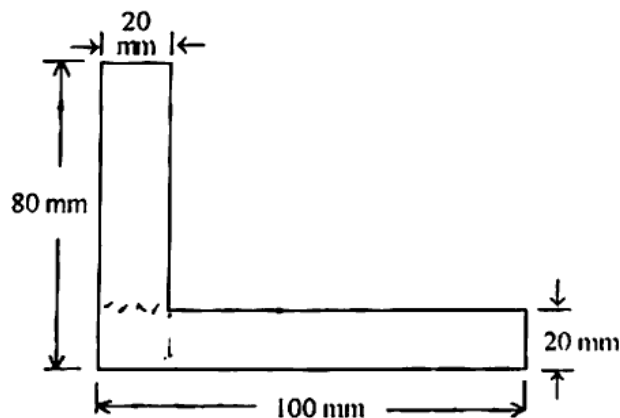
2 × 10

- (a) What is free body diagram ?
- (b) State polygon law of forces.
- (c) State the condition of equilibrium.
- (d) Define angle of friction.
- (e) What do you mean by centroid of a lamina ?
- (f) Write any two disadvantages of friction.
- (g) State the relation between M.A., VR and efficiency of a simple lifting machine.
- (h) State law of machine.
- (i) Define power and write down its S.I. unit.
- (j) State Newton's 2nd law of motion.

2. Answer any six questions :

5 × 6

- (a) A particle is acted on by three forces 2, $2\sqrt{2}$ and 1 kN. The first force is horizontal and towards the right, the second acts at 45° to the horizontal and inclined right upward and the third is vertical. Determine the resultant of the given forces.
- (b) Derive the condition for reversibility of a machine.
- (c) Derive velocity ratio of a compound gear train.
- (d) A body of weight 50 N is pulled along a rough horizontal plane by a force of 18 N acting at an angle of 14° with the horizontal. Find the coefficient of friction.
- (e) Find the position of centroid of a L section as shown in the figure below :



(Turn Over)

SAMPLE QUESTION PAPER

(2)

- (f) A constant force acting on a body of mass 20 kg changes its speed from 2.5 m/sec to 10 m/sec in 15 seconds. What is the magnitude of the force ?
- (g) State and explain d'Alembert's principle.
- (h) State and explain law of conservation of energy.
3. A spherical ball of weight 100 kgf rests on two smooth inclined planes as shown in the figure. Find the reaction of each inclined plane on the ball. 10

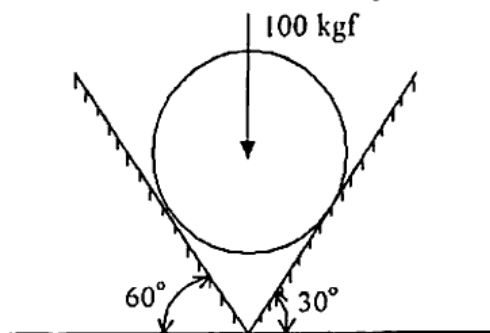
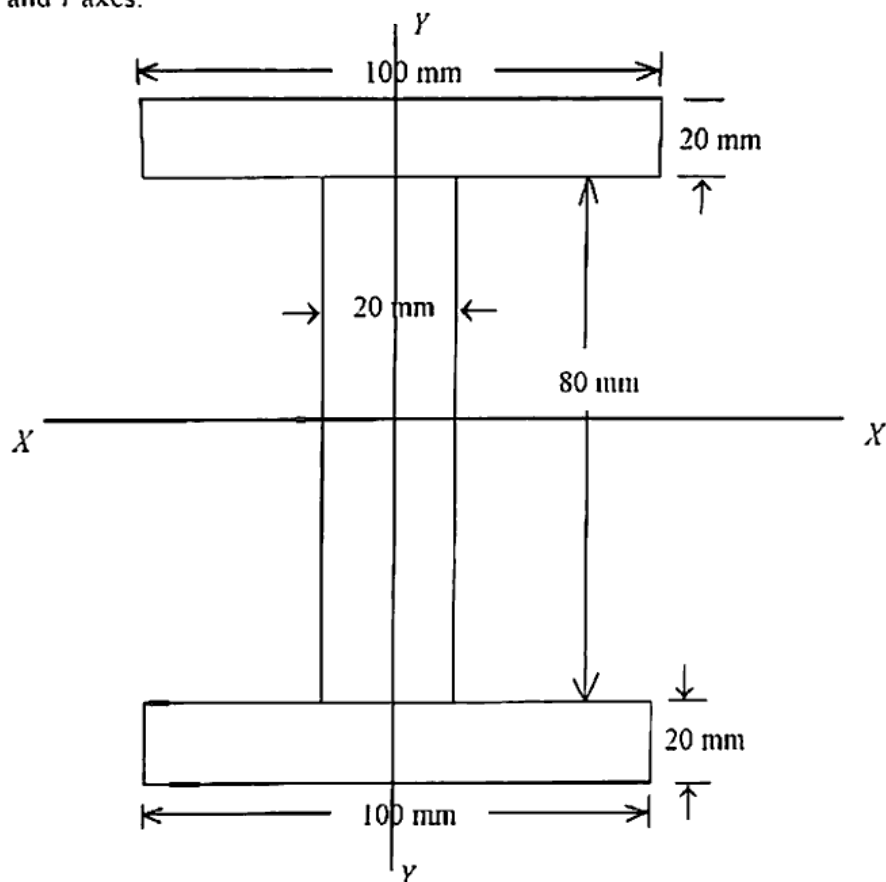


Figure 1

4. Find the moment of inertia of the I-section as shown in the figure 2 about centroidal X and Y axes. 10



SAMPLE QUESTION PAPER

(3)

5. A ladder 5 meter long rests on a horizontal ground and leans against a smooth vertical wall at an angle of 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750 N stands on a rung 1.5 m from the bottom of the ladder. Calculate the co-efficient of friction between the ladder and floor. 10
6. In a certain weight lifting machine, an effort of 15 N can lift a load of 300 N and an effort of 20 N can lift a load of 500 N. Find the law of the machine. Also find the effort required to lift a load of 880 N. 10
7. A ball is dropped from a height $h_0 = 1$ m on a smooth floor. Knowing that the height of the first bounce is $h_1 = 81$ cm determine 10
- (a) coefficient of restitution
- (b) expected height h_2 after the second bounce.
-

II-SEM COMMON/2019(W)/ (New)

Th.4 ENGINEERING MECHANICS

Full Marks: 80

Time : 3 Hours

Answer any FIVE Questions including Q No. 1 & 2

Figures in the right hand margin indicates marks

1	Answer ALL the Questions: (a) What are various characteristics of a force? (b) A rectangular block of weight W is resting over a smooth horizontal surface. Draw free body diagram of rectangular block. (c) State and explain the Triangle law of forces. (d) Two forces are acting at 120°. The greater force is 40N and the resultant is acting at 90° to the smaller force. Find the magnitude of the smaller force. (e) State and explain the Varignon's theorem (Principle of moments). (f) What are conditions of equilibrium for a co-planar non-concurrent force system? (g) A rectangular block of 10 kg rests on a horizontal plane. The coefficient of static and kinetic friction are 0.4 and 0.3 respectively. Calculate limiting value of frictional force at the point of impending motion. (h) State Parallel Axis Theorem in calculating moment of inertia. (i) State Mechanical Advantage and Velocity Ratio of a simple machine. (j) A screw jack has a thread of 10 mm pitch, length of handle is 400mm. Calculate effort applied P to lift a load of 2kN if efficiency of screw jack is 45%.	2×10
2.	Answer any SIX Questions: (a) Two forces are applied at point B of beam AB as shown in the Figure 1. Determine the magnitude and direction of their resultant force using the Parallelogram law. (b) Two forces are applied at point B of beam AB as shown in the Figure 1. Determine the magnitude and direction of their resultant force using the Triangle law of forces. (c) At a point, the following forces act: 20kN North East, 25kN North, 30kN North-West, 35kN South-West. Determine magnitude and direction of resultant force. (d) Two like parallel forces of 50 N and 200N acts at the ends of a rod 260mm long. Find magnitude of the resultant force and the point where it acts. (e) A hollow circular section has an external diameter 80mm and internal diameter 60mm. Find it's moment of inertia about an axis passing through its centroid and polar moment of inertia.	5×6

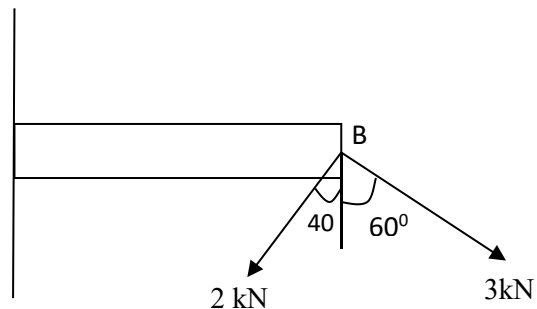


Figure 1

(Turn over)

1/2

- (f) In an lifting machine it is observed that a 120N effort raises a load of 1296N and 200N force is required for a load of 2600N. Determine:
- Law of machine
 - Effort required to raise a load of 3.5kN.
- (g) A truck of mass 15tonne travelling at a speed of 1.6m/sec impacts with a buffer spring, which gets compressed 1.25mm/kN. Applying the concept of conservation of energy, calculate maximum compression of spring.
- 3 Two cylinders of diameter 60mm and 30mm weighing 160N and 40N respectively are placed between two vertical walls as shown in the Figure 2. All the contact surfaces are smooth. Find reactions at 'A', 'B' & 'C'.
- 4 A semicircular area is removed from a trapezoidal lamina as shown in the Figure 3. Determine co-ordinate of centroid of the remaining area.
- 5 A block of weight 1600 N is in contact with a plane inclined at 30° to the horizontal. A force 'P' parallel to the plane and acting up the plane is applied to body. Coefficient of friction between contact surfaces is 0.2 Find
- Magnitude of 'P' just impend the block up along plane
 - 'P' required to prevent the block slide down the plane
- 6 Motion of a particle moving along a straight line is expressed as $x = t^3 - 4.5t^2 + 5$ where "x" is expressed in meter and "t" in second. Plot the following for a time period of 0 sec to 5 sec in an interval of 1 sec:
- Displacement-time curve
 - Velocity-time curve
 - Acceleration-time curve
- 7 Write short note on any TWO:
- Coefficient of Restitution
 - Worm & Worm Wheel
 - Couple with example and neat sketch

10

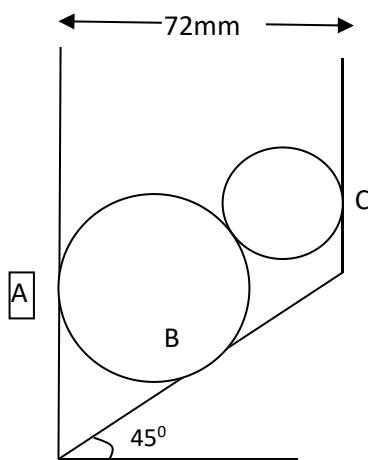


Figure 2

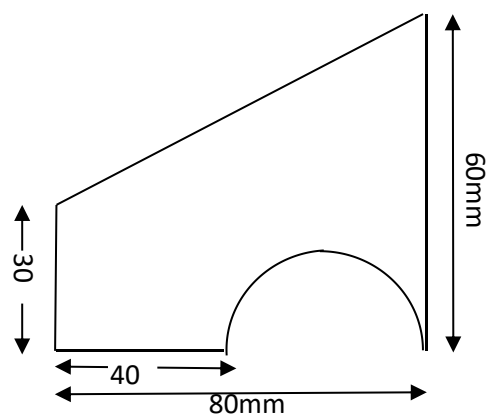


Figure 3

II-SEM /COMMON/ 2019(W)/ (Old)

BET-103

(ENGINEERING MECHANICS)

Full Marks: 80

Time: 3 Hours

Answer any FIVE Questions including Q No. 1 & 2

Q.1	Answer ALL the Questions: - What is Free Body Diagram? - Define any two systems of forces. - State Varignon's Principle of Moments. - What are the conditions for equilibrium of forces ? - Define Angle of Friction and Angle of Repose. - State Parallel Axis Theorem. - What do you mean by reversibility of a machine ? - State Gear train and its types. - What is impulse ? - Define Coefficient of restitution.	2X10
Q.2	Answer any SIX Questions: A beam of length 1 m is supported at end A and B as shown in the Figure 1. It is acted upon by an inclined force of 70 N at a distance of 65 cm from A. Find the reactions at A & B. Figure 1 Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, direction and position of the resultant force. A load of 500N is lying on an inclined plane, whose inclination with the horizontal is 30° as shown in the Figure 2. If the coefficient of friction between the load and the plane is 0.4, find the minimum and maximum horizontal force, which will keep the load in equilibrium. Figure 2 State and Prove the Lami's Theorem.	5X6

e)	Find the centroid of an unequal angle section 120mm x 90mm x 30mm.	
f)	What is Worm and Worm wheel? Derive the expression for its velocity ratio.	
g)	State and Explain D'Alembert's Principle.	
Q.3	A uniform ladder of 6 m length rests against a vertical wall with which, it makes an angle of 45° . The coefficient of friction between the ladder and the wall is 0.3 and that between ladder and the floor is 0.4. If a man, whose weight is one-half of that of the ladder ascends it, how high will it be when the ladder slips ?	10
Q.4	Find the moment of inertia of a T-section with flange as 120mm $\times$ 40mm and web as 120mm $\times$ 40mm about X-X axes through the centre of gravity of the section.	10
Q.5	What is a lifting machine? Define Mechanical Advantage, Velocity Ratio and Efficiency of a Simple Lifting Machine. In a lifting machine, whose velocity ratio is 60, an effort of 120N is required to lift a load of 6kN. Is the machine reversible ? If so, what effort should be applied, so that the machine is at the point of reversing ?	10
Q.6	State Newton's Law of Collision of two bodies. Two balls of masses 3kg and 4kg are moving with velocities 3m/sec and 4m/sec respectively towards each other. If the coefficient of restitution is 0.5, find the velocity of the two balls after impact.	10
Q.7	Write Short Notes on: (2.5X4) a) Resultant Force with example. b) Couple with neat sketch and example. c) Conservation of Energy and Linear Momentum. d) Different types of equilibrium of forces with neat sketch.	10

(4)

6. Find the C.G. of an angle section (L) of $100 \text{ mm} \times 60 \text{ mm} \times 20 \text{ mm}$. 10
7. A stone is dropped from the top of a tower 120 m high. Another stone is projected upward at the same time from the foot of the tower and meets the first stone at a height of 50 m. Find the velocity with which the second stone is projected upwards. 10
-

II/SEM/COMMON/2018 (W)/
NEW

ENGINEERING MECHANICS

(BET - 103)

Full Marks : 80

Time : 3 hours

Answer any five questions including Q.Nos. 1 & 2.

The figures in the right-hand margin indicate marks.

1. Answer the following : 2 × 10
- (a) Define 'concurrent and coplanar' force system.
 - (b) State 'Triangle Law' of forces.
 - (c) Define friction.
 - (d) What do you mean by centre of gravity of a body?
 - (e) Define M.A. and V.R. of a machine.
 - (f) Define momentum.
 - (g) Define 'Recoil of Gun'.

(2)

- (h) State Newton's First Law of Motion.
- (i) State law of conservation of Momentum.
- (j) Define coefficient of restitution.

2. Answer any six questions : 5 × 6

- (a) State Lami's theorem and prove it.
- (b) State and prove Varignon's theorem.
- (c) State the laws of Friction.
- (d) State and prove 'perpendicular axis theorem'.
- (e) What do you understand by the term 'Reversibility' of a machine ? Explain the difference between a reversible and a self-locking machine.
- (f) State and explain Newton's law of motion.
- (g) State law of conservation of Momentum, show how it can be deduced from third law of motion.

(3)

- (h) A ball of mass 8 kg moving with a velocity of 10 m/s, impinges directly on another of mass 24 kg moving at a speed of 2 m/sec in the opposite direction. If $e = \frac{1}{2}$, find the velocities of the balls after impact.

- 3. The resultant of two concurrent forces is perpendicular to the smaller force and angle between the forces is 120° . If the bigger force is 60 N, find the smaller force. 10
- 4. $ABCDEF$ is a regular hexagon of side $P, 2P, 3P, 4P, 5P, 6P$ act along BA, BC, DC, ED, EF and AF respectively. Show that the six forces are equivalent to a couple and find the moment of this couple. 10
- 5. A body of weight 150 N, lies on a horizontal plane for which $\mu = 0.70$. Determine : 10
 - (i) Normal reaction
 - (ii) Limiting force of friction
 - (iii) Horizontal force required to move it
 - (iv) Angle of friction.

PREVIOUS YEAR QUESTIONS WITH ANSWERS

1ST / SEM / COMMON / 2017 (W) (NEW)
ENGINEERING MECHANICS

Code : BET - 103

Time : 3 Hours

Full Marks : 80

Answer any Five questions including Q. Nos. 1 & 2

Figures in the right-hand margin indicate marks.

1. Answer the following :

[2 × 10]

- Define force.
- State Varignon's theorem.
- Define couple.
- Define coefficient of friction.
- What is the distance of centroid of a semicircular area from the base?
- What is the condition of reversibility of a lifting machine?
- What do you mean by concurrent forces?
- State Newton's first law of motion.
- State law of conservation of momentum.
- Define coefficient of restitution.

2. Answer any six :

[5 × 6]

- The resultant of two forces P and 15N is 50N inclined at 60° to the 15N force, Find the magnitude and direction of P.
- A system of parallel forces acting on a lever is shown in the figure-1. Determine the magnitude, direction and position of the resultant.

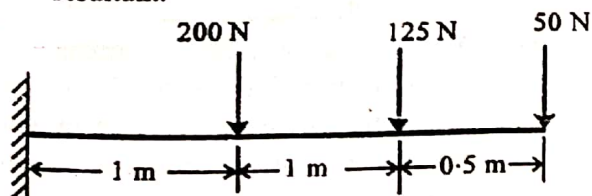


Figure -1

- Locate the centroid of the shaded area shown in Figure-2, created by cutting a Semicircle of dia. 12cm from a quarter circle of radius 12 cm.

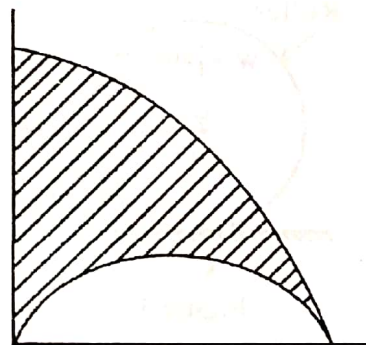


Figure-2

- A body resting on a rough horizontal plane required a pull of 18N inclined at 30° to the plane just to move it. It was found that a push of 22 N inclined at 30° to the plane just moved the body. Determine the weight of the body and coefficient of friction.
- What is gear train? Derive the velocity ratio of a compound gear train.
- In a differential wheel and axle, the dia of the wheel is 2m and the dia of axles are 30cm and 20cm respectively. Find the V.R. If a load of 800N be lifted at an efficiency of 70%, what effort will be required to lift this load?
- A body of mass 200 kg, is found to move with a velocity of 20 m/s when a force of 100 N acts on it for 90s. Determine the velocity of the body :
 - When the force acts in the direction of motion.
 - When the force acts in the opposite direction on motion.
- A ball of mass 8 kg moving with a velocity of 10 m/s, impinges directly on another of mass 24 kg moving at a speed of 2 m/s in the opposite direction. If $e = \frac{1}{2}$, find the velocity of the balls after impact.

- A roller 400 mm in diameter and weighing 400 N is to be subjected to pivoting motion about an edge

A, by applying a force P as shown in the Figure 3. Determine the magnitude of horizontal force P passing through the centre of the roller and corresponding reaction R_A exerted by the edge A. [10]

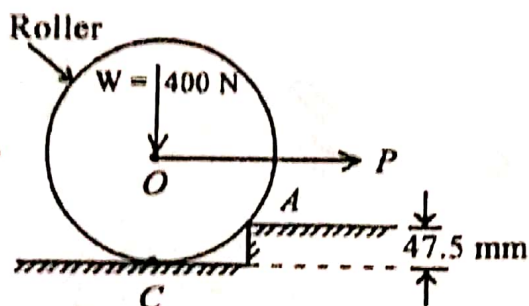


Figure 3

4. A body of weight 500 N is lying on a plane inclined at an angle of 30° with the horizontal. It is supported by an effort P parallel to the plane as shown in the figure 4. Determine the minimum and maximum values of P for which equilibrium can exist, if the coefficient of friction is 0.35. [10]

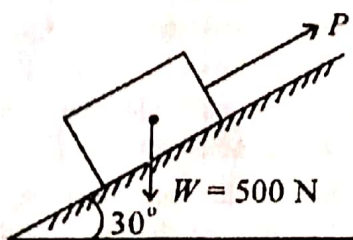


Figure-4

5. Find the moment of inertia about the centroidal axes XX and YY of the section shown in the Figure 5. [10]

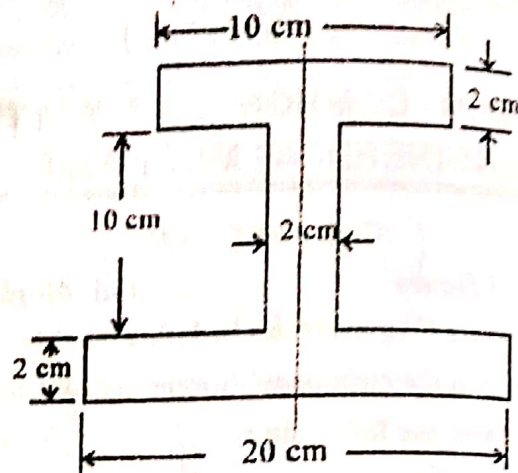


Figure-5

6. In a weight lifting machine, an effort of 40 N can lift a load of 1000 N and an effort of 55 N can lift a load of 1500 N. Find the law of the machine. Also find maximum mechanical advantage and maximum efficiency of the machine. Take velocity ratio of the machine as 48. [10]
7. A pile driver of mass of 250 kg falls 3.0 m on a pile of mass 1200 kg. The pile is driven 0.2 m into the ground. Find the common velocity of the pile and the driver after the impact and average resistance of the ground to the penetration of the pile. [10]

ANSWERS TO - 2017 (W)

1. (a) Force is that which produces change in the state of rest or of uniform motion in a straight line of a body.
- (b) The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.
- (c) Two equal unlike parallel forces whose lines of action are not the same is known as couple.
- (d) It is the ratio of limiting friction to the normal reaction between the two bodies.

$$\mu = \frac{F}{R} = \tan \theta$$

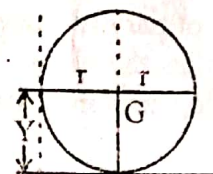
where $\mu \rightarrow$ coeff. of friction

$F \rightarrow$ frictional force.

$R \rightarrow$ Normal reaction.

$\phi \rightarrow$ angle of friction.

(e) $\bar{Y} = \frac{4r}{3\pi}$



- (f) A reversible machine is a machine that can perform reversible cycles. These cycles are cycles that happens through infinitesimal changes or a infinity number of quasi static steps making the process adiabatic and reversible.

The condition for a machine to be reversible is that the efficiency should be more than 50%.

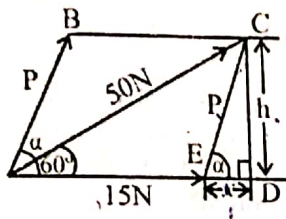
- (g) All the forces pass through a common point are known as concurrent forces.

- (h) Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external force.
- (i) For a collision occurring between object 1 and object 2 in isolated system the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision.
- (j) The coefficient of restitution is a number which indicates how much kinetic energy remains after collision of two objects.

2.(a) $\theta = 60^\circ$

One force = P

another force = 15N



Resultant = 50N

$$\text{In } \Delta ACD \cos 60^\circ = \frac{15 + x}{50}$$

$$\Rightarrow 15 + x = 50 \times \frac{1}{2}$$

$$\Rightarrow x = 25 - 15 = 10 \text{ N}$$

$$AD = 15 \text{ N} + 10 \text{ N} = 25 \text{ N}$$

$$\sin 60^\circ = \frac{h}{50}$$

$$h = 50 \times \sin 60^\circ$$

$$= 50 \times \frac{\sqrt{3}}{2} = 43.3 \text{ N}$$

$$\text{In } \Delta CED, \tan \alpha = \frac{h}{x}$$

$$= \frac{43.3}{10}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{43.3}{10} \right)$$

$$\Rightarrow \alpha = 76.99^\circ$$

According to Law of parallelo gram method

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$\Rightarrow 50 = \sqrt{P^2 + 15^2 + 2 \times P \times 15 \cos 76.99^\circ}$$

$$\Rightarrow (50)^2 = P^2 + 225 + 6.753 P$$

$$\Rightarrow P^2 + 6.753P - 2275 = 0$$

$$\Rightarrow P = \frac{-6.753 \pm \sqrt{6.753^2 - 4 \times 1 \times (-2275)}}{2 \times 1}$$

$$= \frac{-6.753 \pm \sqrt{45.6 - 9100}}{2}$$

$$= \frac{-6.753 \pm \sqrt{9145.6}}{2}$$

$$= \frac{-6.753 \pm 95.632}{2}$$

$$= \frac{95.632 - 6.753}{2}$$

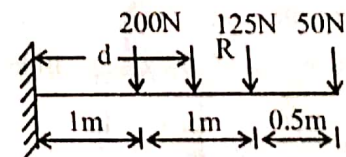
$$= \frac{88.879}{2} = 44.439 \text{ N. (Ans)}$$

(b) A Resultant (R) = ΣF

$$R = 200 \text{ N} + 125 \text{ N} + 50 \text{ N} = 375 \text{ N.}$$

Taking moment,

$$R \times d = 50 \times 2.5 + 125 \times 2 + 200 \times 1$$



$$= 125 + 250 + 200$$

$$\Rightarrow 375d = 575$$

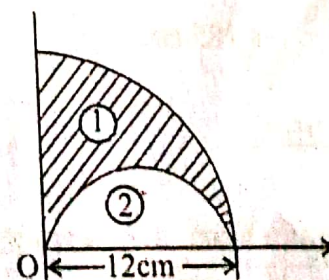
$$\Rightarrow d = \frac{575}{375}$$

$$\Rightarrow d = 1.53 \text{ m.}$$

So position of resultant is 1.53m from left.

(c) Area of the quadrant is taken as +ve and the area of the semicircle is taken as -ve.

a_1 = area of the quadrant



$$a_1 = \frac{1}{4} \pi r^2 = \frac{1}{4} \times \pi \times (12)^2$$

$$= 113.142 \text{ cm}^2 (+)$$

a_2 = area of the semicircle.

$$a_2 = \frac{1}{2} \pi r_1^2 = \frac{1}{2} \times \pi \times (6)^2$$

$$= 56.571 \text{ cm}^2 (-)$$

x_1 = distance of C.G. of the quadrant from OY axis.

$$x_1 = \frac{4r}{3\pi} = \frac{4 \times 12}{3\pi}$$

$$x_1 = 5.093 \text{ cm.}$$

x_2 = distance of the C.G. of the semicircle from OX axis.

$$x_2 = \frac{12}{2} = 6 \text{ cm}$$

y_1 = distance of the C.G. of the quadrant from OY axis.

$$y_1 = \frac{4r}{3\pi} = \frac{4 \times 12}{3\pi}$$

$$y_1 = 5.093 \text{ cm.}$$

y_2 = distance of C.G. of the semicircle from OX axis

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 6}{3\pi}$$

$$y_2 = 2.546 \text{ cm}$$

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{(113.142 \times 5.093) - (56.571 \times 6)}{113.142 - 56.571}$$

$$= \frac{576.232 - 339.426}{56.571}$$

$$= \frac{236.806}{56.571} = 4.185 \text{ cm.}$$

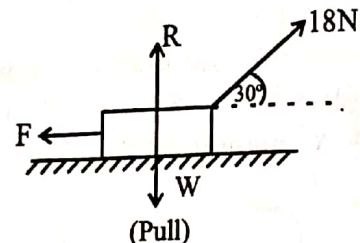
$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$\bar{y} = \frac{(113.142 \times 5.093) - (56.571 \times 2.546)}{113.142 - 56.571}$$

$$= \frac{576.232 - 144.029}{56.571}$$

$$= \frac{432.206}{56.571} = 7.64 \text{ cm.}$$

(d) For Pull :



Resolving the forces horizontally,

$$F = 18 \cos 30^\circ$$

$$= 18 \times 0.866$$

$$F = 15.58 \text{ N}$$

Resolving the forces vertically,

$$R = W - 18 \sin 30^\circ$$

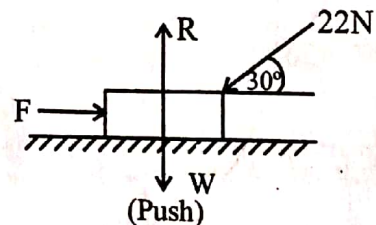
$$= W - 18 \times 0.5$$

$$\Rightarrow R = (W - 9)$$

$$F = \mu R$$

$$\Rightarrow 15.58 = \mu(W - 9) \quad \dots(1)$$

For Push case :



Resolving the forces horizontal

$$F = 22 \cos 30^\circ$$

$$= 19.05 \text{ N}$$

Resolving the forces vertically

$$R = W + 22 \sin 30^\circ$$

$$= W + 22 \times 0.5$$

$$\Rightarrow R = (W + 11)$$

$$F = \mu R$$

$$\Rightarrow 19.05 = \mu(W + 11) \quad \dots(2)$$

Dividing equation (1) by equation (2)

$$\frac{15.6}{19.05} = \frac{\mu(W - 9)}{\mu(W + 11)}$$

$$\Rightarrow 15.6(W + 11) = 19.05(W - 9)$$

$$\Rightarrow 15.6W + 171.6 = 19.05W - 171.45$$

$$\Rightarrow 19.05W - 15.6W = 171.6 + 171.45$$

$$\Rightarrow W(19.05 - 15.6) = 343.05$$

$$\Rightarrow W = \frac{343.05}{(19.05 - 15.6)}$$

$$\Rightarrow W = 99.43 \text{ N}$$

Putting the value of W in equation (1),

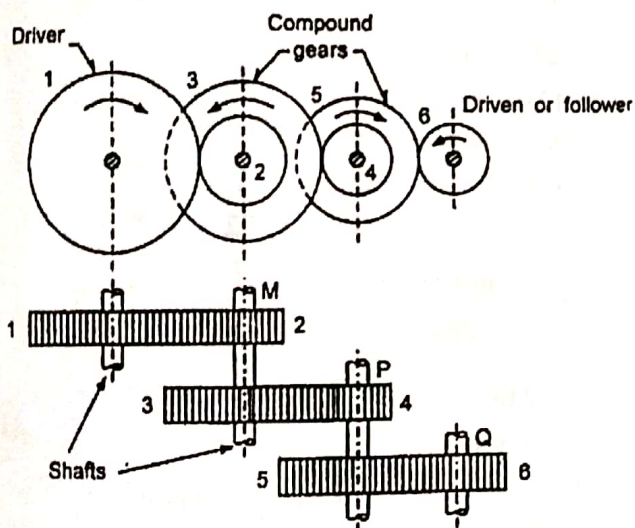
$$15.58 = \mu(99.43 - 9)$$

$$\Rightarrow \mu = \frac{15.58}{(99.43 - 9)}$$

$$\Rightarrow \mu = 0.172 \text{ (Ans)}$$

(e) **Great train** : The combination of gear wheels by means of which motion is transmitted from one shaft to the another shaft is called a great train.

Velocity ratio of compound gear train :



Let N_1 = Speed of driving gear (1)

T_1 = Number of teeth on driving gear (2)

N_2, N_3, N_4, N_5, N_6

→ Speed of respective gears in rpm.

T_2, T_3, T_4, T_5, T_6

→ Number of teeth on respective gears.

Since gear (1) meshes with gear (2)

$$\text{V.R.} = \frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \dots(1)$$

Similarly for gears (3) and (4)

$$\text{V.R.} = \frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \dots(2)$$

For gears (5) and

$$\text{V.R.} = \frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \dots(3)$$

The velocity ratio of the compound gear train is obtained by multiplying equations (1), (2) and (3)

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_1}{T_2} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$N_2 = N_3$$

(∵ gears (2) and (3) are mounted on same shaft)

$$N_4 = N_5$$

(∵ gears (5) and (4) are mounted on same shaft)

$$\text{So } \frac{N_1}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\text{Velocity ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the first driven}}$$

$$= \frac{\text{Product of the no. of the on driven}}{\text{Product of the no. of teeth on driver}}$$

$$(f) D = 2m \quad d_1 = 30 \text{ cm} = 0.3m$$

$$d_2 = 20 \text{ cm} = 0.2m.$$

$$W = 800 \text{ N, efficiency} = 70\%$$

$$\text{V.R.} = \frac{2D}{d_1 - d_2}$$

$$= \frac{2 \times 2}{0.3 - 0.2}$$

$$\text{V.R.} = \frac{4}{0.1} = 40$$

$$\text{efficiency} = \frac{M.A}{V.R}$$

$$\Rightarrow 0.7 = \frac{800/P}{40}$$

$$\Rightarrow \frac{800}{P} = 28$$

$$\Rightarrow P = \frac{800}{28}$$

$$\Rightarrow P = 28.57 \text{ N (P} \rightarrow \text{effort)}$$

(g) $m = 200 \text{ kg}$ $u = 20 \text{ m/s}$
 $F = 100 \text{ N}$ $t = 90 \text{ s}$

(i) acceleration (f) = $\frac{F}{M}$

$$f = \frac{100}{200} = 0.5 \text{ m/s}^2$$

After 90 second $V = u + ft$

$$= 20 + 0.5 \times 90$$

$$= 20 + 45 = 65 \text{ m/s.}$$

(ii) $V = u + ft$

$$= 20 - 0.5 \times 90$$

$$= 20 - 45 = -25 \text{ m/s (Ans.)}$$

(h) $m_1 = 8 \text{ kg}$ $m_2 = 24 \text{ kg}$
 $u_1 = 10 \text{ m/s}$ $u_2 = 2 \text{ m/s}$

$$e = \frac{1}{2} = 0.5$$

momentum before impact = momentum after impact

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow (8 \times 10) + (24 \times 2) = 8v_1 + 24v_2$$

$$\Rightarrow 8v_1 + 24v_2 = 128 \quad \dots(1)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow 0.5 = \frac{v_2 - v_1}{10 - 2}$$

$$\Rightarrow v_2 - v_1 = 4 \quad \dots(2)$$

equation (1) $\rightarrow 8v_1 + 24v_2 = 128$

equation(2) $8 = 8v_2 - 8v_1 = 32$

$$\text{Adding} \quad 32v_2 = 160$$

$$\Rightarrow v_2 = \frac{160}{32} = 5 \text{ m/s}$$

Putting the value of v_2 in equation (1)

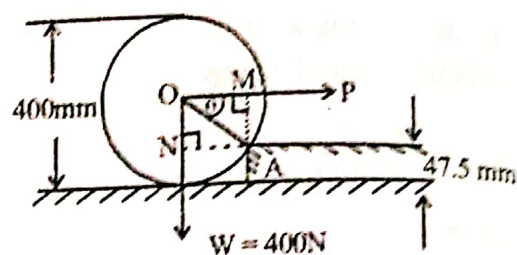
$$8v_1 + 24 \times 5 = 128$$

$$\Rightarrow 8v_1 + 120 = 128$$

$$\Rightarrow 8v_1 = 128 - 120$$

$$\Rightarrow v_1 = \frac{8}{8} = 1 \text{ m/s. (Ans)}$$

3.



$$OA = 200 \text{ mm} = 20 \text{ cm.}$$

$$AM = OA \sin \theta$$

$$= 20 \sin \theta$$

$$OE = 20 \text{ c.m.}$$

$$EN = 47.5 \text{ mm}$$

$$= 4.75 \text{ cm.}$$

$$ON = 20 - 4.75$$

$$= 15.75 \text{ cm.}$$

$$AN = \sqrt{20^2 - 15.75^2}$$

$$= \sqrt{400 - 248.062}$$

$$= \sqrt{151.938}$$

$$= 12.32 \text{ cm.}$$

Taking moments about 'A'.

$$P \times AM = 400 \times AN$$

$$\Rightarrow P \times 20 \sin \theta = 400 \times 12.32$$

$$\Rightarrow P = \frac{400 \times 12.32}{20 \sin \theta}$$

$$P = \frac{246.4}{\sin \theta}$$

'P' will be Least when $\sin \theta$ greatest.

When $\sin \theta = 1$

$$\theta = 90^\circ$$

Putting the value of θ in equation (1)

$$\text{Least value of 'P'} = \frac{246.4}{\sin 90^\circ}$$

$$P = 246.4 \text{ N}$$

$$ON = 15.75 \text{ cm.}$$

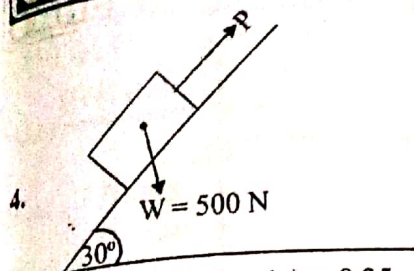
$$OA = 20 \text{ cm.}$$

$$\cos \angle AON = \frac{ON}{OA} = \frac{15.75}{20} = 0.7875$$

Resolving along OA,

$$R = 400 \times 0.7875$$

$$= 315 \text{ N (Ans)}$$



Coeff. of friction (μ) = 0.35

$$\tan \phi = 0.35$$

$$\phi = \tan^{-1}(0.35)$$

$$\phi = 19.29^\circ$$

$$\alpha = 30^\circ \quad W = 500 \text{ N}$$

$$\theta = 0^\circ$$

$$P_{\max} = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

$$P_{\max} = 500 \times \frac{\sin(30^\circ + 19.29^\circ)}{\cos 19.29^\circ}$$

$$= 500 \times \frac{\sin 49.29^\circ}{\cos 19.29^\circ}$$

$$= 500 \times \frac{0.758}{0.943}$$

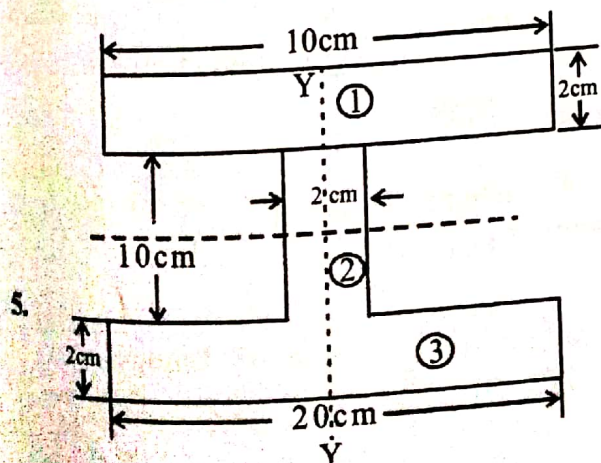
$$= \frac{379}{0.943} = 401.9 \text{ N}$$

$$P_{\min} = W \times \frac{\sin(\alpha - \phi)}{\cos \phi}$$

$$= 500 \times \frac{\sin(30^\circ - 19.29^\circ)}{\cos 19.29^\circ}$$

$$= 500 \times \frac{\sin 10.71^\circ}{\cos 19.29^\circ}$$

$$= 500 \times \frac{0.185}{0.943} = 98.09 \text{ N. (Ans)}$$



$$a_1 = 10 \times 2 = 20 \text{ cm}^2$$

$$a_2 = 10 \times 2 = 20 \text{ cm}^2$$

$$a_3 = 20 \times 2 = 40 \text{ cm}^2$$

$$y_1 = 12 + \frac{2}{2} = 13 \text{ cm}$$

$$y_2 = 2 + \frac{10}{2} = 7 \text{ cm}$$

$$y_3 = \frac{2}{2} = 1 \text{ cm}$$

$$a_1 y_1 = 20 \times 13 = 260 \text{ cm}^2$$

$$a_2 y_2 = 20 \times 7 = 140 \text{ cm}^2$$

$$a_3 y_3 = 40 \times 1 = 40 \text{ cm}^2$$

$$\Sigma a y = a_1 y_1 + a_2 y_2 + a_3 y_3$$

$$= 260 + 140 + 40$$

$$= 440 \text{ cm}^3$$

$$\Sigma a = a_1 + a_2 + a_3$$

$$= 20 + 20 + 40 = 80 \text{ cm}^2$$

$$\bar{Y} = \frac{\Sigma a y}{\Sigma a}$$

$$= \frac{440}{80} = 5.5 \text{ cm.}$$

$$I_{xx1} = \frac{10 \times 2^3}{12} + (10 \times 2) \times (13 - 5.5)^2$$

$$= 6.66 + 20 \times (7.5)^2$$

$$= 6.66 + 1125 = 1131.66 \text{ cm}^4.$$

$$I_{xx2} = \frac{2 \times 10^3}{12} + (2 \times 10) \times (7 - 5.5)^2$$

$$= 166.66 + 20 \times (1.5)^2$$

$$= 166.66 + 45$$

$$= 211.66 \text{ cm}^4.$$

$$I_{xx3} = \frac{20 \times 2^3}{12} + (20 \times 2) \times (5.5 - 1)^2$$

$$= 13.33 + 40 \times (4.5)^2$$

$$= 13.33 + 810$$

$$= 823.33 \text{ cm}^4$$

$$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$$

$$= 1131.66 + 211.66 + 823.33$$

$$= 2166.65 \text{ cm}^4 \text{ (Ans)}$$

6. $P = 40 \text{ N}$ $W = 1000 \text{ N}$
 $P = 55 \text{ N}$ $W = 1500 \text{ N}$

Law of the machine

$$P = mW + C$$

$$40 = m \cdot 1000 + C \quad \dots (1)$$

$$55 = m \cdot 1500 + C \quad \dots (2)$$

$$\text{Subtracting } -15 = -500 \text{ m}$$

$$\Rightarrow 500 \text{ m} = 15$$

$$\Rightarrow m = \frac{15}{500}$$

$$\Rightarrow m = 0.03$$

Putting the value of m in equation (1)

$$1000 \times 0.03 + c = 40$$

$$\Rightarrow 30 + c = 40$$

$$\Rightarrow c = 40 - 30$$

$$\Rightarrow c = 10$$

$$\text{Law of the machine } P = 0.03 W + 10$$

$$\text{V.R.} = 48$$

$$\text{Maximum M.A.} = \frac{1}{m}$$

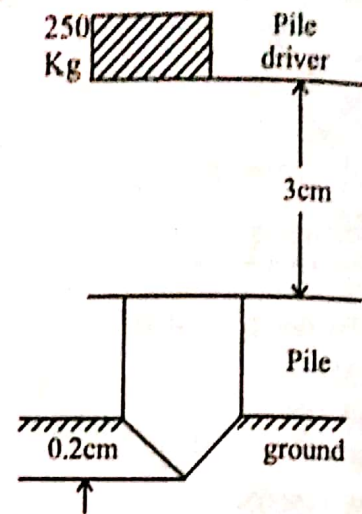
$$= \frac{1}{0.03} = 33.33$$

$$\text{Maximum efficiency} = \frac{1}{m \times \text{V.R.}}$$

$$= \frac{1}{0.03 \times 48} = 0.6944$$

$$= 69.44\%$$

7. Mass of pile driver (M) = 250 kg
 $h = 3 \text{ m}$.



Mass of the pile (m) = 1200 kg

$x = 0.2 \text{ m}$ momentum of driver pile after impact
 = momentum of driver and pile before impact

$$(M + m)v = M \times \sqrt{2gh}$$

Velocity of driver after falling $3 \text{ m} = \sqrt{2gh}$

$$= \sqrt{2 \times 9.81 \times 3}$$

$$u = 7.672 \text{ m/s}$$

$$(M + m)v = M \sqrt{2gh}$$

$$\Rightarrow (250 + 1200)v = 250 \times 7.672$$

$$\Rightarrow V = \frac{250 \times 7.672}{(250 + 1200)}$$

$$= \frac{1918}{1450}$$

$$\Rightarrow V = 1.322 \text{ m/s}$$

Average resistance of the ground to the penetration of the pile

$$R = (M + m) + \frac{m^2 h}{x(M + m)}$$

$$= (250 + 1200) + \frac{250^2 \times 3}{0.2(250 + 1200)}$$

$$= 1450 + \frac{187500}{290}$$

$$= 1450 + 646.55$$

$$= 2096.55 \text{ kg (Ans)}$$

1ST / SEM / COMMON / 2016 (W) (NEW)
ENGINEERING MECHANICS

Code : BET - 103

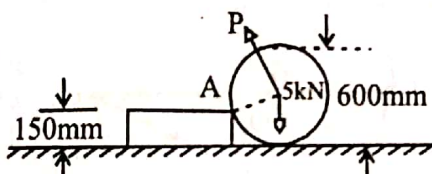
Time : 3 Hours

Full Marks : 70

Answer any five questions.

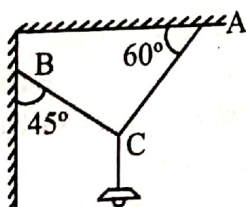
Figures in the right-hand margin indicate marks.

- Define force and state its unit in S.I. System [2]
 - Explain principles of transmissibility and superposition. [5]
 - Two forces act at angle of 120° . The bigger force is 40N and the resultant is perpendicular to the smaller one. Find the smaller force. [7]
- Define Moment of a Force and state its unit in S.I. system [2]
 - State and explain Varignon's theorem [5]
 - A uniform wheel 600 mm diameter weighing 5 kN rests against a right rectangular block of 150 mm height as shown below. [7]



Find the least pull force through the centre of the wheel required just to turn the wheel over the corner A of block. (Assume surfaces to be smooth).

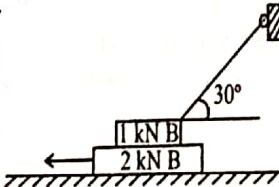
- State the law of Parallelogram of forces [2]
 - State and Proof Lami's theorem [5]
 - An electric light fixture weighs 15 N hangs from a point 'C' by two strings AC and BC. The string is inclined at 60° to the horizontal and BC is at 45° to the horizontal as shown in the figure below.



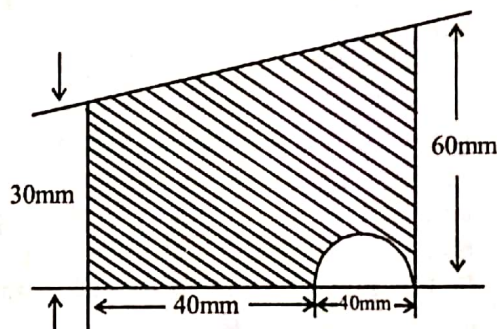
Determine the forces on string AC and BC using Lami's theorem. [7]

- Define Friction and coefficient of Friction [2]
 - Explain laws of friction [5]
 - Two blocks 'A' and 'B' of weight 1kN and 2kN are in equilibrium position as shown below. [7]

If the coefficient of friction μ between two blocks and block 'B' and Floor is 0.3, Find force 'P' required to move the block.



- Define centroid of a plane Figure [2]
 - Find the expression for centroid of a semi circle of radius (r) from 1st principle. [5]
 - Find the centroid of the Figure shown notched as below. [7]



- Define a simple lifting Machine [2]
 - Find velocity ratio of a compound gear train [5]
 - In a lifting machine, an effort of 40N raised a load of 1kN. The efficiency of the machine is 0.5 what is the velocity ratio? For an effort of 74N the load lifted is 2kN what will be the efficiency? What will be the effort required to raise a load of 5 kN? [7]
- State Newton's law of collision of two bodies. [2]
 - State and explain d'Alembert's principle. [5]
 - A ball of mass '1kg' moves with a velocity of 2mt/sec impinges directly on a ball of mass '2kg' at rest. The 1st ball after impinging comes to rest. Find the velocity of the second ball after impact and coefficient of restitution. [7]

ANSWERS TO - 2016 (W)

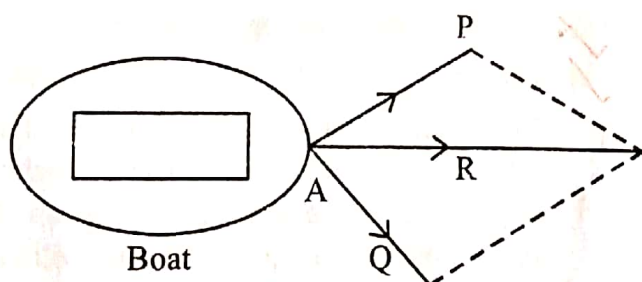
1.(a) Force is that which produces change in the state of rest or of uniform motion in a straight line of a body. The unit of force in SI system is neutron (N), kilonewton (KN) etc.

1.(b) Principle of transmissibility :

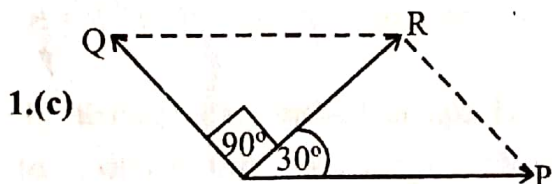
It states that the conditions of equilibrium or conditions of motion of a rigid body. The rigid body will remain unchanged if a force acting at a given point of the rigid body. The rigid body is replaced by a force of the same magnitude and same direction but acting at a different point, provided that the two forces have the same line of action. The effect of a force acting on a rigid body does not change if the force is moved along its line of action to another point on the body.

Principle of superposition

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces consider two forces P and Q acting at A on a boats shown in figure. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.



[Principle of Supurpostion]



1.(c)

Let P → bigger force

Q → smaller force

$$\tan \theta = \frac{P \sin \alpha}{Q + P \cos \alpha}$$

$$\tan 90^\circ = \frac{P \sin 120^\circ}{Q + P \cos 120^\circ}$$

$$\text{or, } \frac{1}{0} = \frac{40 \times \sqrt{3}/2}{Q + 40(-1/2)}$$

$$\text{or, } Q + 40(-1/2) = 0$$

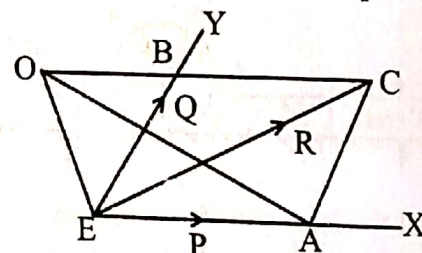
$$\text{or, } Q - 20 = 0$$

$$\text{or } Q = 20\text{N}$$

∴ The smaller force is 20N

2.(a) The turning effect of a force is known as the moment. It is the product of the force multiplied by the perpendicular distance from the line of action of the force to the pivot or point where the object will turn. The unit of moment of a force in S.I. system - Newton meter (Nm) - Kilonewton meter (KN-m)

2.(b) The algebraic sum of of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.



Let the forces P and Q acting along the line EX and EY intersect at E. Let O be any point in the plane of the forces.

Through O draw OBC parallel to EX, meeting EY at B. Choose the scale so that EB represents Q. On the same line cut off EA from EX to represent P.

Complete the parallelogram EACB.

Then EC represents R

Join OE and OA.

$$\Delta OEC = \Delta OEB + \Delta BEC$$

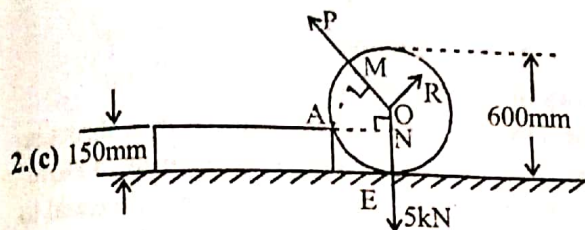
But $\Delta BEC = \Delta EAC$ (∵ each is one half of the area of the parallelogram EACB)

$$\Delta EAC = \Delta OEA$$

$$\text{or, } 2\Delta OEC = 2\Delta OEB + 2\Delta OEA$$

∴ moment of force represented by EC about O
moment of force represented by EB about O
moment of force represented by EA about O.

or, moment of R = moment of P + moment of Q
(about O)



Let R be the reaction at 'A' when the wheel is about to turn about 'A' it will lose contact with the ground and the reaction at E = 0. Then the forces on the wheel are = 5 kN = 5,000 N

P → pull applied

OA = 300 mm (radius of the wheel) = 30 cm

AM = OA sin θ

= 30 × sin θ cm

OE = 30 cm, EN = 15 cm (∵ EN = 150 mm = 15 cm)

ON = 30 cm - 15 cm = 15 cm

AN = $\sqrt{30^2 - 15^2} = 15\sqrt{3}$ cm

Taking moment about 'A',

P × AM = 5000 × AN

$$\text{or, } P = \frac{5000 \times 15\sqrt{3}}{30 \sin \theta}$$

$$\text{or, } P = \frac{2500\sqrt{3}}{\sin \theta} \quad \dots(1)$$

'P' will be least when sin θ is greatest.

when sin θ = 1 θ = 90°

Putting the value of 'θ' in equation (1)

Least value of 'P' = $2500\sqrt{3} = 4330.127$ N

Since ON = 15 cm, OA = 30 cm

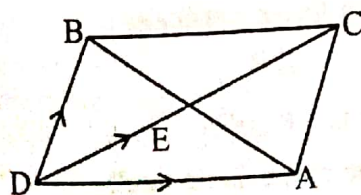
$$\cos \angle AON = \frac{1}{2}$$

Resolving along OA,

R = 5000 × cos ∠AON

$$= 5000 \times \frac{1}{2} = 2500 \text{ N}$$

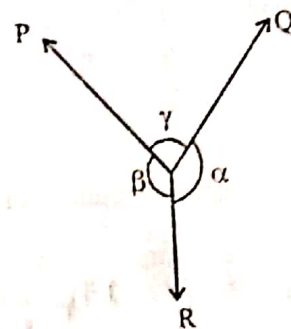
3.(a)



If two forces acting at a point O, be represented in direction and magnitude by straight lines OA and OB and parallelogram OACB be completed, then their resultant acts through O and is represented in magnitude and direction by the diagonal OC.

3.(b) It states "If three coplanar or forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two."

Mathematically



$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Proof - Let P, Q and R are three forces acting at O.

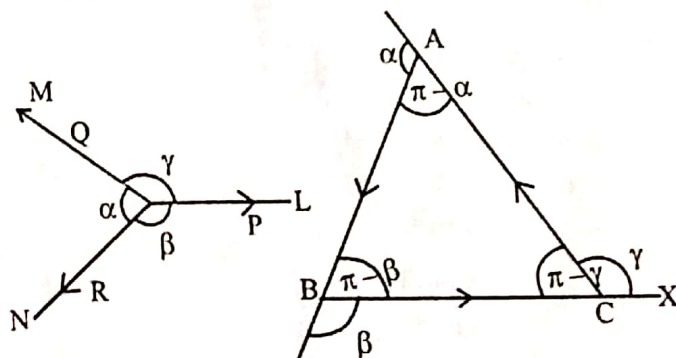
∠POQ = γ, ∠POR = β, ∠QOR = α

construct a triangle ABC, whose sides are parallel to OL, OM and ON.

By the converse of the triangle of forces.

$$\frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}$$

(1)



Produce BC to X. Now CX is parallel to OL and CA is parallel to OM.

∴ ∠ACX = ∠MOL = γ

∴ ∠ACB = (π - γ)

Similarly ∠CAB = (π - α)

∠ABC = (π - β)

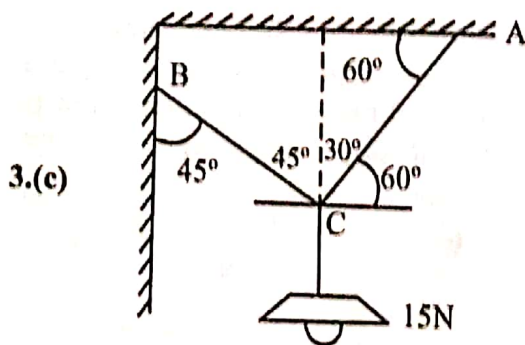
$$\frac{BC}{\sin(\pi - \alpha)} = \frac{CA}{\sin(\pi - \beta)} = \frac{AB}{\sin(\pi - \gamma)}$$

$$\text{or, } \frac{BC}{\sin \alpha} = \frac{CA}{\sin \beta} = \frac{AB}{\sin \gamma}$$

...(2)

From (1) and (2)

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} \quad (\text{proved})$$



Applying Lami's theorem

$$\frac{15}{\sin(30^\circ + 45^\circ)} = \frac{AC}{\sin(180^\circ - 45^\circ)} = \frac{BC}{\sin(180^\circ - 30^\circ)}$$

$$\text{or, } \frac{15}{\sin 75^\circ} = \frac{AC}{\sin 45^\circ} = \frac{BC}{\sin 30^\circ}$$

$$\text{So } \frac{AC}{\sin 45^\circ} = \frac{15}{\sin 75^\circ}$$

$$\text{or, } AC = \frac{15 \times \sin 45^\circ}{\sin 75^\circ}$$

$$= \frac{15 \times 0.707}{0.965}$$

$$\text{or, } AC = 10.989 \text{ N}$$

$$\frac{BC}{\sin 30^\circ} = \frac{15}{\sin 75^\circ}$$

$$\text{or, } BC = \frac{15 \times \sin 30^\circ}{\sin 75^\circ}$$

$$= \frac{15 \times 0.5}{0.965} = 7.772 \text{ N}$$

4.(a) Friction : When one of the blocks moves or tends to move tangentially with respect to the surface on which it rests, the interlocking property of projecting particles oppose the motion. This opposing force which acts in the opposite direction of the movement of the block is called as friction.

Coefficient of friction

It is the ratio of limiting friction to the normal reaction between the two bodies

$$\mu = \frac{F}{R} = \tan \phi$$

$\mu \rightarrow$ coefficient of friction

$F \rightarrow$ frictional force

$R \rightarrow$ normal reaction

$\phi \rightarrow$ angle of friction

4.(b) Laws of static friction

- (1) The force of friction acts always in a direction opposite to that in which the body tends to move.
- (2) The magnitude of the friction is exactly equal to the force, which tends to move the body.
- (3) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces

Mathematically : $\frac{F}{R} = \text{constant}$

$F \rightarrow$ Limiting friction

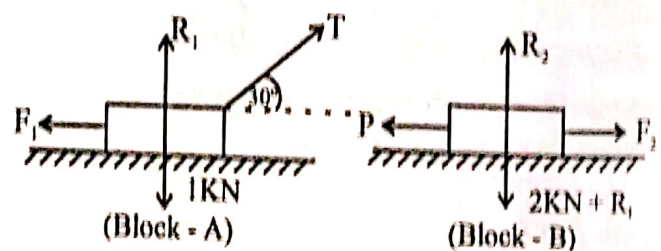
$R \rightarrow$ normal reaction

- (4) The force of friction is independent of the area of contact between the two surfaces.
- (5) The force of friction depends upon the roughness of the surface.

Laws of kinetic friction

- (1) The force of friction always acts in a direction opposite to that in which the body is moving.
- (2) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
- (3) For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

4.(c)



weight of the block A (w_A) = 1 kN

weight of the block B (w_B) = 2 kN

Coefficient friction (μ) = 0.3

Consider Block A

Resolving the forces vertically

$$R_1 + T \sin 30^\circ = 1 \text{ kN}$$

$$\therefore T \sin 30^\circ = (1 - R_1) \text{ kN} \quad \dots (1)$$

Resolving the forces horizontally,

$$T \cos 30^\circ = F_1 = \mu R_1 = 0.3 R_1$$

Dividing equation (1) by equation (2)

$$\frac{T \sin 30^\circ}{T \cos 30^\circ} = \frac{(1 - R_1)}{0.3 R_1}$$

$$\text{or, } \tan 30^\circ = \frac{(1 - R_1)}{0.3 R_1}$$

$$\text{or } 0.577 = \frac{(1 - R_1)}{0.3 R_1}$$

$$\text{or, } 0.173 R_1 = (1 - R_1)$$

$$\text{or, } 0.173 R_1 + R_1 = 1$$

$$\text{or, } R_1 (0.173 + 1) = 1$$

$$\text{or, } R_1 = \frac{1}{(0.173 + 1)}$$

$$\text{or, } R_1 = 0.852 \text{ kN}$$

$$F_1 = 0.3 R_1 = 0.3 \times 0.852$$

$$\text{or, } F_1 = 0.255 \text{ kN}$$

Now consider the block B

Resolving the forces vertically,

$$R_2 = 2 \text{ kN} + R_1$$

$$= 2 \text{ kN} + 0.852 \text{ kN}$$

$$= 2.852 \text{ kN}$$

$$F_2 = \mu R_2$$

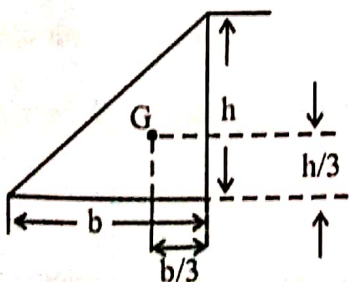
$$= 0.3 \times 2.852 = 0.855 \text{ kN}$$

Resolving the forces horizontally,

$$P = F_1 + F_2$$

$$= 0.255 \text{ kN} + 0.855 \text{ kN} = 1.11 \text{ kN}$$

5.(a)

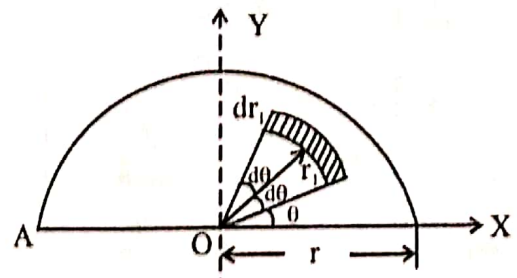


The centroid of a plane figure is the arithmetic mean position of all the points in shape. Any object in n-dimensional space, its centroid is the mean position of all the points in all of the co-ordinate directions.

(2)

- The centroid is the point through which whole area of the Lamina is supposed to act.
- The centroid is used for bodies which does not have mass.

5.(b)



Consider a semicircle of radius r consider an elemental area at a distance r_i from the centre O of semicircle Radial with dr bounded by radii at θ and $(\theta + d\theta)$.

$$\text{Area of element} = r_i d\theta dr$$

Its moment about diametral axis x is given by

$$r_i d\theta \times dr \times r \sin \theta = r_i^2 \sin \theta dr d\theta$$

Total moment of area about diametral axis

$$\int_0^\pi \int_0^r r_i^2 \sin \theta dr_i d\theta$$

$$= \int_0^\pi \left[\frac{r_i^3}{3} \right]_0^r \sin \theta d\theta = \frac{r^3}{3} [-\cos \theta]_0^\pi$$

$$= \frac{r^3}{3} [-\cos \pi - (-\cos 0)]$$

$$= \frac{r^3}{3} [1 + 1] = \frac{2r^3}{3}$$

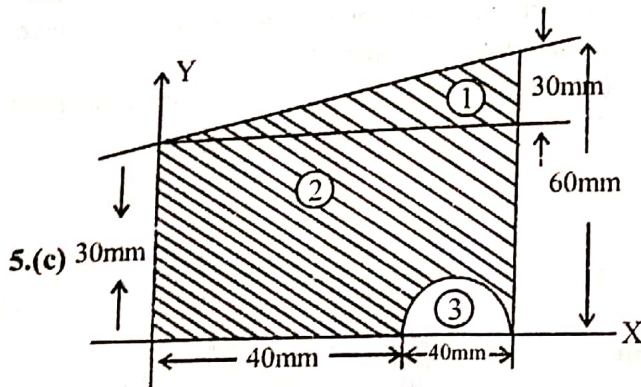
$$\text{Area of semicircle (A)} = \frac{1}{2} \pi r^2$$

$$\bar{y} = \frac{\text{moment of area}}{\text{total area}}$$

$$= \frac{2r^3/3}{\frac{1}{2} \pi r^2} = \frac{2r^3}{3} \times \frac{2}{\pi r^2}$$

$$\bar{y} = \frac{4r}{3\pi}$$

$\bar{y} \rightarrow$ Distance of centroid from point o along vertical line.

**Triangle**

$$a_1 = \frac{1}{2} \times 80 \times 30 = 1200 \text{ mm}^2$$

Distance of C.G. from OY axis

$$x_1 = \frac{2}{3} \times 80 = 53.34 \text{ mm}$$

Distance of C.G. from OX axis

$$y_1 = 30 + \frac{30}{3} = 40 \text{ mm}$$

Rectangle

$$a_2 = 30 \times 80 = 2400 \text{ mm}^2$$

$$x_2 = \frac{80}{2} = 40 \text{ mm}$$

$$y_2 = \frac{30}{2} = 15 \text{ mm}$$

Semicircle :

$$a_3 = \frac{1}{2} \times \pi \times (20)^2 = 628.4 \text{ mm}^2$$

$$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$$

$$y_3 = \frac{4r}{3\pi} = \frac{4 \times 20}{4 \times \pi} = 8.487 \text{ mm}$$

$$\bar{x} = \frac{\sum ax}{\sum a}$$

$$= \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1200 \times 53.34) + (2400 \times 40) - (628.4 \times 60)}{1200 + 2400 - 628.4}$$

$$= \frac{64008 + 96000 - 37704}{3600 - 628.4} = \frac{122304}{2971.6} = 41.157 \text{ mm}$$

$$\bar{Y} = \frac{\sum ay}{\sum a}$$

$$= \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

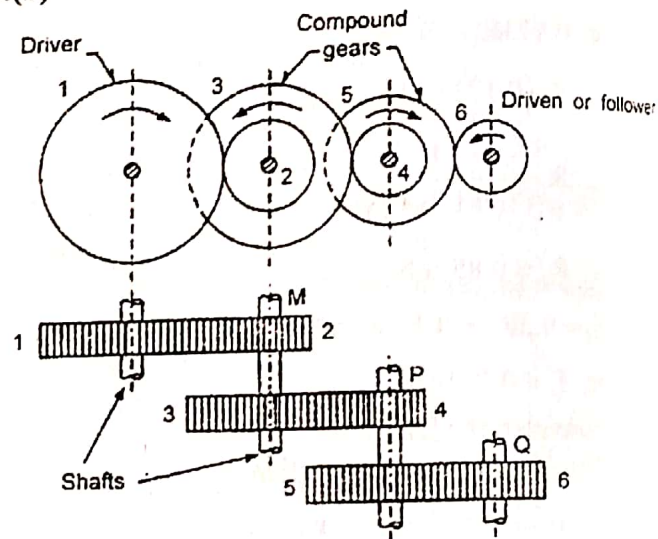
$$= \frac{(1200 \times 40) + (2400 \times 15) - (628.4 \times 8.487)}{1200 + 2400 - 628.4}$$

$$= \frac{48000 + 36000 - 5333.23}{3600 - 628.4}$$

$$= \frac{84000 - 5333.23}{2971.6} = \frac{78666.77}{2971.6} = 26.472 \text{ mm}$$

6.(a) It is a device which enables us to lift a heavy load (w) by applying a comparatively smaller effort.

6.(b)



The gear 1 is driving gear mounted on shaft L, gears 2 and 3 are compound gears which are mounted on shaft M. The gears 4 and 5 are also compound gears which are mounted on shaft P.

The gear 6 is driven gear mounted on shaft Q.

Let $N_1 \rightarrow$ velocity of the driving gear 1 in rpm

$T_1 \rightarrow$ number of teeth on driving gear 1

N_2, N_3, N_4, N_5, N_6 - velocity of the respective gears in rpm

T_2, T_3, T_4, T_5, T_6 - Number of teeth on respective gears.

since gear 1 meshes with gear 2 therefore velocity ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \dots(1)$$

similarly for gears (3) and (4) velocity ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \dots(2)$$

For gears 5 and 6 the velocity ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \dots(3)$$

The velocity ratio of the compound gear train is obtained by multiplying equation (1), (2) and (3)

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$N_2 = N_3$ ($\because$ gears 2 and 3 are mounted on a common shaft m)

$N_5 = N_4$ ($\because$ gears 4 and 5 are mounted on a common shaft p)

$$\frac{N_1}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Velocity ratio = $\frac{\text{velocity of the first driver}}{\text{velocity of the last driver}}$

$\frac{\text{product of the number of teeth on drives}}{\text{Product of the number of teeth on the drivers}}$

6.(c) effort (P) = 40N

Load (w) = 1 kN = 1000N

efficiency = 0.5

$$\text{efficiency} = \frac{w}{p} \times V.R$$

$$\text{or, } 0.5 = \frac{1000 / 40}{V.R} \quad \text{or, } V.R = \frac{1000 / 40}{0.5} = 50$$

For an effort of 74N the load lifted is 2kN

P = 74N w = 2kN = 2000N

$$\text{efficiency} = \frac{W / P}{V.R} \times 100$$

$$= \frac{2000 / 74}{50} \times 100 (\because V.R = 50)$$

$$\therefore \text{efficiency} = 0.54.05 \times 100 = 54.05\%$$

what will be the effort required to raise the load of 5kN

W = 5kN = 500N P = ?

By applying law of machine

P = mw + c

$$40 = m \times 1000 + c \quad \dots(1)$$

$$74 = m \times 2000 + c \quad \dots(2)$$

subtracting

$$-1000 m = -34$$

$$m = -\frac{34}{-1000}$$

$$m = 0.034$$

Putting the value of m in equation (1)

$$40 = 0.034 \times 1000 + c$$

$$\text{or, } c = 40 - 34 \text{ or } c = 6$$

The new equation is

$$P = mw + c$$

$$= 0.034 \times 5000 + 6 = 170 + 6$$

$$\text{or } P = 176 \text{ N}$$

7.(a) Newtons 3rd law of motion is naturally applied to collisions between two bodies. In a collision between two bodies, both bodies experience forces that are equal in magnitude and opposite in direction. Such force often cause one body to speed up (gain momentum) and the other body to slow down (lose momentum)

7.(b) It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be findout by the methods of graphic static.

$$p = mf \quad (1)$$

where p $\rightarrow$ force applied on body

m $\rightarrow$ mass of the body

f $\rightarrow$ acceleration of the body

The equation (1) can be written as p - mf = 0 (2)

It may be noted that equation (1) is the equation of dynamics whereas the equation (2) is the equation of statics. The equation (2) is also the equation of dynamic equilibrium under the action of the real force P. This principle is known as D'Alembert's principle.

7.(c) $m_1 \rightarrow$ mass of the 1st ball

$u_1 \rightarrow$ initial velocity of the 1st ball

$u_2 \rightarrow$ final velocity of the 1st ball

$m_1 = 1 \text{ kg}$

$$u_1 = 2 \text{ m/s } u_2 = 0$$

$m_2 \rightarrow$ mass of the second ball

$v_1 \rightarrow$ initial velocity of the 1st ball

$v_2 \rightarrow$ final velocity of the 2nd ball

$$m_2 = 2 \text{ kg, } v_1 = 0, v_2 = ?$$

Momentum before impact = momentum after impact

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{or, } 1 \times 2 + 2 \times 0 = 1 \times 0 + 2 v_2$$

$$\text{or, } 2 = 2 v_2 \quad \text{or, } v_2 = \frac{2}{2} = 1 \text{ m/s}$$

coefficient of restitution

$$(e) = \frac{v_2 - v_1}{u_1 - u_2} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

$$\text{or, } e = 0.5$$

**1ST / SEM / COMMON / 2015 (W) (NEW)
ENGINEERING MECHANICS**

Code : BET - 103

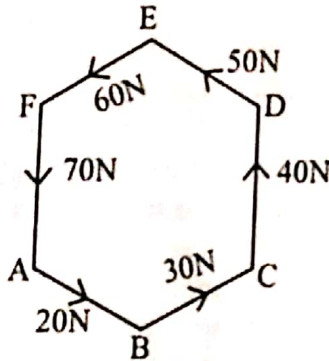
Time : 3 Hours

Full Marks : 70

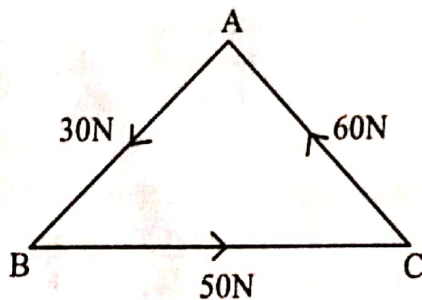
Answer any five questions.

Figures in the right-hand margin indicate marks.

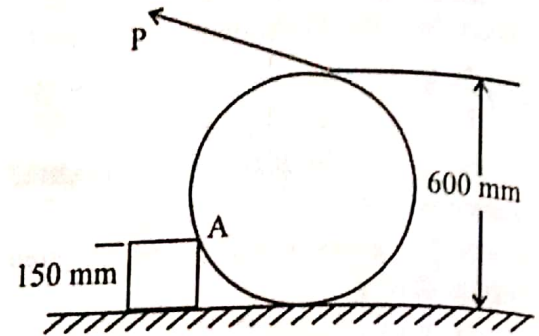
1. (a) What is resultant force? [2]
 (b) Find the magnitude and direction of force, which when combines with a 8N vertical force, will give 6N horizontal force. [5]
 (c) ABCDEF is a regular hexagon. Forces of 20N, 30N, 40N, 50N, 60N and 70N are acting along the sides AB, BC, CD, DE, EF and FA respectively as shown in Fig - 1. Find the magnitude and direction of resultant force. [7]



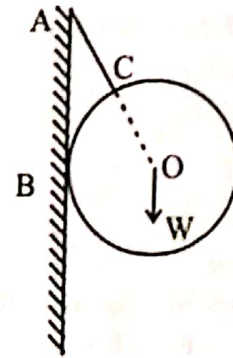
2. (a) State varignon's theorem [2]
 (b) Force of 30N, 50N and 60N are acting along the sides of an equilateral triangle ABC of side 50cm as shown in figure - 2. Find the magnitude, direction and position of resultant force. [5]



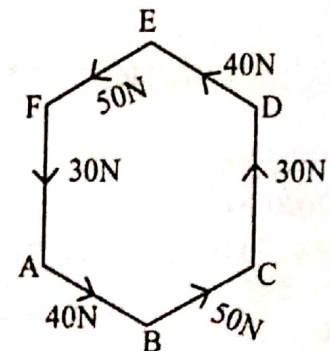
- (c) A uniform wheel of 600mm diameter, weighing 10kN rests against a rigid rectangular block of 150mm height as shown in fig - 3. Find the least pull required at the top of the wheel just to turn the wheel over the corner 'A' of the block. Also find the reaction at 'A'. Assume all the surfaces to be smooth. [7]



3. (a) State the condition of equilibrium for nonconcurrent forces. [2]
 (b) A smooth sphere of weight 1000 N is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point B on the wall as shown in fig - 4. If the Length of the string AC is equal to the radius of the sphere, find the tension in the string and reaction of the wall. [5]



- (c) A regular hexagon has sides equal to 20cm. Forces of 30N each act along AF and DC, 40N each along AB and DE, 50N each along BC and EF as shown in fig - 5. Find the moment of the couple required to keep the system in equilibrium. [7]

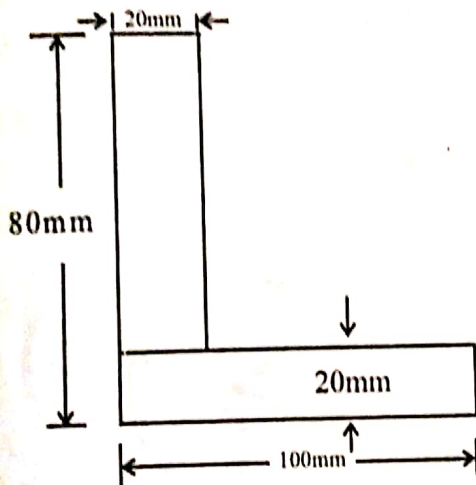


4. (a) Define Limiting angle of friction. [2]
 (b) A body of weight 50N is pulled along a rough horizontal plane by a force of 18N acting at [7]

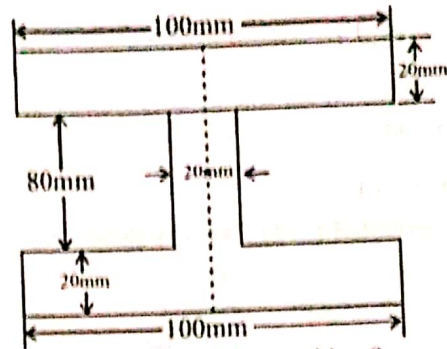
an angle of 14° with the horizontal. Find the coefficient of friction. [5]

- (c) A Ladder 5m Long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750 N stands on a rung 1.5m from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor. [7]

5. (a) State parallel axis theorem. [2]
 (b) Find the position of centroid of a L section as shown in fig - 6 [5]



- (c) Find the moment of inertia of a I-section as shown in fig - 7 about the central axis parallel to the base and perpendicular to the base. [7]



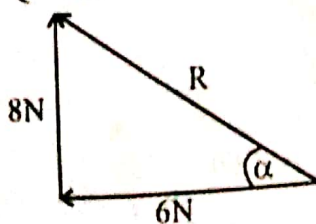
6. (a) What is reversible machine? [2]
 (b) In a lifting machine, an effort of 50N is required to lift a load W. The distance moved by the load and effort are 20mm and 500mm respectively. Determine the magnitude of the load W if the efficiency of the machine is 80%. [5]
 (c) In a worm and worm wheel the number of teeth in the worm wheel is 25. The effort handle is 300mm long and the load drum is 150mm diameter. Find the efficiency of the machine, if an effort of 30N can lift a load of 345N and the worm is double threaded. [7]
 7. (a) Define impulse. [2]
 (b) A constant force acting on a body of mass 20kg changes its speed from 2.5m/sec to 10m/sec in 15 seconds. What is the magnitude of the force? [5]
 (c) Two balls of masses 2kg and 3kg are moving with velocities 2m/sec and 3m/sec towards each other. If the coefficient of restitution is 0.5, find the velocity of the two balls after impact. [7]

ANSWERS TO - 2015 (W)

- 1.(a) If a number of forces are acting simultaneously on a particle then a single force which will produce the same effect as that of all the given forces is known as a resultant force. The resultant force of all the components of forces can be found out analytically or graphically.

- (b) $P = 8\text{N}$

$Q = 6\text{N}$



$$\begin{aligned} \text{(Magnitude)} R &= \sqrt{P^2 + Q^2 + PQ \cos \theta} \\ &= \sqrt{8^2 + 6^2 + 8 \times 6 \times \cos 90^\circ} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} \\ &= \sqrt{100} \\ &= 10\text{N} \end{aligned}$$

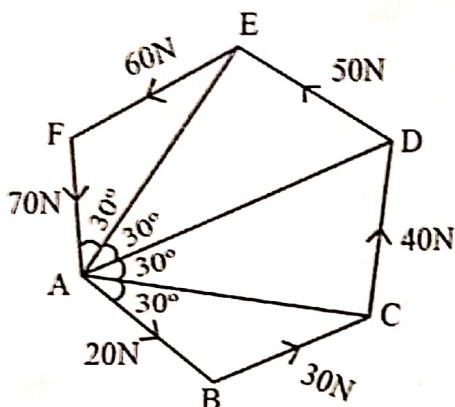
$$\begin{aligned} \text{(Direction)} \tan \alpha &= \frac{P}{Q} \\ &= \frac{8}{6} \end{aligned}$$

$$\text{or, } \tan \alpha = \frac{4}{3}$$

$$\text{or, } \alpha = \tan^{-1} \left(\frac{4}{3} \right)$$

$$\text{or, } \alpha = 53.13^\circ \text{ (Ans)}$$

(c) The system of given forces is shown in this figure.



Resolving all the forces horizontally $\Sigma H = 20 \cos 0^\circ + 30 \cos 30^\circ + 40 \cos 60^\circ + 50 \cos 90^\circ + 60 \cos 120^\circ + 70 \cos 0^\circ$

$$= 105.98 \text{ N}$$

$$\Sigma V = 20 \sin 0^\circ + 30 \sin 30^\circ + 40 \sin 60^\circ + 50 \sin 90^\circ + 60 \sin 120^\circ + 70 \sin 0^\circ = 151.60 \text{ N}$$

Magnitude of the resultant force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(105.98)^2 + (151.60)^2}$$

$$= 184.97 \text{ N}$$

Direction of the resultant force

$$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{151.60}{105.98}$$

$$\text{or, } \theta = \tan^{-1} \left(\frac{151.60}{105.98} \right)$$

$$\text{or, } \theta = 55.03^\circ \text{ (Ans)}$$

2. (a) The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point

(b) Resolving the forces horizontally

$$\Sigma H = 50 \cos 0^\circ + 60 \cos 120^\circ - 30 \cos 0^\circ$$

$$= 50 + (-30) - 15$$

$$= 50 - 45 = 5 \text{ N}$$

Resolving the forces vertically

$$\Sigma V = 50 \sin 0^\circ + 60 \sin 120^\circ - 30 \sin 60^\circ$$

$$= 30\sqrt{3} - 15\sqrt{3}$$

$$= 5\sqrt{3} \text{ N}$$

(Direction)

$$\tan \theta = \frac{\Sigma V}{\Sigma H}$$

$$= \frac{5\sqrt{3}}{5}$$

$$\theta = \tan^{-1} (\sqrt{3})$$

$$= 60^\circ$$

Magnitude

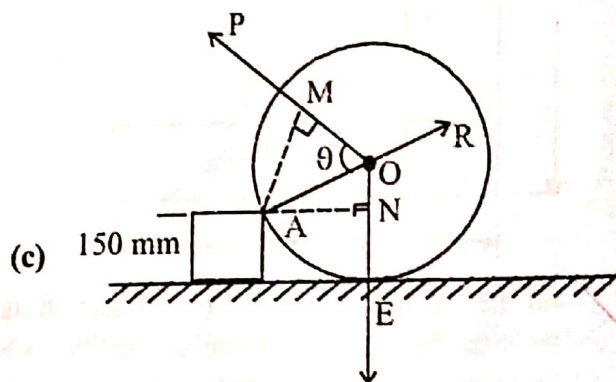
$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(5)^2 + (5\sqrt{3})^2}$$

$$= \sqrt{25 + 75}$$

$$= \sqrt{100}$$

$$= 10 \text{ N (Ans)}$$



Let R be the reaction at 'A' when the wheel is about to turn about 'A' it will lose contact with the ground and the reaction at E = 0. Then the forces on the wheel are = 10 kN = 10,000 kgt

P → Pull applied

$$OA = 300 \text{ mm (radius of the wheel)}$$

$$= 30 \text{ cm}$$

$$AM = OA \sin \theta$$

$$= 30 \sin \theta \text{ cm.}$$

$$OE = 30 \text{ cm, } EN = 15 \text{ cm. } (\because EN = 150 \text{ mm} = 15 \text{ cm})$$

$$ON = 30 - 15 = 15 \text{ cm}$$

$$AN = \sqrt{30^2 - 15^2}$$

$$= 15\sqrt{3} \text{ cm}$$

Taking moments about 'A'

$$P \times AM = 10,000 \times AN$$

$$\text{or, } P = \frac{10,000 \times 15\sqrt{3}}{30 \sin \theta}$$

$$\text{or } P = \frac{5000\sqrt{3}}{\sin \theta} \quad \dots (1)$$

'P' will be least when $\sin \theta$ is greatest.

$$\text{When } \sin \theta = 1 \\ \theta = 90^\circ$$

Putting the value of ' θ ' in eqⁿ (1)

$$\text{Least value of } P = 5000\sqrt{3} \\ = 8660 \text{ kgf}$$

Since $ON = 15\text{cm}$, $OA = 30\text{cm}$

$$\cos \angle AON = \frac{1}{2}$$

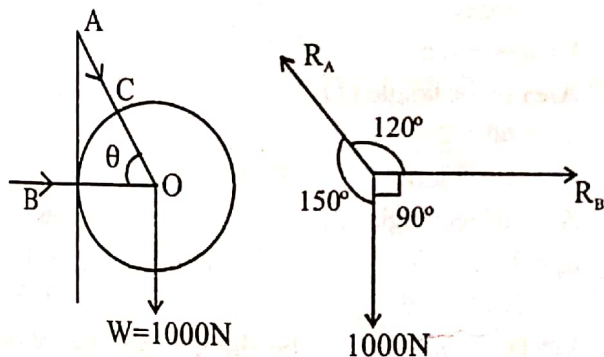
Resolving along OA

$$R = 10,000 \cos \angle AON$$

$$= 10,000 \times \frac{1}{2} \\ = 5000 \text{ kg wt. (Ans)}$$

- 3.(a) (1) The sum of all the forces in the x - direction or horizontal is zero.
(2) The sum of all the forces in the y- direction or vertical is zero.
(3) The sum of moments at any point is zero.

(b)



In ΔAOB (r = radius of sphere)

$$BO = CO = r$$

$$\text{SO } AO = 2CO = 2r$$

$$\therefore \cos \theta = \frac{BO}{AO} = \frac{r}{2r} = \frac{1}{2}$$

$$\therefore \theta = 60^\circ$$

Applying Lami's theorem

$$\frac{R_A}{\sin 90^\circ} = \frac{R_B}{\sin 150^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\frac{R_A}{\sin 90^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\text{or, } R_A = \frac{1000 \times \sin 90^\circ}{\sin 120^\circ} \\ = \frac{1000 \times 1}{0.866} = 1154.73 \text{ N}$$

$$\text{or, } R_A = 1154.73 \text{ N.}$$

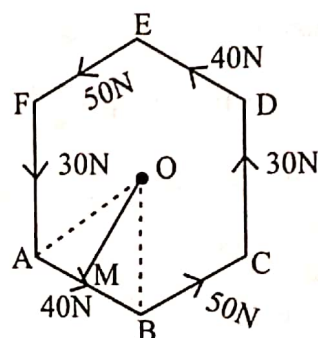
$$\frac{R_B}{\sin 150^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\text{or, } R_B = \frac{1000 \times \sin 150^\circ}{\sin 120^\circ}$$

$$= \frac{1000 \times 0.5}{0.866}$$

$$\text{or, } R_B = 577.36 \text{ N (Ans)}$$

(c)



Let 'O' be the centre of the hexagon. Draw $OM \perp r$ to AB. Join O with A and O with B.

Triangle OAB is an equilateral triangle.

So $OA = 20\text{cm}$

$$AM = 10\text{cm}$$

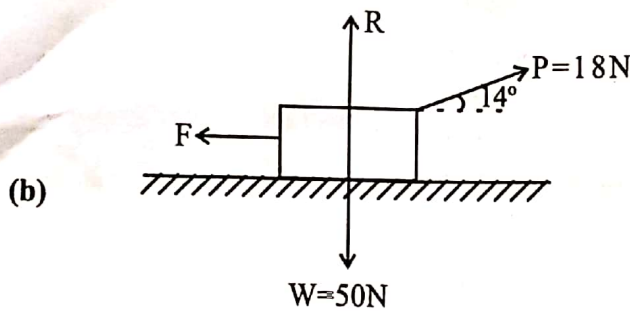
$$OM = \sqrt{20^2 - 10^2} \\ = \sqrt{400 - 100} \\ = \sqrt{300} \\ = 17.32 \text{ cm}$$

Taking anticlockwise moments as +ve and clockwise moments as -ve.

Algebraic sum of the moments about 'O'

$$= (40 \times 17.32) + (50 \times 17.32) - (30 \times 17.32) + (40 \times 17.32) + (50 \times 17.32) - (30 \times 17.32) \\ = 692.8 + 866 - 519.6 + 692.8 + 866 - 519.6 \\ = 692.8 + 692.8 + 866 + 866 - 519.6 - 519.6 \\ = 3117.6 - 1039.2 \\ = 2078.4 \text{ N - cm. (Ans)}$$

- 4.(a) It is defined as the angle which the resultant reaction (R) makes with the normal reaction (RN).



Resolving the forces horizontally

$$F = 18 \cos 14^\circ$$

$$= 17.46 \text{ N}$$

Resolving the forces Vertically

$$R = W - 18 \sin 30^\circ$$

$$= W - 4.354$$

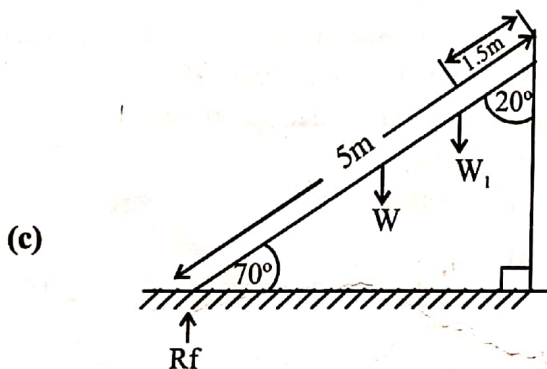
Force of friction (F)

$$17.46 = \mu (W - 4.354) \quad \left(\because \mu = \frac{F}{R} \right)$$

$$\text{or, } 17.46 = \mu (50 - 4.354)$$

$$\text{or, } \mu = \frac{17.46}{45.646} = 0.38$$

$$\mu = 0.38 \text{ (Ans)}$$



Weight of the Ladder (w) = 900N.

Length of the Ladder (L) = 1.5m

Weight of the man (w_1) = 750N

Length between the bottom and the man (L_1) = 1.5m.

$\mu_f \rightarrow$ Coefficient of Ladder and floor

$$R_f = 900 + 750$$

$$= 1650 \text{ N}$$

Force of friction at A $= F_f = \mu_f R_f$

$$\Rightarrow F_f = 1650 \mu_f$$

Now taking moments about 'B' $R_f \times 5 \sin 20^\circ$

$$= (F_f \times 5 \cos 20^\circ) + 900 \times 2.5 \sin 20^\circ + (750 \times 3.5 \sin 20^\circ)$$

$$\text{or, } R_f \times 5 \sin 20^\circ = F_f \times 5 \cos 20^\circ + 4700 \sin 20^\circ$$

$$\text{or, } R_f \times 5 \sin 20^\circ = \mu_f \times 1650 \times 5 \cos 20^\circ$$

$$+ 4700 \sin 20^\circ$$

Dividing both sides by $5 \sin 20^\circ$

$$\frac{R_f \times 5 \sin 20^\circ}{5 \sin 20^\circ} = \frac{\mu_f \times 1650 \times 5 \cos 20^\circ}{5 \sin 20^\circ} + \frac{4700 \sin 20^\circ}{5 \sin 20^\circ}$$

$$\text{or, } R_f = 1650 \mu_f \cot 20^\circ + 940$$

$$\text{or, } 1650 = 1650 \mu_f \cot 20^\circ + 940$$

$$\text{or, } 1650 \mu_f \cot 20^\circ = 1650 - 940$$

$$\text{or, } \mu_f = \frac{710}{1650 \cot 20^\circ}$$

$$= \frac{710}{1650 \times 2.754}$$

$$= 0.15$$

$$\text{or, } \mu = 0.15 \text{ (Ans)}$$

5.(a) It states that the m.I of a rigid body about any axis is equal to its m.I about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the $\perp r$ distance between the two parallel axis.

Let $m \rightarrow$ total mass of the body

$h \rightarrow$ distance between the two parallel axis.

$I \rightarrow$ moment of inertia.

$I_{cm} \rightarrow$ moment of inertia about the centre of mass.

$$I = I_{cm} + mh^2.$$

(b) Area of rectangle (1)

$$a_1 = 60 \times 20$$

$$= 1200 \text{ mm}^2$$

Area of rectangle (2)

$$a_2 = 100 \times 20$$

$$= 2000 \text{ mm}^2$$

Let (x_1, y_1) & (x_2, y_2) be the Co-ordinates of the centroids of rectangles (1) & (2) respectively.

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

$$x_2 = \frac{100}{2} = 50 \text{ mm}$$

$$y_1 = 20 + \frac{60}{2}$$

$$= 20 + 30$$

$$= 50 \text{ mm}$$

$$y_2 = \frac{20}{2} = 10 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{(1200 \times 10) + (2000 \times 50)}{1200 + 2000}$$

$$= \frac{12000 + 100000}{3200}$$

$$= \frac{112000}{3200}$$

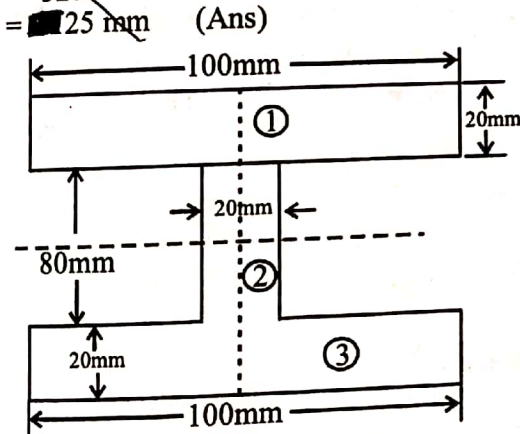
$$= 35 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(1200 \times 50) + (2000 \times 10)}{1200 + 2000}$$

$$= \frac{60000 + 20000}{3200}$$

$$= \frac{26000}{3200} = \frac{80,000}{3200} = 25 \text{ mm} \quad (\text{Ans})$$



$$a_1 = 100 \times 20$$

$$= 2000 \text{ mm}^2$$

$$a_2 = 80 \times 20$$

$$= 1600 \text{ mm}^2$$

$$a_3 = 100 \times 20$$

$$= 2000 \text{ mm}^2$$

$$y_1 = 100 + \frac{20}{2}$$

$$= 100 + 10$$

$$= 110 \text{ mm}$$

$$y_2 = 20 + \frac{80}{2}$$

$$= 20 + 40$$

$$= 60 \text{ mm}$$

$$y_3 = \frac{20}{2} = 10 \text{ mm}$$

$$a_1 y_1 = 2000 \times 110 = 220000 \text{ mm}^3$$

$$a_2 y_2 = 1600 \times 60 = 96000 \text{ mm}^3$$

$$a_3 y_3 = 2000 \times 10 = 20000 \text{ mm}^3$$

$$\Sigma a y = a_1 y_1 + a_2 y_2 + a_3 y_3 = 220000 \text{ mm}^3 + 96000 \text{ mm}^3 + 20000 \text{ mm}^3 = 249600 \text{ mm}^3$$

$$\Sigma a = 2000 \text{ mm}^2 + 1600 \text{ mm}^2 + 2000 \text{ mm}^2 = 5600 \text{ mm}^2$$

$$\bar{Y} = \frac{\Sigma a y}{\Sigma a}$$

$$= \frac{249600}{5600}$$

$$= 44.57 \text{ mm}$$

$$I \times x_1 = \frac{100 \times 20^3}{12} + (100 \times 20) \times (110 - 44.57)^2$$

$$= 66666.66 + 2000 \times (65.43)^2$$

$$= 66666.66 + 8562169.8$$

$$= 8628836.46 \text{ mm}^4$$

$$I \times x_2 = \frac{20 \times 80^3}{12} + (80 \times 20) \times (60 - 44.57)^2$$

$$= 853333.33 + 1600 \times 238.08$$

$$= 853333.33 + 380935.84$$

$$= 1234269.17 \text{ mm}^4$$

$$I \times x_3 = \frac{100 \times 20^3}{12} + (100 \times 20) \times (44.57 - 10)^2$$

$$= 66666.66 + 2000 \times 1195.08$$

$$= 66666.66 + 2390160$$

$$= 2456826.66 \text{ mm}^4$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{xx} = 8628836.46 \text{ mm}^4 + 1234269.17 \text{ mm}^4 + 2456826.66 \text{ mm}^4$$

$$= 12319932.29 \text{ mm}^4 \quad (\text{Ans})$$

6. (a) A reversible machine is a machine that can perform reversible cycles. These cycles are cycles that happens through infinitesimal changes or a infinity number of quasti static steps making the process adiabatic and reversible.

(b) $P(\text{effort}) = 50 \text{ N}$

$$m.A = \frac{w}{p} = \frac{w}{50}$$

$$V.R = \frac{\text{distance moved by the effort}}{\text{distance moved by the Load}}$$

$$V.R = \frac{500}{20} = 25$$

$$\text{efficiency} = \frac{M.A}{V.R} = \frac{W/50}{25}$$

$$\text{or, } 80\% = \frac{W}{50} \times \frac{1}{25}$$

$$\text{or, } 0.8 = \frac{W}{1250}$$

$$\text{or, } W = 1250 \times 0.8 = 1000 \text{ N (Ans)}$$

(c) T (number of teeth) = 25

The effort handle Length $L = 300\text{mm}$

Load drum dia (d) = 150mm

$$V.R = \frac{LT}{2 \times n \times r} \quad (\because \text{Double threaded})$$

$$= \frac{300 \times 25}{2 \times 2 \times 75} \quad \left(r = \frac{d}{2} = \frac{150}{2} \text{mm} = 75\text{mm} \right)$$

$$= \frac{7500}{150} = 25$$

$$M.A = \frac{W}{p} = \frac{\text{Load lifted}}{\text{effort applied}}$$

$$= \frac{345\text{N}}{30\text{N}} = 11.5$$

$$\text{efficiency} = \frac{M.A}{V.R} \times 100$$

$$= \frac{11.5}{25} \times 100$$

$$= 46\% \quad (\text{Ans})$$

7.(a) Impulse is the change of momentum of an object when the object is acted upon by a force for an interval of time.

(b) (Mass) $m = 20\text{kg}$, u (initial velocity) = 2.5m/sec
Final velocity (v) = $10/\text{sec}$ time ($\pm$) = 15sec

$$\text{acceleration (f)} = \frac{v - u}{t}$$

$$= \frac{10 - 2.5}{15}$$

$$\text{or, } f = 0.5 \text{ m/sec}^2$$

$$\text{force (p)} = mf$$

$$= 20 \times 0.5$$

$$= 10 \text{ kg m/sec}^2$$

$$= 10\text{N} \quad (\text{Ans})$$

(c) DATA : $m_1 = 2\text{kg}$. (mass of the ball)
 $m_2 = 3\text{kg}$. (mass of the other ball)
 $u_1 = 2\text{m/sec}$ (velocity of ball)
 $u_2 = -3\text{m/sec}$ (velocity of other ball)

Coeff. of restitution (e) = 0.5

Solve :

Total momentum before impact = total momentum after impact

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{or, } 2 \times 2 + 3 \times (-3) = 2v_1 + 3(-v_2)$$

$$\text{or, } 4 - 9 = 2v_1 - 3v_2$$

$$\text{or, } 2v_1 - 3v_2 = -5 \quad \dots (1)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\text{or, } 0.5 = \frac{-v_2 - v_1}{2 - (-3)}$$

$$\text{or, } 0.5 = \frac{-v_2 - v_1}{5}$$

$$\text{or, } -v_2 - v_1 = 2.5 \quad \dots (2)$$

$$\text{Eq}^n (1) \rightarrow 2v_1 - 3v_2 = -5$$

$$\text{Eq}^2 (2) \times 2 \rightarrow -2v_1 - 2v_2 = 5$$

$$(+)\quad -5v_2 = 0 \quad \text{or, } v_2 = 0\text{m/sec}$$

Putting the value of v_2 in eqⁿ (1) $2v_1 - 3 \times 0 = -5$

$$\text{or, } 2v_1 = -5$$

$$\text{or, } v_1 = \frac{-5}{2} = -2.5\text{m/sec}$$

After impact of two balls

$$v_1 = -2.5 \text{ m/sec}$$

$$v_2 = 0 \text{ m/sec} \quad (\text{Ans})$$

1ST / SEM / COMMON / 2014 (W) (NEW)
ENGINEERING MECHANICS

Code : BET - 103

Time : 3 Hours

Full Marks : 70

Answer any five questions

Figures in the right-hand margin indicate marks

Use of calculator is allowed

1. (a) Define triangle law of forces. [2]
(b) Two forces one of which is double the other, has resultant of 260 N. If the direction of the larger force is reversed and the other remain unaltered, the resultant reduces to 180 N. Determine the magnitude of the forces and the angle between them. [5]
(c) The forces 30N, 45N, 60N, 20N and 70N are acting at one of the angular point of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force. [7]
2. (a) State law of moment. [2]
(b) ABCD is a rectangle, in which AB = CD = 25cm and BC = DA = 45cm. Forces of 150N each act along AB and CD and forces of 200N each act along BC and DA. Find the resultant moment of two couple. [5]
(c) A beam AB of length 5m supported horizontally at A and B carries point loads of 5kN, 8kN and 3kN at a distance of 1m, 2m and 4m from the support A respectively, Find the reactions at A and B. [7]
(a) State Lami's theorem. [2]
(b) A cord supported at A and B carries a load of 10kN at 'D' and a load of W at 'C' as shown in figure. Find the value of W so that CD remains horizontal. [5]

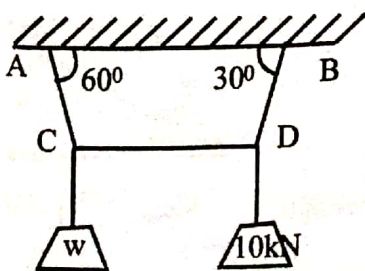


Fig. - 1

- (c) Two cylinders A and B of diameter 100mm and 50mm respectively are placed in trough as shown in Figure.2. Weight of A is 200N and that of B is 50N. Neglecting friction find the reactions at the contact surfaces. [7]

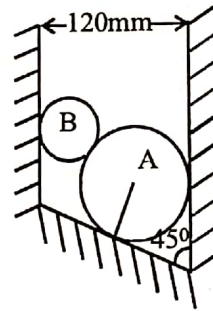


fig.- 2

4. (a) Define limiting angle of friction. [2]
(b) The block 'A' as shown in Figure.3 weighs 2000N. The cord attached to 'A' passes over a frictionless pulley and supports a weight equal to 800N. The value of the coefficient of friction between A and the horizontal plane is 0.35. Find the value of P, if motion is impending towards left. [5]

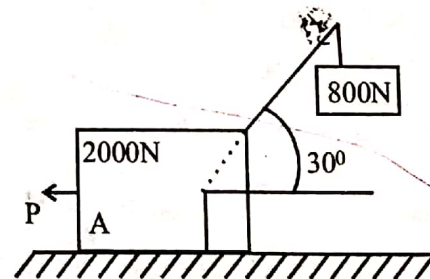
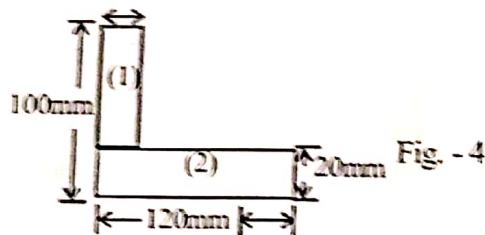


fig. - 3

- (c) A 4m ladder weighing 250N is placed against a smooth vertical wall its lower end 1.5m away from the wall. If the coefficient of friction between the ladder and the floor is 0.3, show that the ladder will remain in equilibrium in this position. [7]
5. (a) What do you mean by centroid? [2]
(b) State and prove parallel axis theorem. [5]
(c) Find the M.I. of a L-section as shown in figure 4 about the centroidal horizontal axis and vertical axis. [7]



6. (a) What is lifting machine? [2]
 (b) What do you mean by reversibility of a lifting machine? Derive the condition of reversible machine. [5]
 (c) A single purchase crabwinch has 300mm long handle and 120mm diameter drum. Number of teeth on the pinion are 25 and that on wheel 130. If an effort of 20N lifts a load of 300N,

find the MA, VR and efficiency of the crabwinch. [7]

7. (a) State law of conservation of linear momentum. [2]
 (b) A bullet weighs 0.5 N and moving with a velocity of 400m/sec hits centrally a 30 N block of wood moving away at 15m/sec and gets embedded in it. Find the velocity of the bullet and block after impact. [5]
 (c) A motorist travelling at a speed of 70km per hour suddenly applies brakes and comes to a stop after skidding 50m. Determine : [7]
 (i) time required to stop the car.
 (ii) the coefficient of friction between the tyres and the road.

ANSWERS TO - 2014 (W)

1.(a) Triangle law forces : It states that if two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle taken in same order then their resultant may be represented in magnitude and direction by third side of the triangles taken in opposite order.

(b) Let first force $f_1 = p$

As question second force $f_2 = 2p$ resultant of two forces $(R_1) = 260N$.

Case - 1 : We know

$$R = \sqrt{p^2 + q^2 + 2pq \cos \theta}$$

$$R_1 = \sqrt{p^2 + (2p)^2 + 2p \cdot 2p \cos \theta}$$

$$\Rightarrow 260 = \sqrt{p^2 + 4p^2 + 4p^2 \cos \theta}$$

$$\Rightarrow (260)^2 = 5p^2 + 4p^2 \cos \theta$$

$$\Rightarrow 6700 = 5p^2 + 4p^2 \cos \theta \dots (i)$$

Case - 2 :

The force is in reverse direction

New resultant $R_2 = 180N$

we know

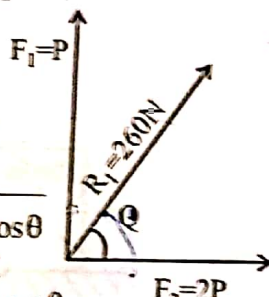
$$R_2 = \sqrt{p^2 + q^2 + 2pq \cos \theta}$$

$$180 = \sqrt{p^2 + 2p^2 + 2p \cdot 2p (\cos 180^\circ)}$$

$$\Rightarrow (180)^2 = p^2 + 4p^2 + 4p^2 - 4p^2 \cos \theta$$

$$\Rightarrow 32400 = 5p^2 - 4p^2 \cos \theta \dots (ii)$$

Adding equation (i) and (ii) then we get



$$\text{equation (i)} \times 1 = 5p^2 + 4p^2 \cos \theta = 67600$$

$$\text{equation (ii)} \times 1 = 5p^2 + 4p^2 \cos \theta = 32400$$

$$10p^2 = 100000$$

$$p = \sqrt{\frac{100000}{10}} = 100$$

$$F_1 = P = 100N, F_2 = 2P = 2 \times 100N = 200N$$

Put the value of f_1 and f_2 in equation (i)

$$67600 = 5p^2 + 4p^2 \cos \theta$$

$$\Rightarrow 67600 = 5(100)^2 + 4(100)^2 \cos \theta$$

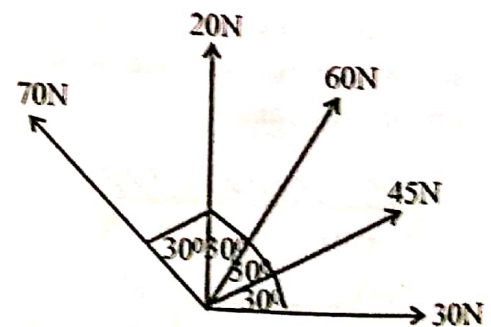
$$\Rightarrow 67600 - 50000 = 40000 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{17600}{40000} = 44$$

$$\theta = \cos^{-1}(.44)$$

$$\theta = 63.896^\circ$$

- (c) Five forces acting at point 'o' 30N, 45N, 60N, 20N, 70N respectively resolving all the forces horizontally, we get



$$\begin{aligned} \sum H &= 30 \cos(0^\circ) + 45 \cos(30^\circ) + 60 \cos(60^\circ) \\ &\quad + 20 \cos(90^\circ) - 70 \cos(120^\circ) = 138.97 N \end{aligned}$$

Resolving all the forces vertically

$$\sum V = 30 \sin(0^\circ) + 45 \sin(30^\circ) + 60 \sin(60^\circ) +$$

$$20 \sin(90^\circ) - 70 \sin(120^\circ) = 155.083 \text{ N}$$

$$\text{Resultant (R)} = \sqrt{(EH)^2 + (EV)^2}$$

$$= 204.935 \text{ N}$$

$$\boxed{R = 204.935 \text{ N}}$$

Direction of resultant force

$$\tan \theta = \frac{FV}{EV}$$

$$\theta = \tan^{-1} = \frac{EV}{EY}$$

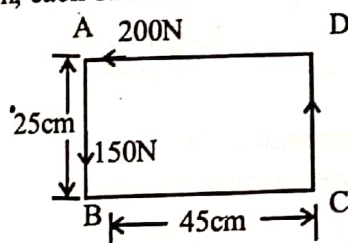
$$= \tan^{-1} \left(\frac{155.083}{133.97} \right) = 49.17$$

$$\boxed{Q = 49.17}$$

2.(a) Law of moment : If a number of co-planer forces are acting simultaneously on a particle then the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about their same point it is also known as varigin is principle of moment.

(b) ABCD is a rectangle

AB = CD = 25cm, each of 150N and BC = AD = 45cm, each of 200N



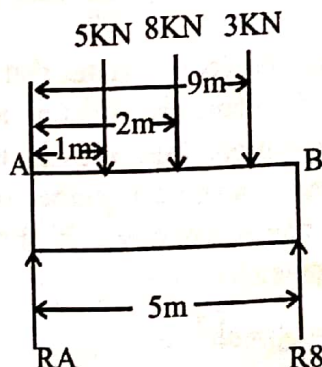
Now taking moment at point 'B'

$$M(B) = -(150 \times 45) - (200 \times 25)$$

$$= -6750 - 5000 = 11750 \text{ nmm.}$$

(c) A beam AB of length 5M.

It carries point loads 5kN, 8kN and 3 kN at a difference of 1m 2m and 4m respectively



Solution we know

$$R_A + R_B = 5 + 8 + 3 = 16 \text{ kN}$$

Taking moment at A

M_A = clockwise moment = anticlockwise

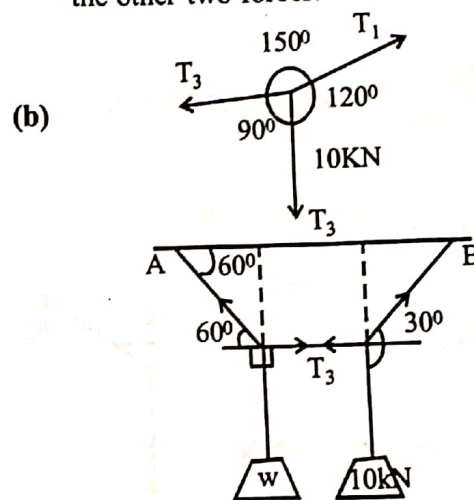
$$\Rightarrow 5 + 16 + 12 = 5R_B$$

$$\Rightarrow R_B = \frac{33}{5} = 6.6 \text{ kN}$$

$$R_A = 16 - R_B$$

$$= 16 - 6.6 = 9.4 \text{ kN}$$

3.(a) Lami's theorem : It states that if three co-planar forces acting at a point be in equilibrium then each force is proportional to the sine of angle between the other two forces.



$$10 \text{ kN} = 10000 \text{ N}$$

According to Lamis theorem

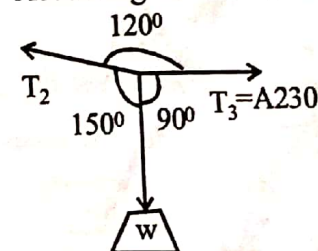
$$\frac{T_1}{\sin 90} = \frac{T_3}{\sin 120} = \frac{10000}{\sin 150}$$

$$\frac{T_3}{\sin 120} = \frac{T_3}{\sin 150}$$

$$\Rightarrow T_3 = \left(\frac{10000}{\sin 150} \right) \sin 120 = 17320.50 \text{ N}$$

$$T_1 = \left(\frac{10000}{\sin 150} \right) \sin 90 = 20000 \text{ N}$$

According to Lami's theorem



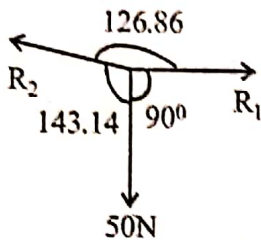
$$\frac{T_2}{\sin 90} = \frac{W}{\sin 120} = \frac{17320}{\sin 150}$$

$$\frac{w}{\sin 120} = \frac{17320}{\sin 150}$$

$$w = \left(\frac{17320}{\sin(150)} \right) \sin(120) = 29999.11 \text{ N}$$

(c) According to Lami's theorem

$$\frac{R_1}{\sin(143.14)} = \frac{R_2}{\sin 90} = \frac{50}{\sin(126.86)}$$



$$R_1 = \left\{ \frac{50}{\sin(126.86)} \right\} \sin(143.14) = 37.48 \text{ N}$$

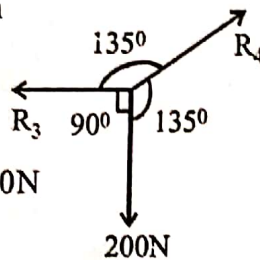
$$R_2 = \left\{ \frac{50}{\sin(126.86)} \right\} \sin 90 = 62.99 \text{ N}$$

According to Lami's theorem

$$\frac{R_3}{\sin 135} = \frac{R_4}{\sin 90} = \frac{200}{\sin 135}$$

$$R_3 \left\{ \frac{200}{\sin(135)} \right\} \sin(135) = 200 \text{ N}$$

$$R_4 = \left\{ \frac{200}{\sin 135} \right\} \sin 90 = 282.84 \text{ N}$$



Reaction at contact surface $R_4 = 282.84 \text{ N}$

4.(a) It states that, it is the ratio of frictional force (f) to the normal reaction (R), it is denoted by μ

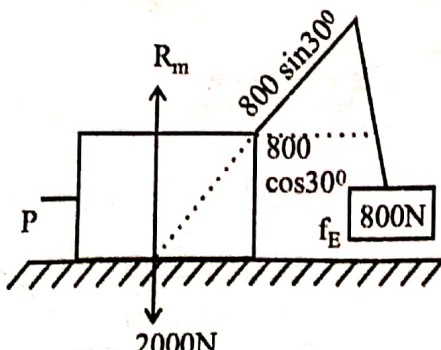
$\Rightarrow$ which is equal to $\tan Q$

$$\mu = \frac{F}{R} = \tan Q$$

where α is known as angle of friction.

(b) Data given, weight of body = 2000N

$$\mu = 0.35$$



Solution : The $\frac{F_f}{R_n} = 0.35$

Resolving the forces horizontally

$$P = f_f + 800 \cos 30 \dots (i)$$

Resolving vertically

$$2000 = R_n + 800 \sin 30$$

$$R_n = 2000 - 800 \sin 30 = 1600$$

from the above equation

$$f_f = \mu R_n$$

$$= (0.35) \times 1600 = 560$$

Putting the value of f_f in equation (i)

$$P = f_f + 800 \cos 30$$

$$= 560 + 800 \cos 30 = 1252.82$$

(c) Data given,

length of ladder = 4m

weight of ladder = 250N

$\mu = 0.3$, distance wall to lower end of ladder = 1.5m

Solution : Here

Left f_f = force of friction of the bottom of ladder

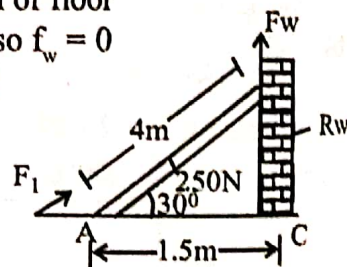
R_s = Normal reaction of floor

as the wall is smooth so $f_w = 0$

Now in triangle ABC

$$BC = \sqrt{AB^2 - AC^2}$$

$$= \sqrt{4^2 - (1.5)^2} = 3.7$$



Now taking moment of 'B'

M_B = clock use moment anticlock wise moment

$$\Rightarrow R_f \times 1.5 = (f_f \times 3.7) + (250 \times 75)$$

$$F_f = \frac{(250 \times 1.5) - (250 \times 75)}{3.7}$$

$$\Rightarrow F_f = 50.67 \text{ N}$$

5.(a) Centroid : The point at which the total area of the lamina is assumed to be concentrated is known as centroid.

(b) Parallel axis theorem : It states that, if the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G , then moment of inertia of the area about any other axis AB, parallel to the first and at a distance 'h' from the centre of gravity is given by

$$I_{AB} = I_G + ah^2$$

where $I_{AB} = ml$ of the area about an axis AB.

$I_G = ml$ of the area about its CG

$a =$ Area of the section and $b =$ distance between CG of section and axis AB.

Proof : Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB, shown in figure below.

Let $8a =$ Area of the strip

$Y =$ Distance between CG of the section and the axis AB

we know that all of the whole section about x-axis passing through the CG $= 8ay^2$

And moment of inertia of the whole section

$$IG = 8ay^2$$

$\therefore$ Moment of inertia of the section about axis

$$AB \quad I_{AB} = E8a(h+y)^2$$

$$= \epsilon 8a(h^2 + y^2 + 2hy)$$

$$= (\epsilon 8ah^2) + (\epsilon 8ay^2) + (\epsilon 8a2hy)$$

$$= ah^2 + la + a$$

$$= IG + ah^2$$

($\therefore$ algebraic sum of moment of CG is 0)

$$I_{AB} = IG + ah^2$$

(c) Divide the L-section in two rectangular section

$as =$ Area of 1st section

$$= \{20 \times (100 - 20)\}$$

$$= 1600mm^2$$

$$= \{853333.22 + (1600 \times 30^2)\} + \{80000 +$$

$$(2400 \times 20^2)\} = 3333333.33mm^4$$

Moment of inertia about yy - axis

$$IG_1 = \frac{db^3}{12} = \frac{80 \times 20^3}{12} = 533333.33$$

$$IG_2 = \frac{d_2b_2^3}{12} = \frac{20 \times 120^3}{12} = 2880000$$

$$h_1 = \bar{x} - x_1 = 40 - 10 = 30mm$$

$$h_2 = x_2 - \bar{x} = 60 - 40 = 20mm$$

$$I_{xx} = (IG_1 + a_1h_1^2) + (IG_2 + a_2h_2^2)$$

$$= (533333.33 + 1600.32^2) + 2880000 + 2400.20^2 = 5333333.33mm^4$$

6.(a) Lifting machine : A device which enable us to lift a heavy load (w) by applying a comparably smaller effort (p)

(b) Reversibility of a lifting machine is means that sometimes a machine is also capable of doing some work in the reverse let $w =$ load lifted by the machine

$p =$ effort applied to lift the load.

$$x_1 = \frac{20}{2} = 10mm$$

$$y_1 = 20 + \left(\frac{100 - 20}{2}\right) = 20 + 40 = 60mm$$

$x_2 =$ Area of 2nd section

$$= 20 \times 12 = 2400mm^2$$

$$x_2 = \frac{120}{2} = 60mm$$

$$y_2 = \frac{20}{2} = 10mm$$

we know that

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(1600 \times 10) + (2400 \times 60)}{(1600 + 2400)} = 40mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{(1600 \times 60) + (2400 \times 10)}{1600 + 2400} = 30mm$$

moment of inertia about xx - axis

$$IG_2 = \frac{b_2d_2^3}{12} = \frac{120 \times 20^3}{12} = 80000$$

$$IG_1 = \frac{b_1d_1^3}{12} = \frac{20 \times 80^3}{12} = 853333.33$$

$$h_1 = (y_1 - \bar{y}) = (60 - 30) = 30$$

$$h_2 = (\bar{y} - y_2) = (30 - 10) = 20$$

$$I_{xx} = (I_{G_1} + a_1h_1^2) + (I_{G_2} + a_2h_2^2)$$

Distance moved by the effort

$x =$ distance moved by the load.

input of machine $= pw \times x$

A little consideration will show that in a reversible, the output of the machine should be more than the machine friction when the effort (p) is zero.

machine friction ($py - wx$)

$$wx > (py - wx)$$

$$\Rightarrow 2wx > py$$

$$\Rightarrow \frac{wx}{py} > \frac{1}{2}$$

$$\Rightarrow \frac{w}{p} / \frac{y}{x} > \frac{1}{2}$$

$$\frac{MA}{VR} > \frac{1}{2}$$

($\therefore$ where MA = mechanical advantage
VR = Velocity ratio of machine)

$$\frac{MA}{VR} > 50\%$$

The condition for a machine to be reversible when the value of efficiency of these machine is greater than 50%.

(c) Data given

T_1 = Number of teeth on wheel = 130

T_2 = Number of teeth of pinion = 25

l = length of handle = 300 mm

r = radius of drum = $\frac{120}{2} = 60\text{mm}$

p = Effort = 20N

w = weight = 300 N

mechanical advantage $MA = \frac{w}{p}$

$$\Rightarrow \frac{300}{20} = 15$$

we know, velocity ratio V.R.

$$= \frac{1}{r} \cdot \frac{T_1}{T_2} = \frac{300}{60} \times \frac{130}{25} = 26$$

we know, efficiency = $\frac{MA}{VR}$

$$= \frac{15}{26} = .57 = .57 \times 100 = 57\%$$

7.(a) Conservation of linear momentum

The total momentum of two bodies remains constant after their collision or any other mutation.

i.e., total momentum before collision total momentum after collision.

(b) Weight of the bullet = 0.5N

$$\text{mass of the bullet} = \frac{0.5}{9.8} \text{ kg} = m_1$$

$$\text{velocity of bullet} = v_1 = 400 \text{ m/sec}$$

$$\text{weight of the block} = 30 \text{ N}$$

$$\text{mass of the block} = \frac{30}{9.8} \text{ kg} = m_2$$

$$\text{velocity of the block} = v_2 = 15 \text{ m/sec}$$

According to law of conservation of momentum.
before collision = Momentum after collision. But
the velocity of the bullet and block after impact
be v .

$$\text{then } m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$\frac{0.5}{9.8} \times 400 + \frac{30}{9.8} \times 15 = \left(\frac{30}{9.8} + \frac{0.5}{9.8} \right) v$$

$$\Rightarrow \frac{200 + 450}{9.8} = \left(\frac{30.5}{9.8} \right) v$$

$$\Rightarrow 650 = 30.5v$$

$$\Rightarrow v = \frac{650}{30.5} = 21.31 \text{ m/s (approx)}$$

(c) Here $u = 70 \text{ km/hr} = 19.44 \text{ m/s}$

$$v = 0, s = 50$$

$$\Rightarrow 8^2 - (19.44)^2 = 2 \times a \times 50$$

$$\Rightarrow a = -3.779 \text{ m/s}^2$$

(-ve sign for retardation)

(i) Again we know, $v = u + at$

$$\Rightarrow v - u = at$$

$$\Rightarrow 0 - 19.44 = -(3.779)t$$

$$\Rightarrow t = \frac{19.44}{3.779} = 5.14 \text{ sec (approx)}$$

(ii) Frictional force = $ma = m(3.779)$
Reaction due to car = $mg = mx(9.8)$
let co-efficient of friction between the tyres
and the road be

$$\mu = \frac{\text{frictional force}}{\text{Reaction force}}$$

$$m = \frac{3 \times 3.779}{m \times 9.8} = 0.3856 \text{ (approx)}$$

1ST / SEM / COMMON / 2013 (W) (NEW)
ENGINEERING MECHANICS

Code : BET - 103

Time : 3 Hours

Full Marks : 70

Answer any FIVE question.

Figure in the right-hand margin indicate marks.

1. (a) Define couple. [2]
(b) A force of 200 N is acting on a bracket OAB hinged at 'O' as shown in figure .I. Find the moment of the force about 'O'. [5]

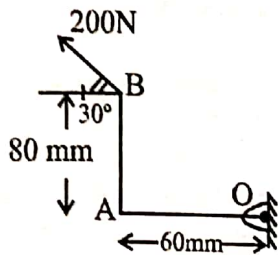


Fig. - 1

- (c) A smooth right circular cylinder of radius 16 cm rests on horizontal plane and is prevented from rolling by an inclined string PC of length 32 cm as shown in figure II. A prismatic bar PQM of length 48 cm and weight 530 N is hinged at P and leans against the cylinder. Find the tension in the string. [7]

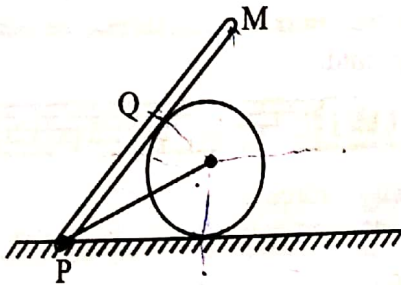


Fig. 2

2. (a) What is free body diagram? [2]
(b) A beam of length 1m is hinged at one end A and rests on a roller support at other end B and shown in figure III. The beam is acted upon by an inclined force of 80 N at a distance of 60 cm from A. Find the reaction at A and B. [5]

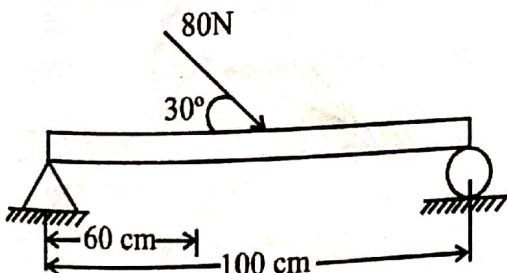


Fig. 3

- (c) A prismatic bar of length 2.1 m and weight 365 N rests within a hemispherical bowl of radius 0.7 m as shown in figure IV. Determine the angle ' α ' for the position of equilibrium. [7]

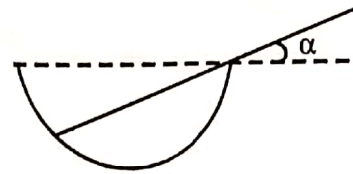


Fig. 4

3. (a) State Lami's theorem. [2]
(b) A body weighing 300 N rests on an inclined plane making an angle of 30° with the horizontal. A horizontal force of 200 N is acting on the body as shown in figure. Find the total force in the direction parallel to the plane and perpendicular to the plane. [5]

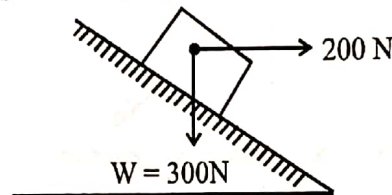


Fig. 5

- (c) Forces of 20 N, 30 N, 40 N, 50 N, 60 N and 70 N are acting along the sides of a regular hexagon as shown in figure. VI. Find the magnitude and direction of the resultant force. [7]

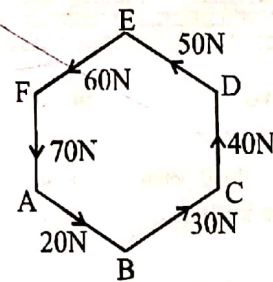


Fig. 6

4. (a) What is limiting angle of friction? [2]
(b) A rectangular block weighing 150 N is laying on an inclined plane, whose inclination with horizontal is 45° . The block is tied up by a horizontal string, as shown in figure-VIII which has a tension of 50 N. Find.
(i) the frictional force on the block
(ii) normal reaction of the inclined plane.
(iii) coefficient of friction between the surfaces of contact. [6]

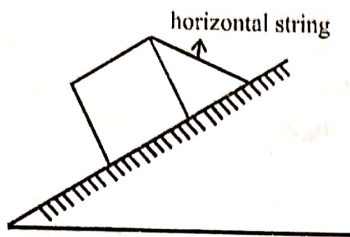


Fig. 7

- (c) A uniform ladder of length 10 m weighing 20 N is placed against a smooth vertical wall with its lower end 4m away from the wall as shown in figure VIII. The coefficient of friction between the ladder and the floor is 0.3. Check whether the ladder will remain in equilibrium in this position or not. [6]

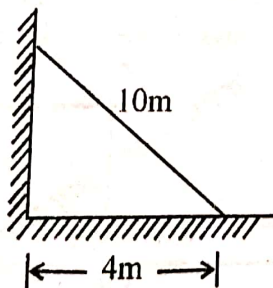


Fig. 8

5. (a) State parallel axis theorem. [2]
 (b) Find the Moment of Inertia of a square section of side 80 mm about its diagonal. [5]

- (c) A triangular section is cut out from a quarter circular section as shown in figure IX. Find the position of the centroid of the cut out section. [7]

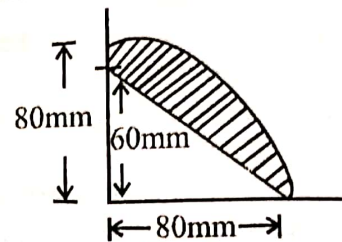


Fig. 9

6. (a) What do you mean by reversibility of a lifting machine? [2]
 (b) Explain worm and worm wheel. Derive its velocity ratio. [6]
 (c) What is a lifting machine? Define velocity ratio, mechanical advantages and efficiency of lifting machine. Explain law of lifting machine. [6]
7. (a) Define coefficient of restitution. [2]
 (b) An elevator is moving upward with an acceleration of 1 m/sec^2 . Find the pressure exerted by the man of mass [5]
 (c) A pile of hammer of mass 1500 kg drops from a height of 60 cm on a pile of mass 75 kg. The pile penetrates 50 mm per blow. Assuming that the pile is resisted by a constant force, find the resistance to penetration of ground. [7]